Nonlinear Landau Damping in a Collisionless Plasma

In his recent Letter [1], Isichenko argues that Landau damping in a collisionless plasma will not be arrested by the nonlinear effects of particle trapping. This contradicts analytical [2,3] and numerical [4–6] results which indicate that in many cases plasma waves damp to nonlinear superpositions of traveling waves [7] as $t \to \infty$. In fact, the contradiction with numerical results already has been discussed in a recent Letter [6]. Hence, we limit our Comment here to an error in Isichenko's analysis. Unfortunately, the error is serious in that it occurs early in the development on which his argument is based.

Isichenko begins his analysis by assuming that the electric field $E(x,t)$ goes to zero as $t \to \infty$, and then proceeds to show that this assumption is self-consistent by deducing a damping rate $E \sim t^{-1}$. Crucial to his analysis is his hypothesis that, as $E \to 0$, each single particle trajectory, which satisfies Newton's equation

$$\ddot{x} = E(x,t), \tag{1}$$

tends to a motion with constant velocity

$$x(a,b,t) = U(a,b)t, \tag{2}$$

where (a,b) is the initial phase-space point for the orbit and $U(a,b)$ is a *constant* velocity. Unfortunately, the damping rate resulting from Isichenko's analysis, namely, $E \sim t^{-1}$ as $t \to \infty$, does not justify Eq. (2). In fact, integrating Eq. (1) once gives the velocity

$$v(a,b,t) = b + \int_0^t d\tau\, E\boldsymbol{(}x(a,b,\tau),\tau\boldsymbol{)}, \tag{3}$$

which tends to a constant value $U(a,b)$ as $t \to \infty$ only if $E\boldsymbol{(}x(a,b,t),t\boldsymbol{)}$ is an *integrable* function of t. However, if $E \sim t^{-1}$ as $t \to \infty$, the function $E\boldsymbol{(}x(a,b,t),t\boldsymbol{)}$ is, *in general*, not integrable, so that $v(a,b,t)$ does not tend to a constant value $U(a,b)$ as $t \to \infty$, and Eq. (2) does not hold. Hence, the transition in Ref. [1] from Eq. (2) there to Eq. (3) there is unjustified, and the analysis that follows there, based on the expansion $x(a,b,t) = Ut + \dots$ and the study of the properties of $U(a,b)$, is inconsistent.

Later in his Letter, Isichenko argues that the integral in Eq. (3) here is bounded as $t \to \infty$ because the decay rate $E \sim t^{-1}$ is supplemented by "another power of t coming from the nearly uniform motion in the coordinate $x \simeq Ut$, over which E is zero average." Unfortunately, this reasoning is circular: The hypothesis that x has nearly uniform time-asymptotic motion is valid only if $v(a,b,t)$ tends to a constant value as $t \to \infty$, which is precisely what Isichenko needs to prove.

Since Isichenko's analysis is not self-consistent, the basis for his general conclusion is incorrect. In fact, if carried to its logical end, Isichenko's conclusion implies that electric fields damp to zero following *any* initial perturbation of a stable Vlasov equilibrium. However, this is directly contradicted by the existence of Bernstein-Greene-Kruskal (BGK) solutions to the nonlinear Vlasov-Poisson (VP) equations [8], which in fact are undamped. Independently of the open questions concerning the stability of these BGK modes, if one of them were given as an initial condition it would provide an exact undamped solution to the VP equations. Other, possibly more important, counterexamples to the general conclusion in Ref. [1] are provided by BGK waves of arbitrarily small amplitude [9] and their nonlinear superpositions [7] which recently have been established as undamped solutions to the nonlinear VP equations. Thus, there clearly are many perturbations—even of arbitrarily small amplitude—of stable Vlasov equilibria for which the electric field does not damp.

In his Letter, Isichenko also reports an analysis that indicates inviscid fluids damp algebraically in time. That analysis does not appear to be affected by the above-mentioned hypothesis, Eq. (2), since he applies it only in the part of his study devoted to collisionless plasmas.

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