Noise Sustained Propagation of a Signal in Coupled Bistable Electronic Elements

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We establish the constructive role of noise on the signal transmission properties of spatially extended, metastable media. Our results are for an experimental system comprised of coupled, nonlinear resonators. The system's capability of reliable signal propagation is enhanced over a finite noise range. We briefly address the reliability of signal detection as a function of system size and other parameters. [S0031-9007(98)06298-X]

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The observation of stochastic resonance (SR) [1] in coupled and spatially extended systems has recently attracted much attention [2–4]. Spatiotemporal patterns and signal processing properties of nonlinear systems can be enhanced by applying uncorrelated, distributed noise. Besides its fundamental appeal, spatiotemporal stochastic resonance promises important insights into the mechanism of information transferal in biological systems. While most research has focused on the synchronization and noise reduction characteristics of spatiotemporal SR, its role in communication by way of coupled, noisy elements remains virtually unexplored. Up to date, the role of noise in the encoding and transmission of information by neurons remains an open and challenging question [5]. It has been shown by Jung *et al.* [4] that noise can aid and optimize the spreading of spiral and single wave fronts in a numerical model of an excitable media. On the experimental side, Kádár *et al.* [6] recently reported noise supported traveling waves in a chemical subexcitable medium. Here we demonstrate the constructive role of noise for the propagation of a signal in a metastable medium.

The experiments are performed using coupled nonlinear electronic resonators operated in a biased bistable state with characteristics similar to that of excitable media. The resonators are initially set in the metastable state and can be kicked into the more stable phase with a sufficiently large perturbation. Unlike excitable elements, which return to their quiescent state after "firing" via a refractory period, the resonators in this setup are reset by a global bias after departure from the metastable phase. This arrangement supports single, nonrepetitive wave fronts and provides a "very clean" experimental system for studying the underlying mechanisms of noise sustained wave propagation in subexcitable biological media [7].

The diffusive coupling and bias are chosen such that, in the absence of random fluctuations, the system is unable to sustain any sort of information transmission. We find that the quality and speed of signal propagation can be greatly enhanced by increasing the ambient noise level. Beyond some optimal level, the "channel" is quickly corrupted by noise, resulting in spurious signals and thus poor reliability. We conclude with a brief discussion on the effects of system size, bias amplitude, and coupling strength.

The experimental setup, shown in block diagram form in Fig. 1, is composed of a one-dimensional chain of 32 symmetrically and locally coupled diode resonators; see Refs. [3] for details. The global, sinusoidal drive serves as a bifurcation parameter for the nonlinear elements, which individually follow the period-doubling cascade to chaos [8]. By operating the drive in the stable period-2 regime and hence giving rise to two stable phases, the diode resonators act as coupled bistable elements. We break this phase symmetry by adding to the drive a second sinusoidal signal at half its frequency. This bias renders one phase more stable. We refer to the less stable phase as the metastable state. The signal is an induced phase change at one end of the chain and the detector outputs a phase change at the other end. Each resonator is coupled to a different noise source, the fluctuations of which are obtained by amplifying shot noise generated by a current through a *pn* junction diode.

Two major forces affect each individual element: local random fluctuations and the influence of its two next neighbors. It is clear that in the absence of noise a local phase jump will lead to a "domino effect" only if the coupling is strong enough. The energetically lower phase then propagates into the metastable phase in the form of a moving kink. The speed of this moving interface depends

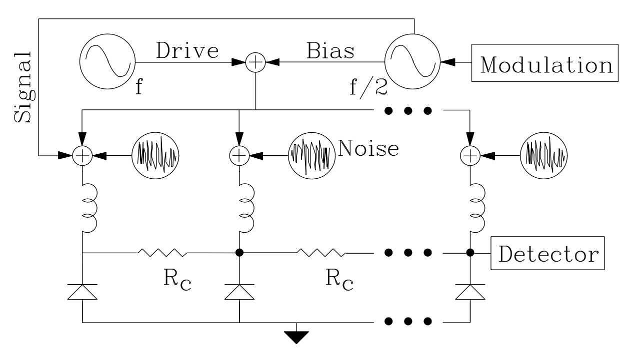

FIG. 1. 32 diode resonators are placed in the period-2 state by the drive at frequency f. An $f/2$ bias serves to make one phase more stable and to force that phase on to the first site. Independent noise sources are applied to each site. Coupling is provided by resistors R_c. The detector outputs the phase of the last site.

on both the coupling strength and the amplitude of the applied bias. If the latter two parameters are chosen low enough, kinks in discrete systems will fail to propagate [9]. Propagation failure of signals due to discreteness of the supporting medium has previously been observed in coupled chemical reactors [10] as well as in theoretical and experimental studies of cardiac tissue [11].

The kink speed for an intermediate coupling strength [3] as a function of bias amplitude is shown in Fig. 2. No kink propagation could be observed for bias values much less than about 1.0 unit. In the experiment, two effects cause the finite cutoff: the discreteness of the system and nonidentical elements. The resonators are loosely matched based on their bifurcation sequence. The variation of important parameters such as energy barrier between the two period-2 phases, and the levels of the noise generators, are within 10%. Since the parameters differ from site to site, kinks get trapped or slowed down at some resonators. The measured kink velocities should therefore be taken as average values over the entire chain, including local variations. Stronger coupling tends to smoothen out differences between elements.

We employed a bias value of 0.55 units, which is roughly $1/2$ of the corresponding threshold. In this parameter regime, the system is not capable of kink propagation in the absence of random fluctuations. To measure the effect of noise on signal propagation, we force the system to reside in its metastable state, i.e., in the less stable period-2 phase. At the same time we induce the resonator at site 1 to switch phase and subsequently measure the phase of all 32 elements as a function of time. Figure 3 shows a staggered space-time plot of the entire system with the addition of 1.3 units noise. The induced phase kink exhibits local variations in speed due to the random forcing and the differences in the energy barriers of individual elements.

Figure 4 displays, for four values of applied noise, the integrated probability of a phase change at site 31 as a function of time after an induced change at site 1. The detector emits a step function which, when averaged over approximately 100 events, produces the smoothly rising curves. In order to measure which part of the output is due to noise nucleated spurious signals, we repeat the same experiment without inducing a kink at the first site. These are shown as the dashed curves. For noise less than 1.0 units no signal gets through to site 31. At 1.1 units the signal always arrives but travels slowly with a wide distribution of arrival times. At 1.5 units there is a shorter transient time and a narrower distribution of arrival times, but spurious signals begin to appear. By 1.8 units about half the signal is caused by the noise. By 2.2 units the early arrival of spurious transitions—some nucleated near the end of the chain—virtually always preclude the detection of the original input signal.

The derivative of an individual curve in Fig. 4 gives the probability distribution of arrival events: the probability that the 31st resonator changes phase at a certain time after the system is placed in the metastable state. Two competing effects corrupt the quality and reliability of signal detection in our system: for very low noise levels, the chances of successful signal propagation are virtually zero, while for very high noise intensities, noise-induced, spurious events contaminate the output. We subtract from

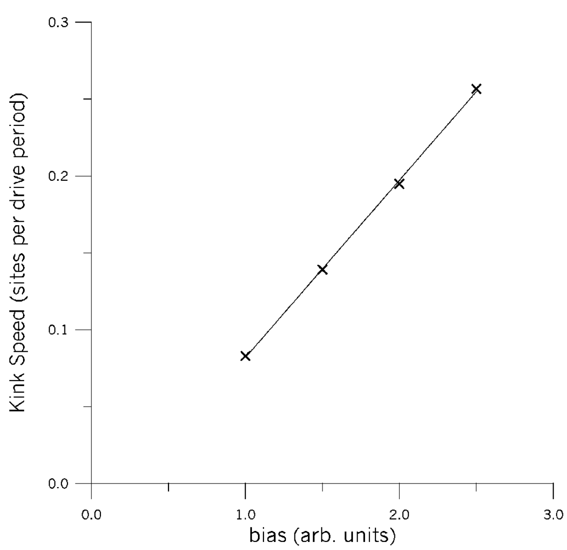

FIG. 2. Kink speed as a function of applied bias. Kinks will not propagate for bias values less than approximately 1.0 unit.

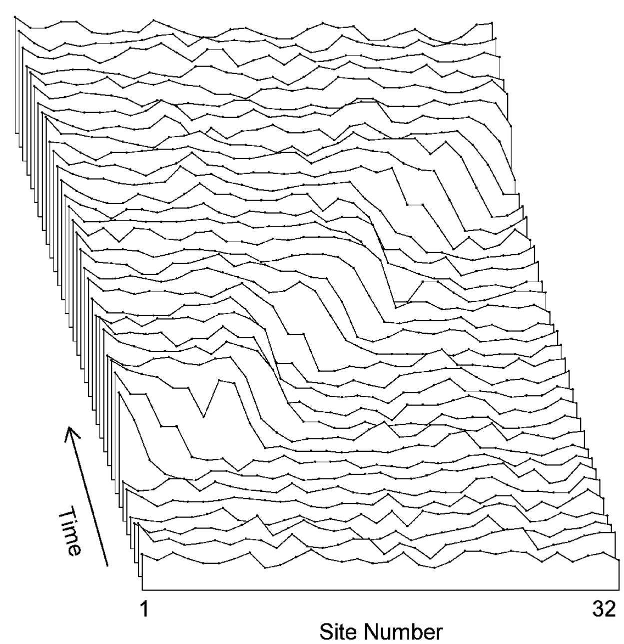

FIG. 3. Staggered snapshots (every 32 drive cycles) of a phase kink moving under the influence of noise across 32 resonators. Local variations in its speed are clearly visible, due to both spatial inhomogeneities and the random forcing. The bias amplitude is 0.55 units and the noise intensity 1.3 units.

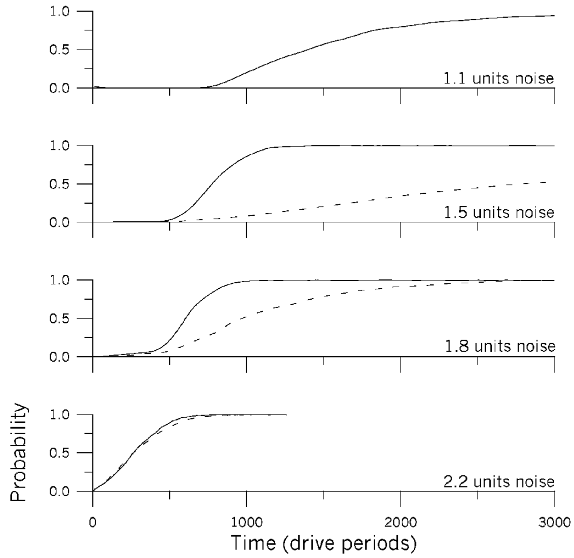

FIG. 4. Probability of a signal at site 31 with (solid line) and without (dashed curves) induced kink at site 1, for four noise levels increasing from top to bottom. The noise level of 1.1 units is too weak to nucleate any transitions.

the distribution with an induced kink the distribution without an induced kink to obtain the probability (as a function of time) that the signal arriving at site 31 is due to the input signal. The integral of this quantity, while positive, gives the probability of successful signal transmission. This quantity, which is plotted in Fig. 5, displays a SR-like sharp rise and subsequent fall behavior. The peak of this curve is assumed at noise intensities between 1.0 and 1.5 units, at which values the information flow is optimized by the presence of noise.

Important information can be gathered from the distributions that include an induced kink at the first site. The skewed distribution functions shift towards shorter times with increasing noise intensity, indicating a higher speed of information transmission. A noise-dependent propagation speed was also found in Ref. [4]. In the absence of noise the kink would arrive at site 31 after approximately 1050 drive periods, as indicated by extrapolating the line in Fig. 2 down to a bias of 0.55 units. In Fig. 6 we plot the mode (probability maximum) as a measure of the arrival times to illustrate the dependency on noise. Also plotted in Fig. 6 are the full widths at half maximum value. The distributions become narrower, denoting a smaller spread or variance in arrival times.

Previous studies of spatiotemporal SR in a similar experimental setting [3] are of a fundamentally different nature. While earlier the system—at least in principle—was translationally invariant, here this spatial symmetry is broken by the application of constant bias and appropriate initial conditions. The resulting unidirectional information flow allows us to define the boundaries as "input" and "output," regarding the system as a one-way communication channel. Furthermore, previous work [2,3] concentrated on synchronization, i.e., *concurrent* noise-induced hopping of the entire array. Here, the stochastic events are strictly *sequential*. The most evident consequence is a greatly different scaling behavior: unlike the former examples, in which the coupling entails partial noise cancellation and improved signal-to-noise ratios, the quality of signal transmission in our setup in fact deteriorates for a higher number of elements. The sequential nature of the process leads to error accumulation instead of noise suppression.

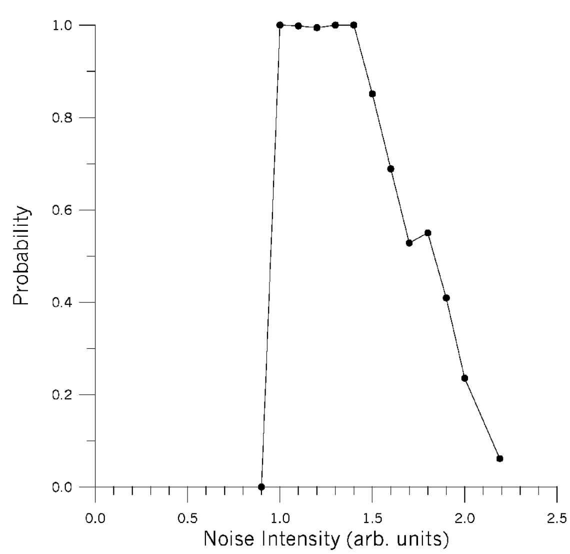

FIG. 5. Maximum of the difference between the probability distributions with and without signal vs noise intensity for thirty coupled diode resonators. The maximum information transmission is assumed for noise levels 1.0–1.5 units.

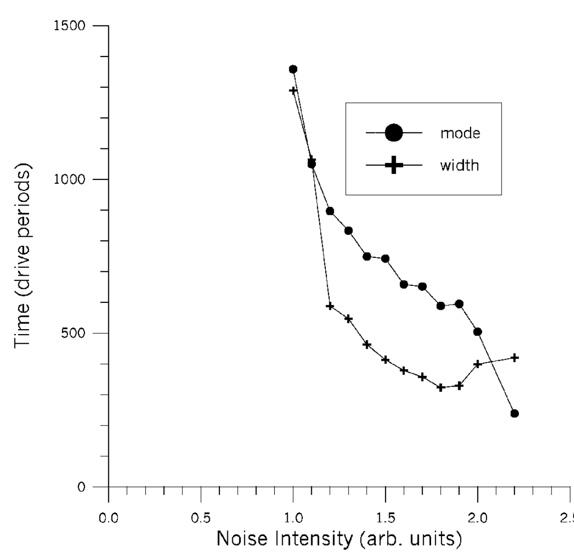

FIG. 6. Mode (circles) and width (crosses represent full width at half maximum) of the probability distribution of arrival events vs noise intensity.

In order to be able to predict parameter values, such as noise intensity, coupling strength, and bias amplitude, which result in satisfactory and reliable signal propagation, we have to look at the underlying mechanism in more detail. There are two relevant time scales, which govern the transition rates of the individual elements. The "unperturbed" escape rate $K_u = \mu \exp(-E_u \beta)$ (where E_u denotes the threshold energy and β the inverse temperature or noise intensity), valid for a resonator with both neighbors in the same, metastable phase. If the left neighbor has performed the transition to the more stable phase, it will modify the energy threshold through the coupling, making the escape more likely: $K_p = \mu \exp(-E_p \beta) > K_u$ (with the perturbed threshold $E_p < E_u$). The degree to which E_u differs from E_p is a function of both the coupling and the bias amplitude; naturally, for zero bias or vanishing coupling we have $E_p = E_u$. If we assume the time for the actual transition to be negligible, it is clear that on the average it will take NK_p^{-1} time units for the kink to traverse N sites. [This also explains the increase in kink speed with growing noise intensity, since $K_p^{-1} \sim \exp(\beta)$.] Over this period of time, the total probability that a resonator "ahead" of the moving front performs a (undesired) transition, is equal to $K_u K_p^{-1} N(N - 1)/2$ [12]. Thus, the signal deterioration will scale with the system size like N^2. From the above considerations it is evident that for a low contamination with spurious signals the ratio K_u/K_p should be as low as possible, i.e., $K_u \ll K_p$ or equivalently: $E_u \gg E_p$. For a fixed bias, stronger coupling will increase the difference between E_u and E_p and thus improve the signal transmission quality.

In conclusion, we establish the beneficial role of fluctuations for signal transmission in a metastable medium. A nonzero noise level optimizes the information flow through coupled, nonidentical metastable elements. The noise contamination of the signal is predicted to increase rapidly with system size. We anticipate that these results could yield further insights into neural information processing. The observation of noise-sustained waves in an experimental setting might provide a new understanding of the impact of intrinsic noise on biophysical and biochemical processes.

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[12] The probability that an unperturbed resonator escapes during the time K_P^{-1} is equal to $P = K_u K_P^{-1}$. If the kink has propagated i sites, there are $(N - i - 1)$ sites left to escape. Thus, after the 0th site is phase flipped, the probability that the remaining $N - 1$ sites nucleate a spurious kink is equal to $P_{\text{total}} = \sum_{i=0}^{N-1} P(N - i - 1) = PN(N - 1)/2$. The kink width is assumed to be negligible in this derivation.