An Explanation for the ρ-π Puzzle of J/ψ and ψ' Decays

Yu-Qi Chen and Eric Braaten

Physics Department, The Ohio State University, Columbus, Ohio 43210

(Received 6 January 1998)

We propose a new explanation for the long-standing puzzle of the tiny branching fraction of $\psi' \to \rho\pi$ relative to that for $J/\psi \to \rho\pi$. In the case of J/ψ, we argue that this decay is dominated by a higher Fock state in which the $c\bar{c}$ pair is in a color-octet ${}^3S_1$ state and decays via the annihilation process $c\bar{c} \to q\bar{q}$. In the case of the ψ', we argue that the probability for the $c\bar{c}$ pair in this higher Fock state to be close enough to annihilate is suppressed by a dynamical effect related to the small energy gap between the mass of the ψ' and the $D\bar{D}$ threshold. [S0031-9007(98)06312-1]

PACS numbers: 13.25.Gv, 12.39.Jh

A long-standing mystery of charmonium physics is the "ρ-π puzzle" of J/ψ and ψ' decays. These particles are nonrelativistic bound states of a charm quark and its antiquark. Their decays into light hadrons are believed to be dominated by the annihilation of the $c\bar{c}$ pair into three gluons. In order to annihilate, the c and $\bar{c}$ must have a separation of order $1/m_c$, which is much smaller than the size of the charmonium state. Thus the annihilation amplitude for an S-wave state like J/ψ or ψ' must be proportional to the wave function at the origin, $\psi(\mathbf{r} = 0)$. The width for decay into any specific final state h consisting of light hadrons is therefore proportional to $|\psi(0)|^2$. The width for decay into e^+e^- is also proportional to $|\psi(0)|^2$. This leads to the simple prediction that the ratio of the branching fractions for ψ' and J/ψ is given by the "15% rule":

$$Q_h \equiv \frac{B(\psi' \to h)}{B(J/\psi \to h)} = Q_{ee} = (14.7 \pm 2.3)\%\,. \qquad (1)$$

The ρ-π puzzle is that the prediction (1) is severely violated in the $\rho\pi$ and several other decay channels. The first evidence for this effect was presented by the Mark II Collaboration in 1983 [1]. They found that $Q_{\rho\pi} < 0.6\%$ and $Q_{K^*K} < 2\%$. Recent data from the BES Collaboration have made the puzzle even sharper. They obtained $Q_{\rho\pi} < 0.23\%$, $Q_{K^{*+}K^-} < 0.64\%$, and $Q_{K^{*0}\bar{K}^0} = (1.7 \pm 0.6)\%$ [2]. Thus the suppression of Q relative to the 15% rule is about 10 for $K^{*0}\bar{K}^0$ and greater than 65 for $\rho\pi$.

A summary of the proposed explanations of the ρ-π puzzle has recently been given by Chao [3]. Hou and Soni [4] suggested that $J/\psi \to \rho\pi$ is enhanced by a mixing of the J/ψ with a glueball $\mathcal{O}$ that decays to $\rho\pi$. Brodsky, Lepage, and Tuan [5] emphasized that $J/\psi \to \rho\pi$ violates the helicity selection rule of perturbative QCD, and argued that the data require $\mathcal{O}$ to be narrow and nearly degenerate with the J/ψ. Present data from BES constrain the mass and width of the glueball to the ranges $|m_{\mathcal{O}} - m_{J/\psi}| < 80$ MeV and 4 MeV $< \Gamma_{\mathcal{O}} < 50$ MeV [6]. This mass is about 700 MeV lower than the lightest $J^{PC} = 1^{--}$ glueball observed in lattice simulations of QCD without dynamical quarks [7]. There are explanations of the ρ-π puzzle that involve the dependence of the decay amplitude on the energy of the charmonium state. Karl and Roberts [8] suggested that the decay proceeds through $c\bar{c} \to q\bar{q}$ followed by the fragmentation of the $q\bar{q}$ into $\rho\pi$. They argued that the fragmentation probability is an oscillatory function of the energy which could have a minimum near the mass of the ψ'. Chaichian and Tornqvist [9] pointed out that the suppression of ψ' decays could be explained if the form factors for two-body decays fall exponentially with the energy as in the nonrelativistic quark model. There are other explanations of the ρ-π puzzle that rely on the fact that there is a node in the radial wave function for ψ', but not for J/ψ. Pinsky [10] suggested that this node makes $\psi' \to \rho\pi$ a "hindered M1 transition" like $J/\psi \to \eta_c\gamma$. Brodsky and Karliner [11] suggested that the decay into $\rho\pi$ proceeds through intrinsic charm components of the ρ and π wave functions. They argued that the $c\bar{c}$ pair in the $|u\bar{d}c\bar{c}\rangle$ Fock state of the ρ^+ or π^+ has a nodeless radial wave function which gives it a larger overlap with J/ψ than ψ'. Finally, Li, Bugg, and Zou [12] have suggested that final-state interactions involving the rescattering of $a_1\rho$ and $a_2\rho$ into $\rho\pi$ could be important and might interfere destructively in the case of the ψ'.

In this Letter, we present a new explanation of the ρ-π puzzle. We argue that the decay $J/\psi \to \rho\pi$ is dominated by a higher Fock state of the J/ψ in which the $c\bar{c}$ is in a color-octet ${}^3S_1$ state. The $c\bar{c}$ pair in this Fock state can annihilate via $c\bar{c} \to g^* \to q\bar{q}$. The amplitude for forming $\rho\pi$ is dominated by the end point of the meson wave functions, where a single q or $\bar{q}$ carries most of the momentum of the meson. The suppression of $\psi' \to \rho\pi$ is attributed to a suppression of the $c\bar{c}$ wave function at the origin for the higher Fock state of the ψ'. Such a suppression can arise from a dynamical effect associated with the small energy gap between the mass of the ψ' and the $D\bar{D}$ threshold.

It is convenient to analyze charmonium decays using nonrelativistic QCD (NRQCD) [13], the effective field theory obtained by integrating out the energy scale of the charm quark mass m_c. In Coulomb gauge, a charmonium state has a Fock state decomposition in which the probability of each Fock state scales as a definite power of v, the typical relative velocity of the charm quark. The dominant Fock state of the J/ψ and the ψ' is $|c\bar{c}_1({}^3S_1)\rangle$,

 0031-9007/98/80(23)/5060(4)$15.00

whose probability $\mathcal{P}$ is of order 1. We denote the color state of the $c\bar{c}$ pair by a subscript (1 for color-singlet, 8 for color-octet), and we put the angular momentum quantum numbers in parentheses. The Fock states $|c\bar{c}_8(^3P_J) + S\rangle$, where S represents dynamical gluons or light quark-antiquark pairs with energies of order $m_c v$ or less, have probability $\mathcal{P} \sim v^2$. The next most important Fock states include $|c\bar{c}_8(^1S_0) + S\rangle$ with $\mathcal{P} \sim v^3$ (or $\mathcal{P} \sim v^4$ if perturbation theory is sufficiently accurate at the scale $m_c v$) and $|c\bar{c}_8(^3S_1) + S\rangle$ with $\mathcal{P} \sim v^4$.

The decay of the J/ψ into light hadrons proceeds via the annihilation of the c and $\bar{c}$, which can occur through any of the Fock states. This is expressed in the NRQCD factorization formula for the inclusive decay rate [13], which includes the terms

$$\Gamma(J/\psi \to \text{light hadrons}) = \left(\frac{20(\pi^2 - 9)\alpha_s^3}{243 m_c^2} + \frac{16\pi\alpha^2}{27 m_c^2}\right)\langle \mathcal{O}_1(^3S_1)\rangle_{J/\psi} + \frac{5\pi\alpha_s^2}{6m_c^2}\langle \mathcal{O}_8(^1S_0)\rangle_{J/\psi} + \frac{19\pi\alpha_s^2}{6m_c^4}\langle \mathcal{O}_8(^3P_0)\rangle_{J/\psi} + \frac{\pi\alpha_s^2}{m_c^2}\langle \mathcal{O}_8(^3S_1)\rangle_{J/\psi} + \ldots, \tag{2}$$

where $\alpha_s = \alpha_s(m_c)$. The matrix elements are expectation values in the J/ψ of local gauge-invariant NRQCD operators that measure the inclusive probability of finding a $c\bar{c}$ in the J/ψ at the same point and in the color and angular-momentum state specified. The matrix element of the $c\bar{c}_1(^3S_1)$ term in (2) is proportional to the square of the wave function at the origin and scales as v^3. Its coefficient includes a term of order α_s^3 from $c\bar{c} \to ggg$ and a term of order α^2 from the electromagnetic annihilation process $c\bar{c} \to \gamma^* \to q\bar{q}$. The color-octet terms in (2) represent contributions from higher Fock states. Their matrix elements scale like v^6, v^7, and v^7, respectively. Their coefficients are all of order α_s^2 and come from $c\bar{c} \to gg$ for the $c\bar{c}_8(^1S_0)$ and $c\bar{c}_8(^3P_0)$ terms and from $c\bar{c} \to g^* \to q\bar{q}$ for the $c\bar{c}_8(^3S_1)$ term. Note that the coefficients of the color-octet matrix elements are 2 orders of magnitude larger than that of $\langle \mathcal{O}_1(^3S_1)\rangle_{J/\psi}$, which suggests that the higher Fock states may play a more important role in annihilation decays than is commonly believed.

Bolz, Kroll, and Schuler have emphasized that contributions from higher Fock states should also be important in exclusive decays of charmonium into light hadrons [14]. They argued that the amplitude for a two-body annihilation decay satisfies a factorization formula which separates the scale m_c from the lower momentum scales. The decay amplitude is expressed in terms of hard-scattering factors $\hat{\mathcal{T}}$ that involve only the scale m_c, initial-state factors that involve scales of order $m_c v$ and lower, and final-state factors that involve only the scale $\Lambda_{\rm QCD}$. If m_c was asymptotically large, the dominant terms in the factorization formula would have the minimal number of partons involved in the hard scattering. Terms involving additional soft partons in the initial state are suppressed by powers of v. Terms involving additional hard partons in the final state are suppressed by powers of $\Lambda_{\rm QCD}/m_c$. As pointed out by Brodsky and Lepage [15], the leading term in the asymptotic factorization formula is strongly constrained by the vector character of the QCD interaction between quarks and gluons. It vanishes unless the sum of the helicities of the mesons is zero. Rotational symmetry then requires the angular distribution to be $1 - \cos^2\theta$. This helicity selection rule is violated by the decay $J/\psi \to \rho\pi$. Parity and rotational symmetry require the helicity of the ρ to be ± 1 and the angular distribution to be $1 + \cos^2\theta$. Since the helicity of the pion is 0, the helicity selection rule is violated. Thus the amplitude for $J/\psi \to \rho\pi$ is suppressed by a factor of $\Lambda_{\rm QCD}/m_c$ relative to that for generic mesons. The leading contribution is a term with a hard scattering factor of order α_s^2 from the process $c\bar{c}_1 \to u\bar{d}d\bar{u}g$.

Since the charm quark mass m_c is less than an order of magnitude larger than $\Lambda_{\rm QCD}$, there can be large corrections to the asymptotic decay amplitude. In particular, there can be significant regions of phase space in which some of the gluons involved in the hard scattering are relatively soft. It might therefore be more appropriate to absorb them into the initial-state or final-state factors. In this case, not all of the soft partons in S need be involved in the hard scattering, and not all the partons that form the ρ and π need be produced by the hard scattering. For example, there can be a contribution from the Fock state $|c\bar{c}_8(^3S_1) + S\rangle$ that involves the hard-scattering process $c\bar{c} \to q\bar{q}$, where $q = u$ or d. This produces a state $|q\bar{q} + S\rangle$ that consists of the soft partons S together with a q and a $\bar{q}$ that are back-to-back and whose momenta are approximately m_c. Such a state has a nonzero overlap with the final state $|\rho\pi\rangle$. The overlap comes from the end points of the meson wave functions, in which most of the momenta of the ρ and π are carried by the q and $\bar{q}$. This contribution to the T-matrix element can be written schematically in the form

$$\mathcal{T}(J/\psi \to \rho^+\pi^-) = \sum_{q\bar{q}} \hat{\mathcal{T}}(c\bar{c}_8(^3S_1) \to q\bar{q}) \times \sum_S \langle \rho\pi \,|\, q\bar{q} + S\rangle \times \langle c\bar{c}_8(^3S_1) + S \,|\, J/\psi\rangle. \tag{3}$$

Note that the initial-state and final-state factors on the right-hand side of (3) cannot be separated, because they are connected by the sum over soft modes S. The T-matrix element (3) would contribute to the $c\bar{c}_8(^3S_1)$ term in the factorization formula (2) for inclusive decays.

The end point contribution in (3) leads to a definite angular distribution. Since the q and $\bar{q}$ carry most of the momenta of the mesons, the angular distribution of the mesons will follow that of the q and $\bar{q}$, which is $1 + \cos^2\theta$. Thus (3) will contribute most strongly to form factors which allow the angular distribution $1 + \cos^2\theta$. It will also contribute most strongly to decays into mesons like ρ and π for which most of the momentum can be carried by a single q or $\bar{q}$. There are also end point contributions involving the hard-scattering process $c\bar{c}_8(^3P_J) \to gg$, which produces a pair of hard gluons with the angular distribution $2 + \cos^2\theta$, and $c\bar{c}_8(^1S_0) \to gg$, for which the angular distribution is isotropic. Their contributions to $J/\psi \to \rho\pi$ are suppressed by the small probabilities for most of the momentum of the ρ or π to be carried by a single gluon and by the mismatch between the angular distribution of the gluons and the $1 + \cos^2\theta$ distribution of the $\rho\pi$.

We argue that the color-octet term in (3) may actually dominate the decay rate for $J/\psi \to \rho\pi$. We compare the various factors in that term with those in the asymptotic amplitude. The hard-scattering factor $\hat{\mathcal{T}}$ in the color-octet term in (3) is only of order α_s, compared to α_s^2 for the asymptotic amplitude. The suppression from the initial-state factor in the color-octet term in (3), including the sum over S, might be as little as a factor of v^2 relative to the asymptotic amplitude. This follows from the fact that the color-octet amplitude contributes in quadrature to the $\langle\mathcal{O}_8(^3S_1)\rangle$ term in (2), which is suppressed by v^4. As for the final-state factors, (3) is suppressed by the end points of the meson wave functions, while the asymptotic amplitude is suppressed by $\Lambda_{\mathrm{QCD}}/m_c$ from the violation of the helicity selection rule. Considering the various suppression factors, it is certainly plausible that the $c\bar{c}_8(^3S_1)$ term in (3) could dominate over the leading contribution from the $c\bar{c}_1(^3S_1)$ Fock state.

We have argued that the decay $J/\psi \to \rho\pi$ may be dominated by the annihilation of the $c\bar{c}$ pair in the $|c\bar{c}_8(^3S_1) + S\rangle$ Fock state via $c\bar{c} \to q\bar{q}$. If this is true, then the ρ-π puzzle can be explained by a suppression of this decay mechanism in the case of the ψ'. This suppression can arise from the initial-state factor $\langle c\bar{c}_8(^3S_1) + S \,|\, J/\psi\rangle$ if the $c\bar{c}$ wave function for the $|c\bar{c}_8(^3S_1) + S\rangle$ Fock state is suppressed in the region in which the separation of the $c\bar{c}$ is less than or of order $1/m_c$. Note that it does not require a suppression of the probability for the higher Fock state, but just a shift in the probability away from the region in which the c and $\bar{c}$ are close enough to annihilate. A possible mechanism for this suppression is a dynamical effect related to the small energy gap between the mass of the ψ' and the $D\bar{D}$ threshold. In the Born-Oppenheimer approximation [16], the $c\bar{c}$ pair in the dominant $|c\bar{c}\rangle$ Fock state moves adiabatically in response to a potential $V_1(R)$ given by the minimal energy of QCD in the presence of a color-singlet $c\bar{c}$ pair with fixed separation R. Similarly, the $c\bar{c}$ pair in the $|c\bar{c}_8(^3S_1) + S\rangle$ Fock state moves adiabatically in response to a potential $V_8(R)$ given by the minimal energy of the soft modes S in the presence of a color-octet $c\bar{c}$ pair with fixed separation R. At short distances, this potential approaches a repulsive Coulomb potential $\alpha_s/6R$. At long distances, the minimal energy state consists of D and $\bar{D}$ mesons separated by a distance R, and $V_8(R)$ therefore approaches a constant $2(M_D - m_c)$ equal to the energy of the $D\bar{D}$ threshold. A charmonium state spends most of the time on the color-singlet adiabatic surface, but it occasionally makes a transition to the color-octet adiabatic surface. Since the J/ψ is 640 MeV below the $D\bar{D}$ threshold, a $c\bar{c}$ pair on the color-octet adiabatic surface is far off the energy shell. The time spent by the J/ψ on this surface is too short for the $c\bar{c}$ pair to respond to the repulsive short-distance potential. Since the wave function for the $|c\bar{c}_1\rangle$ Fock state peaks at the origin, the $c\bar{c}$ wave function for the $|c\bar{c}_8 + S\rangle$ Fock state should have significant support near the origin. However, the mass of the ψ' is only 43 MeV below D^+D^- threshold and 53 MeV below $D^0\bar{D}^0$ threshold. A $c\bar{c}$ pair on the color-octet adiabatic surface can be very close to the energy shell. The ψ' can therefore spend a sufficiently long time on this surface for the $c\bar{c}$ pair to respond to the repulsive short-distance potential. This response can lead to a significant suppression of the $c\bar{c}$ wave function at the origin for the $|c\bar{c}_8 + S\rangle$ Fock state.

If the initial-state factor $\langle c\bar{c}_8(^3S_1) + S \,|\, J/\psi\rangle$ in (3) is suppressed for all soft partons S, the suppression can be expressed in the form of a relation between the NRQCD matrix elements in (2):

$$\frac{\langle\mathcal{O}_8(^3S_1)\rangle_{\psi'}}{\langle\mathcal{O}_1(^3S_1)\rangle_{\psi'}} \ll \frac{\langle\mathcal{O}_8(^3S_1)\rangle_{J/\psi}}{\langle\mathcal{O}_1(^3S_1)\rangle_{J/\psi}}. \tag{4}$$

This inequality can be tested by calculating the matrix elements using lattice NRQCD. Since the ψ' is so close to the $D\bar{D}$ threshold, it would be essential to include dynamical light quarks in the simulations.

Our proposal leads to a prediction for the flavor dependence of the suppression of the decays of ψ' into vector/pseudoscalar final states (VP). Bramon, Escribano, and Scadron [17] have analyzed the decays $J/\psi \to$ VP assuming that the decay amplitude is the sum of a flavor-connected amplitude g, a flavor-disconnected amplitude rg, and an isospin-violating amplitude e. The expressions for the amplitudes are given in Ref. [17]. They allowed for violations of SU(3) flavor symmetry through parameters s and $x - 1$. The authors give two sets of parameters that fit the existing data, one with $x = 1$ and the other with $x = 0.64$. Both sets have e comparable in magnitude to rg and an order of magnitude smaller than g. If the decay $J/\psi \to \rho\pi$ is dominated by end point contributions, we can identify g and e with the two terms in (3). While rg may also have end point contributions from $c\bar{c} \to gg$, we assume for simplicity that it is dominated by subasymptotic contributions from the $c\bar{c}_1(^3S_1)$ Fock state. The amplitudes for the decays $\psi' \to$ VP can be expressed in a similar way in terms of amplitudes g', e', and $(rg)'$. Our explanation of the ρ-π puzzle implies that $|g'|$ is much smaller than $|g|$, and that e' and $(rg)'$ differ from e and rg only by the factor required by the 15% rule.

The unknown amplitude g' is constrained by the BES data on $\psi' \to \rho\pi$ and $\psi' \to K^{*0}\bar{K}^0$. The upper bound on $B(\psi' \to \rho\pi)$ gives an upper bound on $|g' + e'|^2$, which implies that g' lies in a circle in the complex g' plane. The BES measurement of $B(\psi' \to K^{*0}\bar{K}^0)$ gives an allowed range for $|(1 - s)g' - (1 + x)e'|^2$, which constrains g' to an annulus. The intersection of the interior of the circle with the annulus is the allowed region for g'. By varying g' over that region and taking into account the uncertainties in the parameters of Ref. [17], we obtain the predictions for $Q_{\rm VP}$ in Table I. Measurements of the ψ' branching fractions consistent with these predictions would imply that the suppression of the vector/pseudoscalar decays is due to the suppression of g'. This would lend support to our explanation of the ρ-π puzzle.

Our proposal also has implications for the angular distributions of other two-body decay modes. In general, the angular distribution must have the form $1 + \alpha \cos^2\theta$, with $-1 < \alpha < +1$. Our solution to the ρ-π puzzle is based on the suppression of a contribution to ψ' decays that gives the angular distribution $1 + \cos^2\theta$. Thus the parameter α for any two-body decay of the ψ' should be less than or equal to α for the corresponding J/ψ decay.

A solution to the ρ-π puzzle should also be able to explain the pattern of suppression for various $J^{\rm PC}$ states with the same flavors. A preliminary measurement of the axial-vector/pseudoscalar decay mode $\psi' \to b_1\pi$ by the BES Collaboration [18] gives $Q_{b_1\pi} = (24 \pm 7)\%$, consistent with no suppression relative to the 15% rule. A preliminary measurement of the vector/tensor decay mode $\psi' \to \rho a_2$ [18] gives $Q_{\rho a_2} = (2.9 \pm 1.6)\%$, which, though suppressed relative to the 15% rule, is an order of magnitude larger than $Q_{\rho\pi}$. This pattern can be explained by also taking into account the orbital-angular-momentum selection rule for exclusive amplitudes in perturbative QCD [19]. The decay modes $b_1\pi$ and ρa_2 both have form factors that are allowed by the helicity selection rule. They also both have form factors that violate the helicity selection rule, but are compatible with an end point contribution from $c\bar{c} \to q\bar{q}$. However, in the case of $b_1\pi$, the end point contribution is further suppressed by the violation of the orbital-angular-momentum selection rule. Thus we should expect no suppression of $\psi' \to b_1\pi$ and only a partial suppression of $\psi' \to \rho a_2$. A thorough analysis of J/ψ and ψ' decays into axial-vector/pseudoscalar and vector/tensor final states will be presented elsewhere.

TABLE I. Predictions for $Q_{\rm VP}$ in units of 1% for all the vector/pseudoscalar final states. The values for $\rho\pi$ and $K^{*0}\bar{K}^0$ + c.c. were used as input. The columns labeled $x = 1$ and $x = 0.64$ correspond to the two parameter sets of Ref. [17].

VP	$x = 1$	$x = 0.64$
$\rho\pi$	0–0.25	0–0.25
$K^{*0}\bar{K}^0$ + c.c.	1.2–3.0	1.2–3.0
$K^{*+}K^-$ + c.c.	0–0.36	0–0.52
$\omega\eta$	0–1.6	0–1.6
$\omega\eta'$	10–51	12–55
$\phi\eta$	0.8–3.6	0.4–3.0
$\phi\eta'$	0.7–2.5	0.5–2.2
$\rho\eta$	14–22	14–22
$\rho\eta'$	12–20	13–21
$\omega\pi$	11–17	11–17

In conclusion, we have proposed a new explanation of the ρ-π puzzle. We suggest that the decay $J/\psi \to \rho\pi$ is dominated by a Fock state in which the $c\bar{c}$ is in a color-octet ${}^3S_1$ state which decays via $c\bar{c} \to q\bar{q}$. The suppression of this decay mode for the ψ' is attributed to a dynamical effect that suppresses the $c\bar{c}$ wave function at the origin for Fock states that contain a color-octet $c\bar{c}$ pair. Our explanation for the ρ-π puzzle can be tested by studying the flavor dependence of the two-body decay modes of the J/ψ and ψ', their angular distributions, and their dependence on the $J^{\rm PC}$ quantum numbers of the final-state mesons.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-91-ER40690. We thank Steve Olsen for discussions of the data on charmonium decays from the BES Collaboration.

[1] Mark II Collaboration, M. E. B. Franklin *et al.*, Phys. Rev. Lett. **51**, 963 (1983).
[2] Y. S. Zhu, in *Proceedings of the 28th International Conference on High Energy Physics,* edited by Z. Adjuk and A. K. Wroblewski (World Scientific, Singapore, 1997).
[3] K.-T. Chao, in *Proceedings of the 17th International Symposium on Lepton-Photon Interactions,* edited by Z.-P. Zheng and H.-S. Chen (World Scientific, Singapore, 1996).
[4] W. S. Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983).
[5] S. J. Brodsky, G. P. Lepage, and S. F. Tuan, Phys. Rev. Lett. **59**, 621 (1987).
[6] W.-S. Hou, Phys. Rev. D **55**, 6952 (1997).
[7] See, for example, M. Peardon, hep-lat/9710029.
[8] G. Karl and W. Roberts, Phys. Lett. **144B**, 263 (1984).
[9] M. Chaichian and N. A. Tornqvist, Nucl. Phys. **B323**, 75 (1989).
[10] S. S. Pinsky, Phys. Lett. B **236**, 479 (1990).
[11] S. J. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 4682 (1997).
[12] X.-Q. Li, D. V. Bugg, and B.-S. Zou, Phys. Rev. D **55**, 1421 (1997).
[13] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).
[14] J. Bolz, P. Kroll, and G. A. Schuler, Phys. Lett. B **392**, 198 (1997); hep-ph/9704378.
[15] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[16] K. J. Juge, J. Kuti, and C. J. Morningstar, hep-lat/9709131; hep-ph/9711451.
[17] A. Bramon, R. Escribano, and M. D. Scadron, Phys. Lett. B **403**, 339 (1997).
[18] BES Collaboration, S. L. Olsen *et al.*, Int. J. Mod. Phys. A **12**, 4069 (1997).
[19] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).