Atomic Detection and Matter-Wave Coherence

E. V. Goldstein and Pierre Meystre
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721
(Received 28 October 1997)

We analyze several models of atomic detectors in the context of the measurement of coherence properties of matter waves. In particular, we show that an ionization scheme measures normally ordered correlation functions of the Schrödinger field, in analogy with the optical situation. However, it exhibits a sensitivity to exchange processes that is normally absent in optics. [S0031-9007(98)06310-8]

PACS numbers: 03.75.–b, 32.80.–t, 42.50.Vk

Optical coherence theory is based on the observation that most quantum measurements that can be performed on the electromagnetic field yield a signal proportional to normally ordered correlation functions of that field [1]. A quantized multimode field is then said to be coherent to order N if all normally ordered correlation functions up to order N factorize. No such theory is presently available for atomic coherence, probably because until recently it had not been necessary to think of atomic samples as Schrödinger fields. But the experimental work on ultracold atoms, Bose-Einstein condensation (BEC) [2–6], and atom lasers [7] has changed that situation, and the need for a proper theory of atomic coherence is now quite urgent [8].

At least for the case of bosonic fields, it is tempting to simply transpose Glauber's coherence theory [1]. Appealing as it might sound, this approach must be applied with caution, due to the fundamental difference between electromagnetic and matter-wave fields. Most optical experiments detect light by absorption, i.e., by "removing" photons from the light field. This is the reason why normally ordered correlation functions are so important. But atomic detectors work in a number of different ways: One can choose to measure electronic properties, or center-of-mass properties, or both. Additional difficulties arise from the fact that atomic fields are self-interacting, which significantly complicates the propagation of atomic coherence as compared to the case of light. From these remarks, it should be clear that a theory of matter-wave coherence is much richer than its optical equivalent. Yet, like Glauber's coherence theory, it should be operational and based on explicit detection schemes.

The goal of this Letter is to analyze ideal atom detectors and to determine which correlation functions of the matter-wave field they are sensitive to. The systems we explicitly consider are nonresonant atomic imaging systems such as used, e.g., in the MIT BEC experiments, and detectors working via atomic ionization. We show that in contrast to off-resonance imaging, which is known to be sensitive to density correlation functions, narrow-band ionization detectors measure normally ordered correlation functions of the Schrödinger field. This is analogous to the optical case, with the difference that higher-order detection schemes involve additional exchange terms usually absent in optics.

Nonresonant imaging.—To set the stage for our discussion, we first briefly review atomic detection by nonresonant imaging [3,9]. These measurements involve a strongly detuned electromagnetic field interacting with the atoms in the sample in such a way that it induces only virtual transitions. We consider for concreteness ground-state atoms described by the Schrödinger field operator $\hat{\Psi}(\mathbf{r})$ with $[\hat{\Psi}(\mathbf{r}), \hat{\Psi}^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')$ for bosons, and decompose the electromagnetic field into a classically populated mode of wave vector $\mathbf{k}_0$ and polarization $\boldsymbol{\epsilon}_0$ and a series of weakly excited side modes of wave vectors $\mathbf{k}_\ell$ and polarizations $\boldsymbol{\epsilon}_\ell$. After adiabatic elimination of the upper electronic state of the atomic transition under consideration, the interaction between the Schrödinger field and the radiation field is described to lowest order in the side modes by the effective Hamiltonian

$$V = \hbar \int d^3r \left[\frac{|\Omega_0(\mathbf{r})|^2}{\delta_0} + \sum_\ell \left(\frac{\Omega_0(\mathbf{r})\Omega_\ell^\star}{\delta_0} a_\ell^\dagger e^{i(\mathbf{k}_0 - \mathbf{k}_\ell)\cdot\mathbf{r}} e^{-i\omega_0 t} + \frac{\Omega_0^\star(\mathbf{r})\Omega_\ell}{\delta_0} a_\ell e^{-i(\mathbf{k}_0 - \mathbf{k}_\ell)\cdot\mathbf{r}} e^{i\omega_0 t} \right) \right] \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}), \quad (1)$$

where $\mathbf{k}_\ell$ is the wave vector of the ℓth mode of the field, of frequency ω_ℓ, and of polarization $\boldsymbol{\epsilon}_\ell$, the sum is over all field modes in the quantization volume V, $\mathcal{E}_\ell = [\hbar\omega_\ell/2\epsilon_0 V]^{1/2}$, and $[a_\ell, a_{\ell'}^\dagger] = \delta_{\ell,\ell'}$. We have also introduced the Rabi frequencies $\Omega_0(\mathbf{r}) = d\mathcal{E}_0(\mathbf{r})(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_0)/\hbar$, $\boldsymbol{\epsilon}$ being the direction of the atomic dipole, and $\Omega_\ell = d\mathcal{E}_\ell(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_\ell)/\hbar$, and the atom-field detuning $\delta_0 \equiv \omega_a - \omega_0$ is such that $|\delta_0| \gg |\Omega_0(\mathbf{r})|$.

The measurement of specific properties of the Schrödinger field can then be carried out in various ways. For instance, one can detect interferences between a classical incident field and scattered light, as in the MIT experiments [3]. This results in a signal proportional to the density $\langle\hat{\rho}(\mathbf{r}, t)\rangle$, where we have introduced the operator $\hat{\rho}(\mathbf{r}, t) \equiv \hat{\Psi}^\dagger(\mathbf{r}, t)\hat{\Psi}(\mathbf{r}, t)$, whose expectation value is the local density of the sample. Alternatively,

 0031-9007/98/80(23)/5036(4)$15.00

one can measure the spectrum of the scattered light [9], in a fashion familiar from resonance fluorescence experiments. For side modes initially in a vacuum state, the most important nontrivial contribution to the fluorescence signal is proportional to the intensity $|\Omega_0|^2$ of the incident field,

$$w = \frac{|\Omega_0|^2}{\delta_0^2} \sum_\ell |\Omega_\ell|^2 \int d^3r\, d^3r' \int_t^{t+\Delta t} d\tau\, d\tau' \times e^{i[(\mathbf{k}_0-\mathbf{k}_\ell)\cdot(\mathbf{r}-\mathbf{r}')-(\omega_0-\omega_\ell)(\tau-\tau')]} \times \langle \hat{\rho}(\mathbf{r},\tau)\hat{\rho}(\mathbf{r}',\tau')\rangle , \quad (2)$$

and hence is sensitive to the second-order correlation function of the sample density. Indeed, it can be shown that any measurement involving the electromagnetic field scattered by the atomic sample under conditions of off-resonant imaging is determined by correlation functions of the Schrödinger field density.

Ionization.—The reason off-resonant imaging yields a signal dependent on $\hat{\rho}(\mathbf{r},t)$ is that the electric dipole interaction is bilinear in the Schrödinger field operators. This difficulty can, however, be eliminated if, instead of making measurements on the radiation field, one detects the atoms directly [10,11]. One scheme that achieves this goal is the ionization method that we now discuss.

Consider a detector consisting of a tightly focused laser beam that can ionize atoms by inducing transitions from their ground electronic level $|g\rangle$ to a continuum level $|i\rangle$. The corresponding single-particle Hamiltonian is $H = H_{\rm cm} + H_{\rm el} + V(\mathbf{r}) \equiv H_0 + V(\mathbf{r})$, where $H_{\rm cm}$ is the center-of-mass Hamiltonian, $H_{\rm el}$ the electronic Hamiltonian, and $V(\mathbf{r})$ describes the electric dipole interaction between the atom and the ionizing laser field. $H_{\rm el}$ has eigenstates φ_n and eigenfrequencies ω_n, $H_{\rm el}|\varphi_n\rangle = \hbar\omega_n|\varphi_n\rangle$. The corresponding atomic many-body Hamiltonian is $\mathcal{H}_0 = \int d^3r\, \hat{\Psi}^\dagger(\mathbf{r}) H_0 \hat{\Psi}(\mathbf{r})$ where in the Born-Oppenheimer approximation $\hat{\Psi}(\mathbf{r})$ is a multicomponent field with components $\hat{\Psi}_n(\mathbf{r})$, the index n labeling electronic states of the atoms.

We are interested in measuring properties of the ground-state component $\hat{\Psi}_g(\mathbf{r})$ of this field, which is electric dipole coupled to continuum states $\hat{\Psi}_i(\mathbf{r})$. We assume for simplicity that the center-of-mass wave function of these latter states is well described by plane waves of momentum $\mathbf{q}$, so that $\mathcal{H}_0$ may be expressed as $\mathcal{H}_0 = \mathcal{H}_g + \sum_{i\mathbf{q}} \mathcal{H}_{i\mathbf{q}}$, where $\mathcal{H}_{i\mathbf{q}} = \hbar\omega_{iq} b^\dagger_{i,\mathbf{q}} b_{i,\mathbf{q}}$. Here we expanded $\hat{\Psi}_i(\mathbf{r})$ in plane waves as $\hat{\Psi}_i(\mathbf{r}) = \sum_{\mathbf{q}} \phi_{\mathbf{q}}(\mathbf{r}) b_{i,\mathbf{q}}$ with $[b_{i,\mathbf{q}}, b^\dagger_{i'\mathbf{q}'}] = \delta_{\mathbf{q}\mathbf{q}'}\delta_{ii'}$, and $\omega_{iq} = \hbar\mathbf{q}^2/2M + \omega_i$. (Note that the inclusion of ground-state collisions is straightforward and does not affect our conclusions.)

In terms of the components $\hat{\Psi}_n(\mathbf{r})$ of the Schrödinger field, the electric dipole interaction Hamiltonian is

$$\mathcal{V} = \hbar \sum_i \int d^3r\, \Omega_i(\mathbf{r}) \hat{\Psi}_i^\dagger(\mathbf{r}) \hat{\Psi}_g(\mathbf{r}) e^{-i\omega_L t} + \text{H.c.}, \quad (3)$$

where Ω_i is the Rabi frequency between the levels $|g\rangle$ and $|i\rangle$, and the ionizing laser field of frequency ω_L is treated classically.

In this detection scheme, one extracts information about the state of the field $\hat{\Psi}_g(\mathbf{r},t)$ by standard methods, such as, e.g., the detection of the quasifree electrons of the continuum states.

For ground-state atoms cooled well below the recoil temperature and tightly focused laser beams, the spatial size of the atomic wave function is much larger than the laser spot, and we can approximate the electric field $\mathbf{E}(\mathbf{r})$ by $\mathbf{E}(\mathbf{r}) \simeq \mathbf{E}\delta(\mathbf{r}-\mathbf{r}_0)$, so that Eq. (3) becomes $\mathcal{V} = \hbar \sum_i \Omega_i(\mathbf{r}_0) \hat{\Psi}_i^\dagger(\mathbf{r}_0) \hat{\Psi}_g(\mathbf{r}_0) e^{-i\omega_L t} + \text{H.c.}$

We take the atomic system to be initially in the state $|\psi\rangle = |\{\psi_{i,\mathbf{q}}\}, \psi_g\rangle$. To first order in perturbation theory, the transition probability away from that state during the time interval Δt is

$$w \simeq \sum_{i,\mathbf{q},i',\mathbf{q}'} |\Omega_i(\mathbf{r}_0)|^2 \int_t^{t+\Delta t} d\tau \int_t^{t+\Delta t} d\tau' [e^{i\omega_L(t-t')} \langle\psi_g|\hat{\Psi}_g^\dagger(\mathbf{r}_0,\tau)\hat{\Psi}_g(\mathbf{r}_0,\tau')|\psi_g\rangle \langle\{\psi_{i,\mathbf{q}}\}|\hat{\Psi}_i(\mathbf{r}_0,\tau)|\{\phi_{i',\mathbf{q}'}\}\rangle \times \langle\{\phi_{i',\mathbf{q}'}\}|\hat{\Psi}_i^\dagger(\mathbf{r}_0,\tau')|\{\psi_{i,\mathbf{q}}\}\rangle + e^{-i\omega_L(t-t')} \langle\psi_g|\hat{\Psi}_g(\mathbf{r}_0,\tau)\hat{\Psi}_g^\dagger(\mathbf{r}_0,\tau')|\psi_g\rangle \times \langle\{\psi_{i,\mathbf{q}}\}|\hat{\Psi}_i^\dagger(\mathbf{r}_0,\tau)|\{\phi_{i',\mathbf{q}'}\}\rangle \langle\{\phi_{i',\mathbf{q}'}\}|\hat{\Psi}_i(\mathbf{r}_0,\tau')|\{\psi_{i,\mathbf{q}}\}\rangle], \quad (4)$$

where the sum is over all final states $|\{\phi_{i'\mathbf{q}'}\}\rangle$ in the excited state manifold. In this expression, we have neglected contributions involving the product of two creation or annihilation operators, a result of the implicit assumption that any atom in the continuum will be removed from the sample instantaneously. In addition, we explicitly carried out the sum over all final states of the ground-state field, but not for the excited fields manifold. This is because we want to allow for the possibility of selective detection of the ionized atoms. Following Ref. [1], this can easily be achieved by replacing the sum over final states in Eq. (4) by a weighted sum

$$\sum_{i',\mathbf{q}'} \longrightarrow \sum_{i',\mathbf{q}'} \mathcal{R}(i',\mathbf{q}'), \quad (5)$$

where $\mathcal{R}(i',\mathbf{q}')$ is the detector sensitivity to atoms in state $|\phi_{i',\mathbf{q}'}\rangle$. In practice, we have in mind energy-selective detectors, $\mathcal{R}(i',\mathbf{q}') \to \mathcal{R}(E)$, and the degeneracy of the levels must then, of course, be accounted for.

There is a fundamental distinction between the situation at hand and Glauber's photodetection theory, because in the present case both the detected and detector

fields consist of matter waves. There is a complete symmetry between these two fields so far, and their roles are interchangeable. In order to break this symmetry and to truly construct a detector, we now make a series of assumptions on the state of the detector fields $\hat{\Psi}_i(\mathbf{r},t)$. Physically, this amounts to making a statement about the way the detector is prepared prior to a measurement. Specifically, we assume that all atoms are in the ground state, $\Psi_i(\mathbf{r}_0,0)\,|\{\psi_{i,\mathbf{q}}\}\rangle = 0$, and that any atom in an ionized state will be removed from the sample instantaneously, as already mentioned. In that case, the second term in Eq. (4) vanishes.

We concentrate in the following on the example of energy-selective detectors, and consider specifically the limits of *narrow band* and *broad band* detection [1,12]. In the first case the detector bandwidth ΔE_d is assumed to be much narrower than the energy width ΔE_g of the ground-state Schrödinger field, which is determined solely by the spread in center-of-mass momentum (temperature) since all atoms occupy the same internal state. The reverse is true in the second case. We note that in contrast to optical fields detection, the narrow band regime can now be achieved only by manipulating the detector sensitivity $\mathcal{R}(E)$. Indeed, even a monochromatic excitation of the atomic fields results in $\Delta E_g = \Delta E_d$ due to atomic center-of-mass dispersion.

For a narrow band detection and for large enough detection times $\Delta t \gg (\hbar/\Delta E_g)$ the integrals in Eq. (4) can be extended to $\pm\infty$. After substitution of Eq. (5) into Eq. (4) this leads to the following expression for the ionization rate $r_{nb}(\omega) = w_{nb}(\omega)/\Delta t$

$$r_{nb}(\omega) \propto \int_{-\infty}^{\infty} d\tau\, e^{-i(\omega-\omega_L)\tau} G_A(t, t+\tau; \mathbf{r}_0, \mathbf{r}_0) + \text{c.c.}, \tag{6}$$

where $\hbar\omega$ is the energy of the registered photoelectrons, and we introduced the normally ordered first-order correlation function of the ground state Schrödinger field

$$G_A(t, t'; \mathbf{r}_0, \mathbf{r}_0) = \langle \hat{\Psi}_g^\dagger(\mathbf{r}_0, t)\hat{\Psi}_g(\mathbf{r}_0, t')\rangle\,. \tag{7}$$

In this limit, the detector measures the Fourier component of the atomic correlation function $G_A(t,t';\mathbf{r}_0,\mathbf{r}_0)$. For stationary fields, the Wiener-Khintchine theorem implies that tuning the detector sensitivity $\mathcal{R}(E)$ yields the spectrum of the Schrödinger field $\hat{\Psi}_g(\mathbf{r},t)$.

In the case of broad band detection, in contrast, the energy distribution ΔE_c of the ionized states is much broader than ΔE_g. This situation can be realized, e.g., by exciting the ground state with a broad band laser pulse and detecting the resulting electrons (or ions) with a broad band detector, $\mathcal{R}(E) \simeq$ const. Assuming that the spectrum of the ground atoms Schrödinger field is centered at $\bar{\omega}$ we find

$$r_{bb} \simeq \eta(\mathbf{r}_0) G_A(t, t; \mathbf{r}_0, \mathbf{r}_0)\,, \tag{8}$$

where we have introduced in prevision of the following discussion the "detector cross efficiency"

$$\eta(\mathbf{r}_1,\mathbf{r}_2) = \sum_i \Omega_i(\mathbf{r}_1)\Omega_i^\star(\mathbf{r}_2) \times \int_0^{\Delta t} d\tau\, \langle \Psi_i(\mathbf{r}_2, t+\tau)\Psi_i^\dagger(\mathbf{r}_1,t)\rangle e^{-i(\bar{\omega}-\omega_L)\tau}, \tag{9}$$

from which the usual detector efficiency is simply recovered as $\eta(\mathbf{r}_0) \equiv \eta(\mathbf{r}_0,\mathbf{r}_0)$. As expected, a broad band detector is not able to resolve any spectral feature of the Schrödinger field, and only measures the local atomic density, like off-resonant imaging.

Higher-order correlations.—The detection of higher-order correlation functions of the Schrödinger field can be achieved by a straightforward generalization of the ionization detector. For instance, second-order coherence measurements can be carried out by focusing the laser at two locations $\mathbf{r}_1$ and $\mathbf{r}_2$, in which case

$$\mathcal{V} = \hbar \sum_{\mu=1,2} \sum_j \Omega_j(\mathbf{r}_\mu)\hat{\Psi}_j^\dagger(\mathbf{r}_\mu)\hat{\Psi}_g(\mathbf{r}_\mu)e^{-i\omega_L t} + \text{H.c.}$$

The joint probability to ionize an atom at $\mathbf{r}_1$ and another one at $\mathbf{r}_2$ is then

$$w_2 \simeq \sum_{\{j_i\}\{\mathbf{q}_i\}} \int_t^{t+\Delta t} d\tau_1 \int_t^{t+\Delta t} d\tau_2 \int_t^{t+\Delta t} d\tau_3 \int_t^{t+\Delta t} d\tau_4 e^{-i\omega_L(\tau_1+\tau_2-\tau_3-\tau_4)} \Omega_{j_1}^\star(\mathbf{r}_1)\Omega_{j_2}^\star(\mathbf{r}_2)\Omega_{j_3}(\mathbf{r}_2)\Omega_{j_4}(\mathbf{r}_1) \times \langle \hat{\Psi}_{j_1}(\mathbf{r}_1,\tau_1)\hat{\Psi}_{j_2}(\mathbf{r}_2,\tau_2)\hat{\Psi}_{j_3}^\dagger(\mathbf{r}_2,\tau_3)\hat{\Psi}_{j_4}^\dagger(\mathbf{r}_1,\tau_4)\rangle \langle \hat{\Psi}_g^\dagger(\mathbf{r}_1,\tau_1)\hat{\Psi}_g^\dagger(\mathbf{r}_2,\tau_2)\hat{\Psi}_g(\mathbf{r}_2,\tau_3)\hat{\Psi}_g(\mathbf{r}_1,\tau_4)\rangle\,. \tag{10}$$

This joint probability involves two detected atoms, hence it is now necessary to properly account for the quantum statistics of the measured particles. For this purpose, we describe the ionized atoms as ion-electron pairs, whereby the electrons are described by the creation and annihilation operators $c_{\mathbf{k}s}^\dagger$ and $c_{\mathbf{k}s}$ satisfying Fermi commutation relations $[c_{\mathbf{k}s}, c_{\mathbf{k}'s'}^\dagger]_+ = \delta_{ss'}\delta_{\mathbf{k}\mathbf{k}'}$. Here s labels the electron spin and $\mathbf{k}$ its momentum. We similarly introduce ion creation and annihilation operators $a_{\mathbf{k}s}^\dagger, a_{\mathbf{k}s}$, also satisfying Fermi commutation relations (for bosonic atoms). For a spin-zero atom, the atomic mode operators $b_{j,\mathbf{q}}$ can be expressed in terms of the ion and electron operators as

$$b_{j,\mathbf{q}} \equiv |j\mathbf{q}\rangle\langle 0| = \sum_{\mathbf{k}\mathbf{k}'ss'} |\mathbf{k}\mathbf{k}'ss'\rangle\langle \mathbf{k}\mathbf{k}'ss'|j\mathbf{q}\rangle\langle 0| = \sum_{\mathbf{k}s} \varphi_j(\mathbf{k}) a_{\mathbf{q}s} c_{\mathbf{k}-s} = \sum_s a_{\mathbf{q}s} c_{j-s}\,, \tag{11}$$

where $\varphi_j(\mathbf{k})$ are electron wave functions in $\mathbf{k}$ space, $c_{js} = \sum_{\mathbf{k}} \varphi_j(\mathbf{k}) c_{\mathbf{k}s}$, and we have assumed that the center-of-mass wave function is $e^{i\mathbf{q}\cdot\mathbf{r}}$ with $\mathbf{r}$ being the ion (or atomic center-of-mass) position. Because of spin conservation the values of electron and ion spins are clearly opposite.

Substituting this result into Eq. (10) yields, in the case of broad band detection,

$$
\begin{aligned}
w_2 = {} & \eta(\mathbf{r}_1)\eta(\mathbf{r}_2) \int_t^{t+\Delta t} d\tau_1 \int_t^{t+\Delta t} d\tau_2 \langle \Phi_g^\dagger(\mathbf{r}_1,\tau_1)\Phi_g^\dagger(\mathbf{r}_2,\tau_2)\Phi_g(\mathbf{r}_2,\tau_2)\Phi_g(\mathbf{r}_1,\tau_1)\rangle \\
& + \eta(\mathbf{r}_1,\mathbf{r}_2)\eta(\mathbf{r}_2,\mathbf{r}_1) \int_t^{t+\Delta t} d\tau_1 \int_t^{t+\Delta t} d\tau_2 \langle \Phi_g^\dagger(\mathbf{r}_1,\tau_1)\Phi_g^\dagger(\mathbf{r}_2,\tau_2)\Phi_g(\mathbf{r}_2,\tau_1)\Phi_g(\mathbf{r}_1,\tau_2)\rangle \\
& + \eta_x(\mathbf{r}_1,\mathbf{r}_2) \int_t^{t+\Delta t} d\tau_1 \langle \Phi_g^\dagger(\mathbf{r}_1,\tau_1)\Phi_g^\dagger(\mathbf{r}_2,\tau_1)\Phi_g(\mathbf{r}_2,\tau_1)\Phi_g(\mathbf{r}_1,\tau_1)\rangle , \qquad (12)
\end{aligned}
$$

where the detector sensitivity due to processes involving electron exchange $\eta_x(\mathbf{r}_1,\mathbf{r}_2)$ is

$$
\begin{aligned}
\eta_x(\mathbf{r}_1,\mathbf{r}_2) = {} & \int_t^{t+\Delta t} d\tau_2 \int_t^{t+\Delta t} d\tau_3 \int_t^{t+\Delta t} d\tau_4 e^{-i\omega_L(\tau_1+\tau_2-\tau_3-\tau_4)} \\
& \times \sum_{\alpha\beta\boldsymbol{\kappa}\mathbf{q}} [e^{i[\omega_\kappa(\tau_1-\tau_3)+\omega_q(\tau_2-\tau_4)+\omega_\alpha(\tau_1-\tau_4)+\omega_\beta(\tau_2-\tau_3)]} |\Omega_\alpha(\mathbf{r}_1)|^2 |\Omega_\beta(\mathbf{r}_2)|^2 \phi_{\boldsymbol{\kappa}}^\star(\mathbf{r}_1)\phi_{\boldsymbol{\kappa}}(\mathbf{r}_2)\phi_{\mathbf{q}}^\star(\mathbf{r}_2)\phi_{\mathbf{q}}(\mathbf{r}_1) \\
& + \Omega_\alpha^\star(\mathbf{r}_1)\Omega_\alpha(\mathbf{r}_2)\Omega_\beta^\star(\mathbf{r}_2)\Omega_\beta(\mathbf{r}_1) |\phi_{\boldsymbol{\kappa}}(\mathbf{r}_1)|^2 |\phi_{\mathbf{q}}(\mathbf{r}_2)|^2 e^{i[\omega_\kappa(\tau_1-\tau_4)+\omega_q(\tau_2-\tau_3)+\omega_\alpha(\tau_1-\tau_3)+\omega_\beta(\tau_2-\tau_4)]}] . \qquad (13)
\end{aligned}
$$

The first term is familiar from double photodetection, with the usual exchange contributions from the *detected* field. The second term is an additional exchange term due to the fact that the detector field is a single Schrödinger field. Its origin is the interference of the *detector* field at points $\mathbf{r}_1$ and $\mathbf{r}_2$. The last term results from the fact that electrons do not know from which atom they originate. The second and third terms in Eq. (12) are absent in conventional photodetection theory, a result of the distinguishability of two detectors used. A similar comment can be made about the position measurement scheme discussed in Refs. [10,11]. In that case, the absence of the detector exchange contribution can be traced back to the fact that the set of states excited at each location are distinguishable. Alternatively, the last two terms can be eliminated by using, e.g., a gated detection scheme [12] that removes the contribution of the exchange terms in the detected field. In practice, such gating can be achieved by using nonoverlapping short laser pulses.

In summary, we find that in contrast to off-resonant imaging, the ionization scheme is closely related to the detectors familiar from quantum optics. However, it presents new features, and is, in particular, sensitive to exchange terms in the detector. These results show the need to introduce different classes of coherence for matter waves, associated with different classes of measurements. This topic will be addressed in detail in a future publication [13].

This work is supported in part by the U.S. Office of Naval Research Contract No. 14-91-J1205, by the National Science Foundation Grant No. PHY95-07639, by the U.S. Army Research Office, and by the Joint Services Optics Program.

[1] R. Glauber, in *Quantum Optics and Electronics*, edited by C. de Witt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, New York, 1965).
[2] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[3] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] J. R. Ensher, D. S. Jin, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 4984 (1996).
[5] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, and W. Ketterle, Phys. Rev. Lett. **77**, 416 (1996).
[6] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997).
[7] M.-O. Mewes, M. R. Andrews, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **78**, 582 (1997).
[8] W. Ketterle and H. J. Miesner, Phys. Rev. A **56**, 3291 (1997).
[9] J. Javanainen, Phys. Rev. Lett. **75**, 1927 (1995).
[10] J. E. Thomas and L. J. Wang, Phys. Rev. A **49**, 558 (1994).
[11] J. E. Thomas and L. J. Wang, Phys. Rep. **262**, 311 (1995).
[12] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interaction. Basic Processes and Applications* (Wiley, New York, 1992).
[13] E. V. Goldstein, O. Zobay, and P. Meystre, cond-mat/9804089.