"Cosmological" Scenario for *A*-*B* Phase Transition in Superfluid ^{3}He

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During a very rapid superfluid transition in ^{3}He following a reaction with a single neutron, the creation of topological defects (vortices) has been recently demonstrated to be in accordance with the Kibble-Zurek scenario for the cosmological analog. We discuss here the extension of the Kibble-Zurek scenario to the case when alternative symmetries may be broken and different states nucleated independently. We have calculated the nucleation probability of the various states of superfluid ^{3}He during a superfluid transition. Our results can explain the transition from the supercooled *A* phase to the *B* phase triggered by a nuclear reaction. The new scenario is an alternative to the well-known "baked Alaska" scenario. [S0031-9007(98)06200-0]

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Superfluid ^{3}He has an order parameter which describes the simultaneous spin, orbital, and gauge symmetries which are broken at the superfluid transition. This transition can be regarded as the closest condensed matter analogy to the cosmological grand unification transition. This analogy has been utilized in the experimental test of the Kibble cosmological mechanism of cosmic strings creation. According to this mechanism [1], at the transition separate regions of the Universe are independently nucleated with a random orientation of the gauge field in each region. The size of these initial regions (domains) depends strongly on the rapidity with which the transition is traversed. According to Zurek [2] the fundamental distance between the independently created coherent domains (in the language of [2] the distance between the ensuing vortices Z) is of the order of $Z = \xi_0(\tau_Q/\tau_0)^{1/4}$, where ξ_0 is the zero temperature coherence length, $\tau_0 = (\xi_0/v_F)$ is the characteristic time constant of the superfluid, and τ_Q is the characteristic time for cooling through the phase transition. As the domains grow and make contact with their neighbors, the resulting gauge field cannot be uniform. The subsequent order-parameter "glass" forces a distribution of topological defects leading to a tangle of quantized vortex lines. The first quantitative tests of defect creation during a gauge symmetry transformation have been recently performed in superfluid ^{3}He.

The superfluid ^{3}He (at very low temperatures in the Grenoble experiment [3] and at relatively high temperatures in the Helsinki experiment [4]) was heated locally by neutron irradiation via the nuclear reaction:

$$^3\mathrm{He} + \mathrm{n} = {}^3\mathrm{H}^- + \mathrm{p}^+ + 764\ \mathrm{keV}\,.$$

The energy released by the neutron reaction heats a small region of the liquid ^{3}He (about 30 μm) into the normal state. This region recools rapidly through the superfluid transition owing to the rapid outflow of quasiparticles into the surrounding superfluid. For the experimental conditions of both experiments it has been proposed that quasiparticles from the heated region disperse outwards, meaning that the hot bubble is cooled rapidly from its sides and that the cooling rate is so fast that the order parameter of the surrounding superfluid ^{3}He cannot follow the changing temperature front fast enough (see [5] for theoretical details). Consequently internal regions of the hot volume transit into the superfluid phase independently in accordance with the Zurek cosmological scenario. The experimental results of both experiments justify this assumption. In the Grenoble experiment the excess number of quasiparticles created by the reaction has been counted and it was found that a significant fraction of the energy released by the reaction does not appear in the thermal reservoir of quasiparticles. This energy deficit agrees well in magnitude with the energy expected to be trapped as topological defects (in this case vortices) as calculated from Zurek's scenario for the Kibble mechanism.

Under the relatively high temperature conditions of the Helsinki experiment any vortices created by the neutron reaction would be rapidly destroyed via interaction with the quasiparticle gas. However, in the rotating cryostat there is an added bias field, that of rotation. This field can extract a few vortex rings from the bubble which then grow to the dimensions of the cell. After the process the number of vortices can be measured directly by NMR. The number of extracted vortices corresponds well to that calculated from the Zurek scenario.

Our knowledge of superfluid ^{3}He is much better than our knowledge of the Universe. In the case of superfluid ^{3}He we not only know the symmetries broken during the superfluid transition but we also know the Ginzburg-Landau potential exactly and we can calculate quantitatively the dynamics of the order parameter during the transition. There are two different stable phases of ^{3}He, the *A* and *B* phases which correspond to different broken symmetries. The energy difference between these two states is relatively small. Let us say that it is negligible on the time scale of the transition. This means that regions which independently enter the superfluid state should not only have different orientation of the order parameter but may also correspond to states with different symmetries [6]. It is this complication of the Kibble-Zurek scenario which we consider in the calculations below. Ironically, a very similar situation may be relevant to the Universe, where, in addition to the creation of the SU(3) $\times$ SU(2) $\times$ U(1) state, other states may also be created, in particular, the SU(4) $\times$ U(1) state [7]. The first state, we believe, corresponds to the energy minimum of our Universe, whereas the second state has much higher creation probability owing to its higher symmetry. This is exactly the situation in superfluid ^{3}He where the B phase has the lower energy, except in the case of the strong interaction correction for high pressure and temperature.

The rotational and gauge symmetries of ^{3}He are usually represented by a 3×3 matrix of complex numbers A_{ai} which is known as the order parameter. Above the transition all the elements of the matrix have zero values (representing full symmetry). Below the transition, some of these quantities become nonzero. The symmetry of the order parameter after the transition corresponds to the manifold of symmetries which remain unbroken. In the case of superfluid ^{3}He there are 13 possibilities (13 states) corresponding to the various symmetries of the order parameter [8]. The free energy of these states can be expressed in the framework of the phenomenological theory of Ginzburg and Landau by

$$F = -\alpha A^*_{ai}A_{ai} + \beta_1 A^*_{ai}A^*_{ai}A_{bj}A_{bj} + \beta_2 A^*_{ai}A_{ai}A^*_{bj}A_{bj} + \beta_3 A^*_{ai}A^*_{bi}A_{aj}A_{bj} + \beta_4 A^*_{ai}A_{bi}A^*_{bj}A_{aj} + \beta_5 A^*_{ai}A_{bi}A_{bj}A^*_{aj}\,,$$

where $\alpha = \alpha_0(1 - T/T_c)$, which changes sign at the transition temperature T_c, and the quantities β_i are functions of pressure (and also of temperature through the so-called "strong correction") and depend on the details of the microscopic interaction.

The different possible symmetries of the order parameter correspond to local minima and saddle points in this 18-dimensional energy surface. In superfluid ^{3}He we know there are two stable states, the A and B phases. The energy balance between the A and B phases is determined by the relationship between the parameters β_i. At zero pressure, the B phase corresponds to the absolute minimum, while at pressures above 20 bars there is a region of temperature where the A phase becomes the preferred state.

These two states have different order parameter symmetries. In the B phase, relative spin (S) orbit (L) symmetry $\mathrm{SO}(3)_{S+L}$ remains unbroken (such that A_{ai} resembles a rotation matrix). In the A phase (the "axial" state) the symmetry of the spin system is reduced to a gauge symmetry (U_S), which couples to the orbital motion to yield a combined symmetry of the orbital rotation and gauge (G) fields $\mathrm{U}_S \times \mathrm{U}_{L+G}$ [9].

According to the Zurek scenario, regions on a distance scale of Z undergo the superfluid transition separately. We can consider these regions as independent elementary samples of ^{3}He. (Later we shall analyze the influence of the gradient energy between the different regions.) We have numerically modeled the creation of the superfluid phases in a single region during a rapid cooling. For this we have applied the time dependent Ginzburg-Landau equation in the form

$$-\tau\frac{\partial}{\partial t}A_{ai} + \alpha_0\frac{T_c - T(t)}{T_c}A_{ai} - (\beta_1 A^*_{ai}A_{bj}A_{bj} + \beta_2 A_{ai}A^*_{bj}A_{bj} + \beta_3 A^*_{bi}A_{aj}A_{bj} + \beta_4 A_{bi}A^*_{bj}A_{aj} + \beta_5 A_{bi}A_{bj}A^*_{aj}) = 0\,.$$

For the initial conditions we apply temperature $T = T_c$ and a small independent random perturbation of all 18 numbers of the A_{ai} matrix. Then we reduce the temperature over time ($10^{-9} - 10^{-7}$ s) and calculate the time dependence of the order parameter during this "downhill" process. We monitor both the symmetry of the order parameter A_{ai} and the energy (F) during this time evolution and find that both the A and the B phases can develop. The final state depends on the starting perturbation of the order parameter and the profile of the 18-dimensional potential surface. It does not depend on velocity of cooling or the final temperature. That is because we have used the formalism of the Ginzburg-Landau theory, which is not valid far from T_C. In other words our results can be applied to real ^{3}He for relatively high temperatures. For low temperatures more complicated theories should be considered. Nevertheless, our results demonstrate the new explanation of A-B phase transition which we will discuss later. To achieve good statistical resolution on the probability of both A and B phase creation we have performed several thousands calculations for each pressure.

Other metastable states may develop transiently after the application of an initial perturbation which has the exact symmetry of one of these states. However the trajectory of A_{ai} in these cases is unstable and any small perturbation away from the final symmetry leads to the more stable A or B states.

It is important to note that, although according to Zurek the cooling rate determines the dimensions of the independent regions, the trajectory of the order parameter for a single coherent region is rate independent and is determined only by the profile of the Ginzburg-Landau potential. At zero pressure, when we have weak coupling with $\beta_i = (-1, 2, 2, 2, -2)$, the B phase corresponds to the absolute energy minimum. In our computer simulation we find that, even under these conditions, nucleation of the A phase has a high probability. In quantitative terms we find the probability of B phase creation to be 54% $\pm$ 1%, while that of the A phase creation is 46%. It is difficult to visualize the trajectory of the order parameter in 18-dimensional space, but we can monitor the energy during the transition. Figure 1 shows typical trajectories of the superfluid ^{3}He free energy after rapid cooling. In some cases the trajectory approaches a saddle point on the

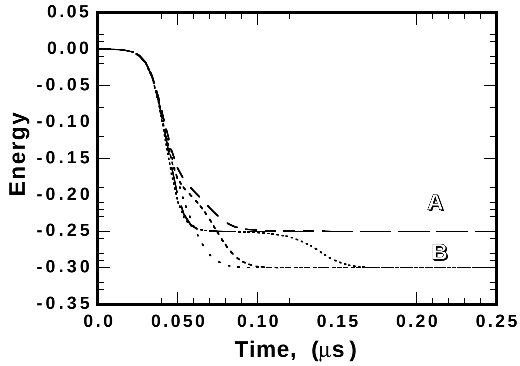

FIG. 1. The time evolution of the free energy density during a superfluid phase transition after a small random perturbations. The temperature was reduced from $T = T_c$ to $T = 0$ in a time of 10^{-8} s.

energy surface. The behavior here is clarified by reducing the rate of energy change.

In order to study the influence of gradient energy on the development of the order parameter we consider a one-dimensional spatial sample of Zurek length Z divided into 100 points. We chose Z to agree with the Grenoble experiment at zero bar (about $8\xi_0$). Two different perturbations are applied, one for the first 50 points and the second for the remaining 50 points. The development of the A_{ai} matrix during the downhill process is calculated at each point, taking into account the gradient energy. The results of these calculations are shown in Fig. 2. We find that the boundary between the two different states remains almost stationary during the main part of the downhill process. However, towards the end of this process the boundary begins to move in the energetically favorable direction. This result looks very natural, since the boundary replacement is determined by the energy difference and the time dependence of the energy is very similar for the two different symmetries at the beginning of the downhill process, as seen in Fig. 1.

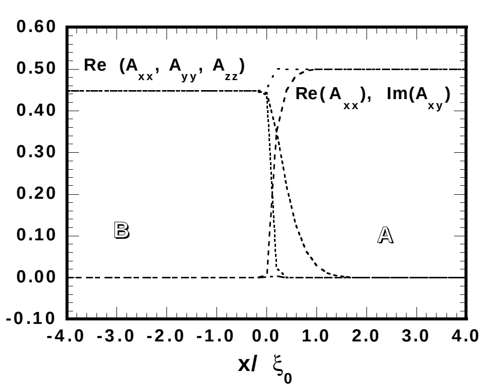

FIG. 2. Computer simulation of the spatial distribution of nonzero terms of the order parameter at 0.25 μs after nucleation of the B phase on the left hand side and of the A phase on the right hand side of the sample.

In the frame of the Ginzburg-Landau approximation the β_i parameters depend only on the pressure. There are a number of theories which suggest somewhat different values for these parameters. We have used the parameters, calculated by Sauls and Serene [10]. In Fig. 3 we show the probability of A phase nucleation as a function of pressure along with the energy balance between the A and B phases. It is important to notice that the probability of A state nucleation may become greater than 50% even in the region where the B phase is stable.

All experimentalists who work with superfluid ^{3}He have noticed the crucial asymmetry of the A-B transition. If one is cooling ^{3}He at a pressure above 20 bars, the A phase may survive as a supercooled metastable state far below the equilibrium A-B transition line. On the other hand, on warming it is difficult to get a superheated B phase. In [11] it was shown that there is some critical temperature at which a transition from the A to B phase will always occur. The pressure dependence of this threshold temperature is parallel to the equilibrium A-B transition line and crosses the T_c temperature line at about 15 bars. This threshold pressure for T_C has a natural explanation in our model. It corresponds to the condition when the probability of B state nucleation exceeds that of A state nucleation.

This observation may supply the critical jigsaw piece of information for the long-running puzzle of the A-B transition in superfluid ^{3}He. As proposed by Leggett and demonstrated in the Stanford experiments (see review [12]) cosmic rays can trigger the transition from a supercooled A phase to B phase. The standard view is that if ^{3}He is locally overheated to a normal state and cools back to the superfluid state by diffusion then the surrounding superfluid state just fills the bubble from outside. If we have the A state all around, then the B state cannot be created inside the hot bubble by cooling from its boundaries. That is the reason why a cooling process with an inverted temperature front (named "baked Alaska") has been proposed to explain the B state nucleation [12]. In this scenario the normal state shell gives the ability to

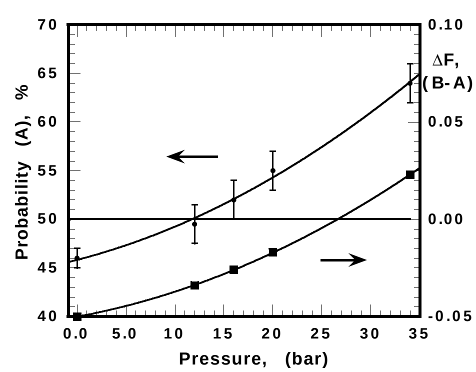

FIG. 3. The probability of A state nucleation as a function of pressure for temperature near T_c, and the difference of energy (F) between the A and B states.

nucleate the *B* state inside the bubble independently from the surrounding *A* state.

From our point of view the baked Alaska model is a rather artificial suggestion. It is likely that the cosmic event creates very energetic quasiparticles. These energetic quasiparticles travel out from the site of the event and create many new low energy quasiparticles on thermalization. It is important to point out that the low energy quasiparticles do not maintain the direction of the primary energetic ones. That is why it is likely that the quasiparticles remain inside the hot bubble and expand by the usual diffusion process.

However, in the framework of the cosmological Kibble-Zurek approach we do not need a normal shell to protect the interior of the hot bubble from the influence of the outside state. The diffusion cooling proceeds so rapidly that many seeds of the *A* and *B* phases are nucleated independently of the surrounding ^{3}He state. The baked Alaska process, if it occurs, will lead to an even larger number of such seeds. The subsequent development of the structure depends first on the relative densities of the two phases and secondly on the energy balance between them and on the domain boundary surface energy. The conditions for equal probability of nucleation of the *A* and *B* phases are different from those corresponding to the equilibrium of their free energies. This is the reason for the asymmetry of the *A*-*B* transition. When the *B* state is energetically preferable, but the *A* state has higher probability of nucleation, the *A* state seeds percolate. Consequently the *B* state seeds shrink due to the *A*-*B* surface tension. In order to pass through the transition the seeds of the *B* phase should percolate up to the critical cluster dimensions. This is possible when the conditions of 50% probability of nucleation are approximately fulfilled.

In the case where there is a possibility of nucleating two distinct phases, then owing to the eventual suppression of one phase (and annealing of its vortices), the distance between the subsequent vortices which remain from the order-parameter glass will be larger than that implied by the straightforward Zurek scenario. A simple argument suggests that the separation increases by the order of $Q^{-0.5}$, where Q is the probability of nucleation of the surviving state. This correction makes the calculated distance between vortices closer to that observed in the Grenoble [3] experiment. Recent experimental data of Helsinki [13] confirm directly the influence of the proximity of the *A* phase on the density of vortices.

Having considered superfluid ^{3}He we should look more carefully at similar possibilities for the early Universe. In other words, the vacuum of the Universe after a grand unification transition may also have had metastable states with different symmetries. For example, vacua with symmetries $[\mathrm{SU}(4) \times \mathrm{U}(1)]$ and $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]$ might have been able to coexist in the early Universe in separate domains. The spatial scale of these domains should be of the order of the parameter Z in Zurek's scenario. The transition of the metastable phase to the stable phase might have given rise to temperature and density inhomogeneities which may have influenced the Universe inhomogeneity observed at present.

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