T Dependence of the Magnetic Penetration Depth in Unconventional Superconductors at Low Temperatures: Can It Be Linear?

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We present a thermodynamics argument against a strictly linear temperature dependence of the magnetic penetration depth, which applies to superconductors with arbitrary pairing symmetry at low temperatures. [S0031-9007(98)06210-3]

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Some evidence for an unconventional $d_{x^2-y^2}$-pairing symmetry in cuprate high-T_c superconductors is provided by recent angle resolved photoemission experiments [1]. A striking proof for the $d_{x^2-y^2}$ symmetry of the Cooper pairs in cuprates arises from the observation of a spontaneously generated *half flux quantum* in Josephson tunneling experiments carried out on tetracrystal substrates [2]. Early support for the possibility of a $d_{x^2-y^2}$ symmetry of the Cooper pairs in cuprate high-T_c superconductors came from the observation of a *linear* T dependence of the magnetic penetration depth [3,4] at low temperatures T:

$$\lambda(T) - \lambda(0) \propto T. \tag{1}$$

Such a linear T dependence of the magnetic penetration depth (MPD) has a topological origin. If the order parameter associated with the Cooper pair condensate vanishes along node lines on the Fermi surface the spectrum $N_s(E)$ of quasiparticle excitations in the superconducting phase is gapless and varies proportional to E at low excitation energies: $N_s(E) \propto E$ for $E \ll \Delta_{\max}$. For this reason a pure $d_{x^2-y^2}$-pairing state (node lines along $k_x = \pm k_y$) should display a strictly *linear* dependence of MPD vs T at *low* temperatures. In previous work this effect was also discussed for the *polar* phase in a triplet pairing superconductor, e.g., [5].

New experiments [6] indicate deviations from this linearity of MPD with temperature, for example, a T^2 dependence of MPD below some crossover temperature T^* was measured. Such a behavior may occur due to various reasons. For example, Kosztin and Leggett [7] explain this behavior in terms of *nonlocal* electrodynamics. Their argument is, that in clean $d_{x^2-y^2}$-pairing superconductors there exist *surface induced* nonlocal effects, which lead to a T^2 dependence of $\lambda_{ab}(T) - \lambda_{ab}(0)$, as extracted from optical and microwave experiments with the magnetic field orientated *parallel* to the $\hat{\mathbf{c}}$ direction. On the other hand, in experiments with the magnetic field orientated *perpendicular* to the $\hat{\mathbf{c}}$ direction the T dependence of MPD cannot be altered by the Kosztin-Leggett effect.

Since the Kosztin-Leggett effect [7] really depends on the existence of a *surface* in the problem it cannot be applied to other measurement techniques of MPD, for example, direct static magnetic measurements, measurements of vortex properties, the lower critical magnetic field B_{c1}, muon spin relaxation. Such techniques of measuring MPD have *bulk* character.

In the following, we present a proof (based on linear response theory), for arbitrary superconductors, that a strictly linear T dependence of MPD at low temperatures violates the third law of thermodynamics. For simplicity, let us consider a uniform system where all properties depend on coordinates $\mathbf{r} - \mathbf{r}'$ only. The current-current correlator,

$$\eta(\mathbf{k},\omega) = k^2 - \frac{\omega^2}{c^2}\varepsilon_{\rm tr}(\mathbf{k},\omega), \tag{2}$$

connects the vector potential $\mathbf{A}(\mathbf{k},\omega)$ to the external current $\mathbf{j}_{\rm ext}(\mathbf{k},\omega)$ via

$$\eta(\mathbf{k},\omega)\mathbf{A}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{j}_{\rm ext}(\mathbf{k},\omega). \tag{3}$$

In turn, the transversal dielectric function, $\varepsilon_{\rm tr}(\mathbf{k},\omega)$, is related to the electromagnetic kernel $Q(\mathbf{k},\omega)$ by the relation

$$\varepsilon_{\rm tr}(\mathbf{k},\omega) = 1 - \frac{4\pi Q(\mathbf{k},\omega)}{\omega^2}. \tag{4}$$

The definition of the operator of inverse MPD is then

$$\frac{1}{\lambda^2(\mathbf{k},T)} = \lim_{\omega\to 0}\frac{\omega^2}{c^2}\{1 - \mathrm{Re}\,\varepsilon_{\rm tr}(\mathbf{k},\omega)\} \equiv \frac{4\pi}{c^2}Q(\mathbf{k},\omega = 0). \tag{5}$$

In the *static* case the additional free energy in the presence of an externally controlled current distribution $\mathbf{j}_{\rm ext}(\mathbf{k})$ (we use a transversal gauge) can be written in the form [8]:

$$\mathcal{F} = -\frac{1}{2c}\int\frac{d^3k}{(2\pi)^3}\mathbf{j}_{\rm ext}(\mathbf{k})\cdot\mathbf{A}(-\mathbf{k},\omega = 0) = -\frac{1}{8\pi}\int\frac{d^3k}{(2\pi)^3}\eta(\mathbf{k},\omega = 0)\,|\mathbf{A}(\mathbf{k},\omega = 0)|^2. \tag{6}$$

By using these relations and Maxwell's equations, it follows

$$\mathcal{F} = -\frac{1}{8\pi}\int\frac{d^3k}{(2\pi)^3}\left[k^2 + \frac{1}{\lambda^2(\mathbf{k},T)}\right]\times\frac{|\mathbf{k}\times\mathbf{B}(\mathbf{k};T)|^2}{k^4}. \tag{7}$$

Here $\mathbf{B}(\mathbf{k},T)$ is the (temperature dependent) induced magnetic field and satisfies the equation:

$$\left[k^2 + \frac{1}{\lambda^2(\mathbf{k},T)}\right]\mathbf{B}(\mathbf{k}\,;T) = \frac{4\pi}{c}\, i\mathbf{k} \times \mathbf{j}_{\rm ext}(\mathbf{k})\,. \qquad (8)$$

Differentiating Eq. (7) with respect to temperature T and calculating the derivative $\frac{\partial}{\partial T}\mathbf{B}(\mathbf{k}\,;T)$ from Eq. (8) we get an expression for the entropy:

$$S[T] = -\frac{\partial \mathcal{F}}{\partial T}$$
$$= -\frac{1}{8\pi}\int \frac{d^3k}{(2\pi)^3}\frac{\partial}{\partial T}\left[\frac{1}{\lambda^2(\mathbf{k},T)}\right]\frac{|\mathbf{B}(\mathbf{k}\,;T)|^2}{k^2}\,. \qquad (9)$$

According to the Nernst principle (*third law of thermodynamics*) the entropy should vanish in the limit $T \to 0$. From the positivity of the integrand we must conclude

$$\lim_{T\to 0}\frac{\partial \lambda(\mathbf{k},T)}{\partial T} = 0\,. \qquad (10)$$

If we wish to avoid a violation of the third law of thermodynamics the T dependence of the magnetic penetration depth in a superconductor *cannot* be of the form $\lambda(T) - \lambda(0) \propto T^n$ with $n = 1$. The argument can be extended to any nonuniform system.

We see that the vanishing of the first derivative of MPD for $T \to 0$ is a consequence of a general principle of thermodynamics. The value of T^* below which a deviation of the linear T dependence of MPD may be observed depends on the exact physical mechanism. It may be nonlocality [7], it may be the effect of impurities (as proposed in Ref. [9]), it may be also the effect of collective excitations (e.g., the influence of vertex corrections on the T dependence of MPD was discussed in Ref. [10] for the case of pure s-wave pairing).

A famous reformulation of the third law of thermodynamics states that it is impossible to reach absolute zero. From this point of view a pure $d_{x^2-y^2}$-pairing symmetry in clean high-T_c superconductors becomes, perhaps, invalid for $T \to 0$. A possibility to avoid the paradox of a linear T dependence of MPD for $T \to 0$ in cuprate superconductors is a *phase transition* (at a temperature T_{c2} much lower than the transition temperature T_c) to a new unconventional pairing state *without* nodes on the Fermi surface [11,12].

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