Overcoming Intrinsic Spin Resonances with an rf Dipole

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(Received 18 December 1997)

A coherent spin resonance excited by an rf dipole was used to overcome depolarization due to intrinsic spin resonances at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory. We found that our data are consistent with a full spin flip of a polarized proton beam, without emittance growth, at $G\gamma = 12 + \nu_z$ and $36 - \nu_z$, by adiabatically exciting a vertical coherent betatron oscillation using a single rf dipole magnet. The interference pattern observed between the intrinsic spin resonance and the coherent spin resonance agrees well with multiparticle spin simulations based on a simple two-resonance model. The interference pattern can be used for beam diagnostics. [S0031-9007(98)06178-X]

PACS numbers: 29.27.Hj, 41.85.–p

In a planar synchrotron, the vertical magnetic guide field causes the polarization vector of a polarized proton beam to precess about the vertical axis $G\gamma$ turns per revolution, where $G = (g - 2)/2 = 1.7928474$ is the proton anomalous magnetic g factor, and γ is the relativistic Lorentz factor. Thus $G\gamma$ is called the spin tune of polarized protons in a synchrotron. On the other hand, horizontal magnetic fields such as those from a tilted dipole magnet, and vertical orbital motion through quadrupoles, can alter the polarization vector away from the vertical direction. When a spin resonance condition is encountered, this effect adds coherently and causes depolarization. Important spin resonances are classified as imperfection resonances resulting from vertical closed orbit errors or intrinsic resonances resulting from vertical betatron motion [1].

In medium or low energy accelerators, imperfection resonances can be corrected by using harmonic orbit correctors [2], or by using a partial Siberian snake [3–5]. A 5% partial snake has been successfully implemented at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory to overcome the imperfection resonances that occur at $G\gamma =$ integers. However, the strength of the 5% partial snake is too weak to overcome the intrinsic spin resonances at $G\gamma = kP \pm m\nu_z$, where k and m are integers, $P = 12$ is the superperiodicity, and $\nu_z \approx 8.70$ is the vertical betatron tune for the AGS [6].

During acceleration of polarized protons in the AGS up to 25 GeV/c, 7 intrinsic spin resonances at $0 + \nu_z$, $24 - \nu_z$, $12 + \nu_z$, $36 - \nu_z$, $24 + \nu_z$, $48 - \nu_z$, and $36 + \nu_z$ are encountered. The resonance strength ϵ_k is defined as the Fourier amplitude of the spin perturbing field. When a polarized beam is uniformly accelerated through such an isolated spin resonance, the final polarization P_f is related to the initial polarization P_i by the Froissart-Stora formula [7]

$$P_f = (2e^{-\pi|\epsilon_K|^2/2\alpha} - 1)P_i\,, \tag{1}$$

where α is the resonance crossing rate given by

$$\alpha = \frac{d(G\gamma - kP \mp m\nu_z)}{d\theta}\,, \tag{2}$$

and θ is the orbital angle in the synchrotron. At the nominal fast acceleration rate at the AGS, four strong spin resonances at $0 + \nu_z$, $12 + \nu_z$, $36 - \nu_z$, and $36 + \nu_z$ are the most harmful ones to the beam polarization.

Since the intrinsic spin resonance strength is proportional to the betatron amplitude, the final polarization is an ensemble average of the Froissart-Stora formula over the betatron amplitude of the beam particles. Assuming a Gaussian beam distribution, the final polarization becomes

$$\langle P_f\rangle = \left(\frac{1 - \pi|\epsilon_{\rm rms}|^2/\alpha}{1 + \pi|\epsilon_{\rm rms}|^2/\alpha}\right)\langle P_i\rangle\,, \tag{3}$$

where $\epsilon_{\rm rms}$ is the spin resonance strength corresponding to an rms emittance [1]. It is difficult to achieve a full spin flip for all particles since the resonance strength of the beam core is small. Furthermore, the spin flip method is not applicable to many weak spin resonances driven by horizontal and vertical coupling, synchrotron motion, field gradient errors, and other sources.

Alternatively, if the beam is kicked to induce a coherent betatron oscillation so that the betatron oscillation amplitudes of all particles are large, a full spin flip can be attained [8–10]. An experiment using a pulsed dipole to increase the beam's vertical betatron amplitude was performed at the IUCF Cooler Ring and demonstrated a sharp

0031-9007/98/80(21)/4673(4)$15.00

spin flip due to the strengthening of the intrinsic spin resonance. However, this method is a nonadiabatic process and causes an emittance growth. To eliminate the problem of the emittance growth, a coherent betatron motion can be adiabatically excited and maintained by an rf dipole (see Ref. [11], in which the rf dipole was referred to as an ac dipole). Essentially, the rf dipole field and the focusing potential of the accelerator form a potential well that preserves the emittance of the beam. Such a controlled coherent betatron oscillation can be obtained by using an rf dipole magnet operating at a frequency close to a betatron sideband. This paper reports the results from recent polarized proton beam experiments at the AGS in July 1997, where an rf dipole was used to induce coherent betatron oscillations for achieving a full spin flip during the acceleration of polarized beam, when an intrinsic spin resonance was encountered.

In a linear approximation, the amplitude of the coherent betatron motion is given by

$$Z_{\rm coh} = \frac{\Theta}{4\pi\delta}\beta_z\,, \tag{4}$$

where Θ is the maximum beam deflection angle from the rf dipole, β_z is the vertical betatron function at the rf dipole, and δ is the difference between the rf dipole tune and the tune of the nearest betatron sideband. Equation (4) shows that the beam is unstable at $\delta = 0$.

Since the betatron coordinate can be expressed as the linear combination of the vertical intrinsic and induced coherent betatron motion [11], the polarized protons experience not only the intrinsic spin resonance, but also a coherent spin resonance at the rf dipole frequency. The resulting polarization, in the limiting case with $\delta = 0$, is given by

$$\left\langle\frac{P_f}{P_i}\right\rangle = \frac{2}{1+\pi|\epsilon_{\rm rms}|^2/\alpha} \times \exp\left\{-\frac{(Z_{\rm coh}^2\hat{\beta}_z/2\beta_z\sigma_z^2)\,(\pi|\epsilon_{\rm rms}|^2/\alpha)}{1+\pi|\epsilon_{\rm rms}|^2/\alpha}\right\} - 1\,, \tag{5}$$

and in the case with δ much larger than the intrinsic spin resonance strength, by

$$\left\langle\frac{P_f}{P_i}\right\rangle = \frac{1-\pi|\epsilon_{\rm rms}|^2/\alpha}{1+\pi|\epsilon_{\rm rms}|^2/\alpha} \times \left(2\exp\left\{-\frac{Z_{\rm coh}^2\hat{\beta}_z}{\beta_z\sigma_z^2}\,\frac{\pi|\epsilon_{\rm rms}|^2}{2\alpha}\right\} - 1\right). \tag{6}$$

Here $\hat{\beta}_z$ is the maximum vertical betatron function in the accelerator, and σ_z is the rms beam size. The more interesting and relevant situation lies between these two cases where rich interference patterns are produced [12]. A multiparticle simulation based on a two spin resonance model was developed to numerically analyze this situation.

In the AGS polarized beam experiments, polarized beam from an atomic H^- source was accelerated to 200 MeV through the LINAC and strip injected into the booster. At the end of the LINAC, the beam polarization was measured by a 200 MeV polarimeter. In the booster, the polarized beam was accelerated to 1.56 GeV or $G\gamma = 4.7$, just below the booster vertical betatron tune which was set at 4.9 to avoid an intrinsic spin resonance. The imperfection resonance at $G\gamma = 4$ was corrected by harmonic orbit correction dipole magnets. The polarized beam was injected into the AGS at $G\gamma = 4.7$, where the polarization was 0.77 ± 0.05, and the polarized beam intensity was about 5×10^9 polarized protons per pulse [5].

In the AGS, the polarization of the circulating beam was measured with the AGS internal polarimeter, which measured the left-right asymmetry of p-p elastic scattering off a nylon fish-line ($C_6H_{11}NO$) target. The background from the quasielastic scattering was estimated by measuring the asymmetry from a carbon fiber target, and was subtracted from the asymmetry measurement with the nylon target to obtain the p-p elastic scattering asymmetry. The beam polarization was calculated from the measured asymmetry normalized by the effective analyzing power for the nylon target, which was obtained from the analyzing power for the p-p elastic scattering data [13]. The beam emittance was measured by an ionization profile monitor (IPM) [14] as a function of time during the AGS cycle of 2.5 s.

The 5% partial Siberian snake was ramped in the AGS to produce a full spin flip at every integer $G\gamma$ [5]. The present experiment employed an rf dipole to overcome the intrinsic spin resonances at $0 + \nu_z$, $12 + \nu_z$, and $36 - \nu_z$. A ferrite dipole magnet was tuned with a parallel capacitor to about 108 kHz which matched a sideband of the vertical betatron tune. The vertical betatron function at the rf dipole location was about $\beta_z = 16.6$ m. The maximum strength was about 2×10^{-3} T m. Between the $0 + \nu_z$ resonance and the $36 + \nu_z$ resonance, the revolution frequency changes from about 363.8 to 371.5 kHz. The AGS horizontal and vertical betatron tunes were set at 8.85 and 8.7, respectively. At each of the three strong intrinsic spin resonances, the rf dipole was linearly ramped up to its full amplitude in 1000 revolutions before the intrinsic spin resonance was encountered, and was kept at this amplitude level for 1000 revolutions during which the spin resonance was crossed. It was then ramped back to zero in another 1000 revolutions. The acceleration rate was $\alpha = 4.8 \times 10^{-5}$ rad^{-1}. In order to keep a fixed modulation tune, the frequency of the rf dipole excitation was phase locked to the AGS rf frequency.

The polarization was measured at a fixed energy flattop above each intrinsic resonance. The measurement flattops were set at $G\gamma = 13.5$, 24.5, and 30.5, respectively. The parameters varied in the experiment were the rf dipole field strength and the separation of the modulation tune from the vertical intrinsic betatron tune. Since the rf dipole is a narrow band magnet tuned at 108 kHz, the rf dipole modulation tune was fixed and the vertical betatron tune was varied to obtain different tune separation.

Figure 1 shows the measured polarization at three energies versus the rf dipole strength, which is converted to the corresponding coherent betatron amplitude. The overall systematic scale error of the beam polarization measurement was estimated to be 0.10 [5], and the statistical error for each point was ± 0.03. The lines shown on the figure correspond to results obtained from numerical spin simulations of a two spin resonance model. The oscillatory behavior of the simulation result is due to the interference between the coherent betatron oscillations and the intrinsic betatron motion. The spin vector of each particle was tracked by multiplying its turn by turn transform matrix. The beam polarization was then obtained from the spin ensemble average of a Gaussian beam distribution. A scaling factor was introduced to convert the simulation results to the corresponding asymmetry measured by the polarimeter. This factor is a combination of the initial polarization and the effective analyzing power.

The simulations for the $12 + \nu_z$ and $36 - \nu_z$ resonances were done with the measured betatron tune. For the $0 + \nu_z$ resonance, where we did not obtain an accurate measurement of betatron tune, the tune separation was obtained by fitting to the data. The best fit corresponded to a tune separation of $\delta = 0.0063 \pm 0.0014$. The beam emittance used in the numerical simulations was obtained from the measured depolarization with the rf dipole off at $G\gamma = 13.5$, 24.5, and 30.5. From Eq. (3), the normalized 95% beam emittance was $36 \pm 13\pi$ mm mrad for the $0 + \nu_z$ resonance and $26 \pm 4\pi$ mm mrad for the $12 + \nu_z$ and $36 - \nu_z$ resonances. This is also consistent with beam profile measurements made with the AGS IPM. However, the IPM measurements were not very reliable due to the low beam intensity. The lower beam emittance for the $12 + \nu_z$ and $36 - \nu_z$ resonances was attributed to a more careful machine tuning later in the experiment.

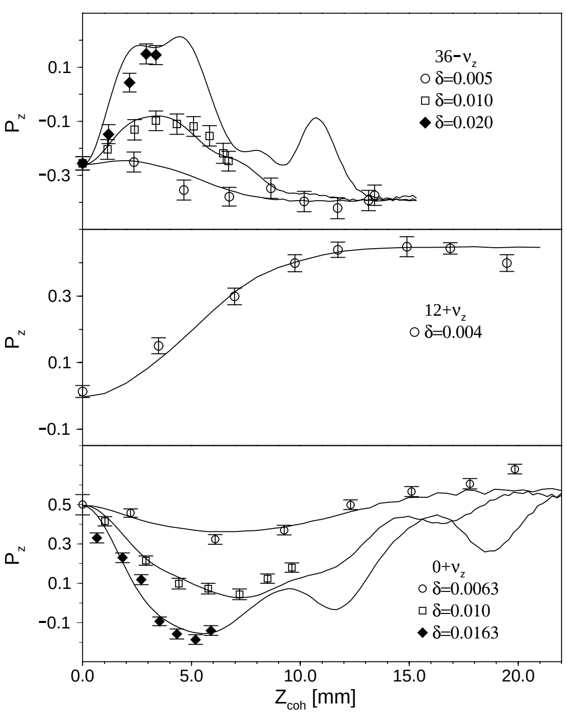

FIG. 1. The measured proton polarization vs the coherent betatron oscillation amplitude (in mm) for different tune separations at spin depolarizing resonances $0 + \nu_z$ (bottom plot), $12 + \nu_z$ (middle plot), and $36 - \nu_z$ (upper plot). Pz stands for the vertical polarization, while Z_{coh} stands for the vertical coherent oscillation amplitude. The error bars show only the statistical errors. The resonance strength of the coherent spin resonance due to the rf dipole is proportional to the coherent betatron amplitude. The lines are the results of multiparticle spin simulations based on a model with two overlapping spin resonances.

Since the spin resonance at $12 + \nu_z$ was relatively weak compared with the other three resonances, it is easier for the rf-induced spin resonance to dominate, giving a smooth dependence of the measured polarization on the rf dipole field strength shown in the middle plot of Fig. 1. On the other hand, since the intrinsic spin resonances at $0 + \nu_z$ and $36 - \nu_z$ were strong enough to partially flip the spin, they strongly interfered with the coherent spin resonance induced by the rf dipole. In agreement with the numerical simulation, the upper and lower plots of Fig. 1 show complicated interference patterns when the tune separation is large. Nevertheless, a full spin flip can eventually be obtained when the strength of the rf-induced spin resonance becomes strong enough.

Since the vertical betatron tune, in this experiment, was not set as its optimal value at the $0 + \nu_z$ resonance for the polarization measurement at higher energy flattops, there was a 15%–20% polarization loss. This was reflected in a lower polarization value at the higher energies for the $12 + \nu_z$ and $36 - \nu_z$ spin resonances. This polarization loss at the $0 + \nu_z$ resonance can be avoided in the future by carefully measuring the betatron tune and by properly setting the vertical betatron tune and the rf dipole field strength amplitude.

Figure 2 shows the measured beam polarization for the $12 + \nu_z$ intrinsic resonance as a function of the tune separation at a fixed rf dipole field strength of 1.85×10^{-3} T m. The solid line shows the result of a multiparticle numerical simulation. When the rf dipole modulation tune is near the intrinsic betatron tune, the polarization reaches a plateau of full spin flip.

In conclusion, we have studied the method of employing an rf dipole field to overcome strong intrinsic spin resonances. We observed the predicted strong interference between intrinsic spin resonances and rf-induced spin resonances. The interference pattern depended upon the tune separation, relative phase, and relative strength of the intrinsic and rf-induced resonances. The experimental data were found to agree well with multiparticle numerical spin simulations. The data indicate that depolarization due to the intrinsic spin resonance can be avoided by the coherent betatron motion induced by an rf dipole.

In general, the application of such a method requires a small beam emittance, because the corresponding intrinsic

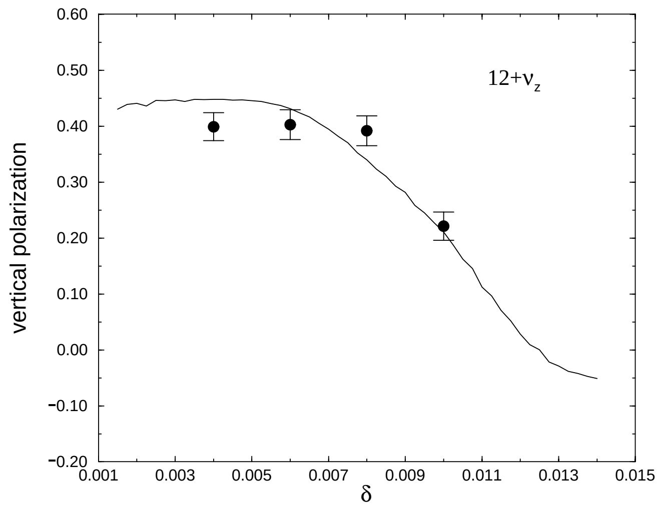

FIG. 2. The measured proton beam polarization, at the spin resonance $12 + \nu_z$, vs the tune separation with a fixed rf dipole field strength of 1.85×10^{-3} T m. The line shows the results of multiparticle spin simulations with a normalized 95% beam emittance of 26π mm mrad, and an initial polarization of 0.45.

spin resonance would be weaker and therefore it would be easier for the coherent spin resonance to dominate and reach full spin flip. The spin tune variation and spread are determined by beam energy variation and spread. For this method, it is only important that the energy of any individual proton does not deviate significantly from the linear energy ramp during the passage through the artificial and intrinsic resonance. This condition was easily fulfilled for the fast acceleration rate of $\alpha = 4.8 \times 10^{-5}$ rad^{-1}. To maintain the stability of the excited coherent oscillation, the variation of the betatron tune should be a small fraction of the resonance proximity parameter δ. In the AGS, the betatron tune was stable to about ± 0.001 which proved to be sufficient for the rf dipole operation. For crossing the $12 + \nu_z$ and $36 - \nu_z$ resonances, the tune spread of the beam was adequate during this experiment. However, it should be improved for crossing the $0 + \nu_z$ resonance in order to ease the beam aperture requirement. To achieve this, the chromaticity and the nonlinear betatron detuning will need to be minimized carefully.

The authors thank D. Gassner, T. Kasprzyk, G. J. Mahler, S. Naase, J. Reich, P. Sampson, R. Sanders, G. Smith, C. Whalen, and K. Zeno for their help during the experiment. The research was supported by grants from the U.S. Department of Energy, Division of High Energy Physics Grant No. DE-FG02-92ER40747 and Contract No. W-31-109-ENG-38.

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