Long-Distance Contributions to the $K_L \to \mu^+\mu^-$ Decay Width

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The dispersive two-photon contribution to the $K_L \to \mu^+\mu^-$ decay amplitude is analyzed, using chiral perturbation theory techniques and large-N_C considerations. A consistent description of the decays $\pi^0 \to e^+e^-$, $\eta \to \mu^+\mu^-$, and $K_L \to \mu^+\mu^-$ is obtained. As a by-product, one predicts $B(\eta \to e^+e^-) = (5.8 \pm 0.2) \times 10^{-9}$ and $B(K_L \to e^+e^-) = (9.0 \pm 0.4) \times 10^{-12}$. [S0031-9007(98)06126-2]

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The rare decay $K_L \to \mu^+\mu^-$ has deserved a significant theoretical interest during the last three decades. It represents a potentially important channel to study the weak interaction within the standard model (SM), as well as possible effects of new physics, mainly in connection with flavor-changing neutral currents and CP violation.

This decay proceeds through two distinct mechanisms: a long-distance contribution from the 2γ intermediate state and a short-distance part, which in the SM arises from one-loop diagrams (*W boxes, Z penguins*) involving the weak gauge bosons. Since the short-distance amplitude is sensitive to the presence of a virtual top quark, it could be used to improve our present knowledge on the quark-mixing factor V_{td}; moreover, it offers a window into new-physics phenomena. This possibility has renewed the interest in the study of the $K_L \to \mu^+\mu^-$ process in the past years.

The short-distance SM amplitude is well known [1]. Including QCD corrections at the next-to-leading logarithm order [2], it implies [3]

$$B(K_L \to \mu^+\mu^-)_{\rm SD} = 0.9 \times 10^{-9}(\rho_0 - \bar\rho)^2 \times \left(\frac{\overline{m}_t(m_t)}{170\ {\rm GeV}}\right)^{3.1}\left(\frac{|V_{cb}|}{0.040}\right)^4, \quad (1)$$

where $\rho_0 \approx 1.2$ and $\bar\rho \equiv \rho(1 - \lambda^2/2)$, with ρ and λ the usual quark-mixing parameters, in the Wolfenstein parametrization [4]. The deviation of ρ_0 from 1 is due to the charm contribution. Using the presently allowed ranges for m_t and the quark-mixing factors, one gets [3] $B(K_L \to \mu^+\mu^-)_{\rm SD} = (1.2 \pm 0.6) \times 10^{-9}$. If this number is compared with the measured rate [5]

$$B(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \times 10^{-9}, \quad (2)$$

it is seen that the decay process is strongly dominated by the long-distance amplitude.

Clearly, in order to extract useful information about the short-distance dynamics it is first necessary to have an accurate (and reliable) determination of the $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$ contribution.

It is convenient to consider the normalized ratios

$$R(P \to l^+l^-) \equiv \frac{B(P \to l^+l^-)}{B(P \to \gamma\gamma)} = 2\beta\left(\frac{\alpha}{\pi}\frac{m_l}{M_P}\right)^2 |F(P \to l^+l^-)|^2, \quad (3)$$

where $\beta \equiv \sqrt{1 - 4m_l^2/M_P^2}$. The on-shell 2γ intermediate state generates the absorptive contribution [6]

$$\mathrm{Im}[F(P \to l^+l^-)] = \frac{\pi}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right). \quad (4)$$

Using the measured branching ratio [7], $B(K_L \to \gamma\gamma) = (5.92 \pm 0.15) \times 10^{-4}$, this implies the so-called *unitarity bound*

$$B(K_L \to \mu^+\mu^-) \geq B(K_L \to \mu^+\mu^-)_{\rm abs} = (7.07 \pm 0.18) \times 10^{-9}. \quad (5)$$

Comparing this result with the experimental value in Eq. (2), we see that $B(K_L \to \mu^+\mu^-)$ is almost saturated by the absorptive contribution.

One immediate question is whether the small room left for the dispersive contribution, $B(K_L \to \mu^+\mu^-)_{\rm dis} = (0.1 \pm 0.5) \times 10^{-9}$, can be understood dynamically. Naively, one would expect a larger value just from the intermediate 2γ mechanism. This has motivated some recent speculations [8] about a possible cancellation between the long- and short-distance dispersive amplitudes, which could allow for additional new-physics contributions at short distances.

The obvious theoretical framework to perform a well-defined analysis of the long-distance part is chiral perturbation theory (ChPT). Unfortunately, the chiral symmetry constraints are not powerful enough to make an accurate determination of the dispersive contribution [9–11]. The problem can easily be understood by looking at the $K_L \to \gamma\gamma$ amplitude,

$$A(K_L \to \gamma\gamma) = c(q_1^2, q_2^2)\varepsilon^{\mu\nu\rho\sigma}\epsilon_{1\mu}\epsilon_{2\nu}q_{1\rho}q_{2\sigma}, \quad (6)$$

which, at lowest order in momenta, proceeds through the chain $K_L \to \pi^0, \eta, \eta' \to 2\gamma$. The lowest-order—$O(p^4)$—chiral prediction, can only generate a constant form factor $c(q_1^2, q_2^2)$; it thus corresponds to the decay into on-shell photons ($q_1^2 = q_2^2 = 0$) [12],

$$c(0,0) = \frac{2G_8 \alpha f_\pi}{\pi(1 - r_\pi^2)} c_{\rm red}\,, \tag{7}$$

$$c_{\rm red} = 1 - \frac{(1 - r_\pi^2)}{3(r_\eta^2 - 1)}(c_\theta - 2\sqrt{2}\, s_\theta)(c_\theta + 2\sqrt{2}\,\rho_n s_\theta) + \frac{(1 - r_\pi^2)}{3(r_{\eta'}^2 - 1)}(2\sqrt{2}\, c_\theta + s_\theta)(2\sqrt{2}\,\rho_n c_\theta - s_\theta)\,,$$

where $r_P^2 \equiv M_P^2/M_{K_L}^2$, $c_\theta \equiv \cos\theta_P$, and $s_\theta \equiv \sin\theta_P$, with $\theta_P \approx -20°$ the η-η' mixing angle. The global parameter $G_8 \equiv 2^{-1/2} G_F V_{ud} V_{us}^* g_8$ characterizes [13] the strength of the weak $\Delta S = 1$ transition $K_L \to \pi^0, \eta, \eta'$.

In Eq. (7) we have factored out the contribution of the pion pole, which normalizes the dimensionless reduced amplitude $c_{\rm red}$. The second and third terms in $c_{\rm red}$ correspond to the η and η' contributions, respectively. Nonet symmetry (which is exact in the large-N_C limit) has been assumed in the electromagnetic 2γ vertices; this is known to provide a quite good description of the anomalous $P \to 2\gamma$ decays ($P = \pi^0, \eta, \eta'$). Possible deviations of nonet symmetry in the nonleptonic weak vertex are parametrized through $\rho_n \neq 1$.

In the standard $SU(3)_L \otimes SU(3)_R$ ChPT, the η' contribution is absent and $\theta_P = 0$; therefore, $c_{\rm red} \propto (3M_\eta^2 + M_\pi^2 - 4M_K^2)$, which vanishes owing to the Gell-Mann–Okubo mass relation. The physical $K_L \to \gamma\gamma$ amplitude is then a higher-order—$O(p^6)$—effect in the chiral counting, which makes it difficult to perform a reliable calculation.

The situation is very different if one uses instead a $U(3)_L \otimes U(3)_R$ effective theory [14], including the singlet η_1 field. The large mass of the η' originates in the $U(1)_A$ anomaly which, although formally of $O(1/N_C)$, is numerically important. Thus, it makes sense to perform a combined chiral expansion [15] in powers of momenta and $1/N_C$, around the nonet-symmetry limit, but keeping the anomaly contribution (i.e., the η_1 mass) together with the lowest-order term. In fact, the usual successful description of the $\eta/\eta' \to 2\gamma$ decays [16] corresponds to the lowest-order contribution within this framework, plus some amount of symmetry breaking through $f_\eta \neq f_{\eta'} \neq f_\pi$. The mixing between the η_8 and η_1 states provides a large enhancement of the $\eta \to 2\gamma$ amplitude, which is clearly needed to understand the data. In the standard $SU(3)_L \otimes SU(3)_R$ framework, the η' is integrated out, and its effects are hidden in higher-order local couplings [17]; since the η_1 and η_8 fields share the same isospin and charge, the singlet pseudoscalar does affect the η dynamics in a significant way, which is reflected in the presence of important higher-order corrections [18]. These corrections are more efficiently taken into account within the $U(3)_L \otimes U(3)_R$ framework [19].

Taking $s_\theta = -1/3$ ($\theta \approx -19.5°$), the η-pole contribution in Eq. (7) is proportional to $(1 - \rho_n)$ and vanishes in the nonet-symmetry limit; the large and positive η' contribution results then in $c_{\rm red} = 1.80$ for $\rho_n = 1$. With $0 \leq \rho_n \leq 1$, the η and η' contributions interfere destructively and $c_{\rm red}$ is dominated by the pion pole. One would get $c_{\rm red} \simeq 1$ for $\rho_n \simeq 3/4$.

The measured $K_L \to \gamma\gamma$ decay rate [7] corresponds to $|c(0,0)| = (3.51 \pm 0.05) \times 10^{-9}$ GeV^{-1}. With $|G_8| = 9.1 \times 10^{-6}$ GeV^{-2}, obtained from the $O(p^2)$ analysis of $K \to 2\pi$ [13], this implies $c_{\rm red}^{\rm exp} = (0.84 \pm 0.11)$. However, the fitted value of $|G_8|$ gets reduced by about 30% when $O(p^4)$ corrections to the $K \to 2\pi$ amplitudes are taken into account [21]. This sizable shift results mainly from the constructive $\pi\pi$ rescattering contribution, which is obviously absent in $K_L \to \gamma\gamma$. Thus, we should rather use the corrected (smaller) $|G_8|$ determination, which leads to $c_{\rm red}^{\rm exp} = (1.19 \pm 0.16)$.

Leaving aside numerical details, we can safely conclude that the physical $K_L \to \gamma\gamma$ amplitude, with on-shell photons, is indeed dominated by the pion pole ($c_{\rm red} \sim 1$). Although the exact numerical prediction is sensitive to several small corrections [22] ($\rho_n \neq 1$, $f_\pi \neq f_\eta \neq f_{\eta'}$, $s_\theta \neq -1/3$) and therefore is quite uncertain, the needed cancellation between the η and η' contributions arises in a natural way and can be fitted easily with a reasonable choice of symmetry-breaking parameters.

The description of the $K_L \to \gamma\gamma$ transition with off-shell photons is *a priori* more complicated because the $q_{1,2}^2$ dependence of the form factor originates from higher-order terms in the chiral Lagrangian. This is the reason why only model-dependent estimates of the dispersive $K_L \to l^+ l^-$ transition amplitude have been obtained so far. At lowest order in momenta, $c(q_1^2, q_2^2) = c(0,0)$; thus, the (divergent) photon loop can be explicitly calculated up to a global normalization, which is determined by the known absorptive contribution [i.e., by the experimental value of $c(0,0)$]. The model dependence appears in the local contributions from direct $K_L l^+ l^-$ terms in the chiral Lagrangian [9,11] (allowed by symmetry considerations), which reabsorb the loop divergence.

It would be useful to have a reliable determination in some symmetry limit. The large-N_C description of $K_L \to \gamma^*\gamma^*$ provides such a possibility. At leading order, this process occurs through the π^0, η, η' poles, as represented in Fig. 1. Therefore, the problematic electromagnetic loop in Fig. 1(a) is actually the same governing the decays $\pi^0 \to e^+ e^-$ and $\eta \to \mu^+ \mu^-$, and the unknown local contribution [Fig. 1(b)] can be fixed from the measured rates for these transitions [23,24]. In fact, the same combination of local chiral couplings shows up in both decays [23], leading to a relation that is well satisfied by the data.

Although the $\eta' \to l^+ l^-$ transition introduces additional chiral couplings, they are suppressed by at least

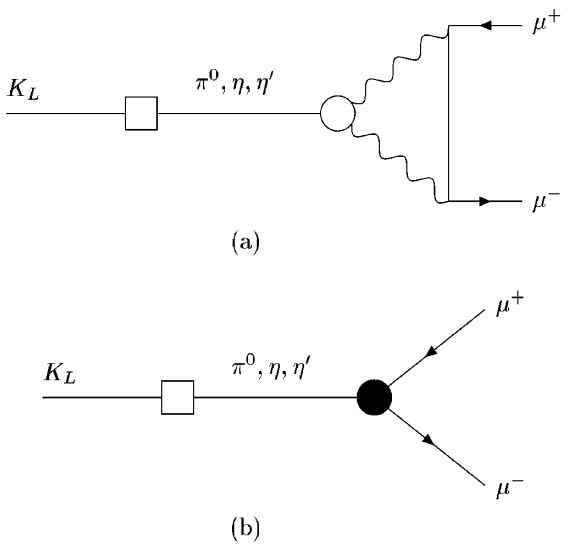

FIG. 1. (a) Photon loop and (b) associated counterterm contributions to the $K_L \to \mu^+\mu^-$ process.

one more power of $1/N_C$. Thus, in the large-N_C limit the different electromagnetic $P \to l^+l^-$ decays get related through the same counterterms [23],

$$\mathcal{L}_{\text{c.t.}} = \frac{3i\alpha^2}{32\pi^2}(\bar{l}\gamma^\mu\gamma_5 l) \times \{\chi_1 \,\text{Tr}(Q^2\{U^\dagger, \partial_\mu U\}) + \chi_2 \,\text{Tr}(QU^\dagger Q\partial_\mu U - Q\partial_\mu U^\dagger QU)\}, \quad (8)$$

where $Q \equiv \text{diag}(2/3, -1/3, -1/3)$ is the quark electromagnetic charge matrix and $U \equiv \exp(i\sqrt{2}\,\Phi/f)$ the usual $\text{U}(3)_L \otimes \text{U}(3)_R$ matrix describing the pseudoscalar nonet.

Nonet symmetry should provide a good estimate of the ratio $R(K_L \to l^+l^-)$. Since $K_L \to \gamma\gamma$ is dominated by the pion pole, we can reasonably expect that symmetry-breaking corrections would play a rather small role. In any case, this symmetry limit allows us to investigate whether the tiny dispersive contribution allowed by the data is what should be expected from the 2γ intermediate state.

In this limit, all $R(P \to l^+l^-)$ ratios are governed by the same dispersive amplitude [25],

$$\text{Re}[F(P \to l^+l^-)] = \frac{1}{4\beta}\ln^2\!\left(\frac{1-\beta}{1+\beta}\right) + \frac{1}{\beta}\,\text{Li}_2\!\left(\frac{\beta-1}{\beta+1}\right) + \frac{\pi^2}{12\beta} + 3\ln\!\left(\frac{m_l}{\mu}\right) + \chi(\mu), \quad (9)$$

where $\chi(\mu) \equiv -[\chi_1^r(\mu) + \chi_2^r(\mu) + 14]/4$ is the relevant local contribution, with $\chi_i^r(\mu)$ $(i = 1, 2)$ the corresponding chiral couplings renormalized in the $\overline{\text{MS}}$ scheme. The μ dependence of the $\chi(\mu)$ and $\ln(m_l/\mu)$ terms compensate each other, so that the total amplitude is μ independent.

Table I shows the fitted values of $\chi(M_\rho)$ from the three measured ratios $R(\pi^0 \to e^+e^-)$, $R(\eta \to \mu^+\mu^-)$, and $R(K_L \to \mu^+\mu^-)$. Subtracting the known absorptive contribution, the experimental data provide two possible solutions for each ratio; they correspond to a total positive (solution 1) or negative (solution 2) dispersive amplitude. We see from the table that the second solution from the decay $\pi^0 \to e^+e^-$ is clearly ruled out; owing to the smallness of the electron mass, the logarithmic loop contribution dominates the dispersive amplitude, which has then a definite positive sign [an unnaturally large and negative value of $\chi(M_\rho)$ is needed to make it negative]. The large experimental errors do not allow one to discard at this point any of the other solutions: the remaining value from $\pi^0 \to e^+e^-$ is consistent with the results from the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ decays, and these are also in agreement with each other if the same solution (either the first or the second) is taken for both. We see that, in any case, the three experimental ratios are well described by a common value of $\chi(M_\rho)$. In this way, the experimentally observed small dispersive contribution to the $K_L \to \mu^+\mu^-$ decay rate fits perfectly well within the large-N_C description of this process.

We have not considered up to now the short-distance contribution to the $K_L \to \mu^+\mu^-$ decay amplitude [3]. This can be done through a shift of the effective $\chi(M_\rho)$ value [27],

$$\chi(M_\rho)_{\text{eff}} = \chi(M_\rho) - \delta\chi_{\text{SD}} \quad (10)$$

$$\delta\chi_{\text{SD}} \approx 1.7(\rho_0 - \bar{\rho})\left(\frac{\overline{m}_t(m_t)}{170\ \text{GeV}}\right)^{1.56}\left(\frac{|V_{cb}|}{0.040}\right)^2.$$

For the allowed range $|\bar{\rho}| \le 0.3$, one has $\delta\chi_{\text{SD}} \approx 1.8 \pm 0.6$, which allows one to exclude solution 2 for $\chi(M_\rho)$ obtained from $\eta \to \mu^+\mu^-$. Solution 1, on the contrary, is found to be compatible with the results from $K_L \to \mu^+\mu^-$, and can be used to get a constraint for $\delta\chi_{\text{SD}}$. Indeed, taking as the best determination

$$\chi(M_\rho) = 5.5^{+0.8}_{-1.0}, \quad (11)$$

the first solution for $K_L \to \mu^+\mu^-$ leads to

$$\delta\chi_{\text{SD}} = 2.2^{+1.1}_{-1.3}, \quad (12)$$

in agreement with the $\delta\chi_{\text{SD}}$ value quoted above. The second solution for $K_L \to \mu^+\mu^-$ appears to be less favored, yielding $\delta\chi_{\text{SD}} = 3.6 \pm 1.2$; this shows a discrepancy of about 1.4σ with the short-distance estimate. Notice

TABLE I. Fitted values of $\chi(M_\rho)$ from different $R(P \to l^+l^-)$ ratios. The numbers quoted for $K_L \to \mu^+\mu^-$ refer to the difference $\chi(M_\rho) - \delta\chi_{\text{SD}}$.

	$\chi(M_\rho)$ [Solution 1]	$\chi(M_\rho)$ [Solution 2]
$\pi^0 \to e^+e^-$	4^{+4}_{-6}	-24 ± 5
$\eta \to \mu^+\mu^-$	$5.5^{+0.8}_{-1.0}$	$-0.8^{+1.0}_{-0.8}$
$K_L \to \mu^+\mu^-$	$3.3^{+0.9}_{-0.7}$	$1.9^{+0.7}_{-0.9}$

that the precision of the result in (12) is still relatively low. However, the errors could be reduced by improving the measurements of the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ branching ratios.

Finally, once the local contribution to the $P \to l^+l^-$ decay amplitude has been fixed, it is possible to obtain definite predictions for the decays into e^+e^- pairs,

$$\begin{aligned} B(\pi^0 \to e^+e^-) &= (8.3 \pm 0.4) \times 10^{-8}, \\ B(\eta \to e^+e^-) &= (5.8 \pm 0.2) \times 10^{-9}, \\ B(K_L \to e^+e^-) &= (9.0 \pm 0.4) \times 10^{-12}. \end{aligned} \tag{13}$$

In the same way, the amplitudes corresponding to the η' decays are found to be $B(\eta' \to e^+e^-) = (1.5 \pm 0.1) \times 10^{-10}$ and $B(\eta' \to \mu^+\mu^-) = (2.1 \pm 0.3) \times 10^{-7}$. However, in view of the large mass of the η', these predictions could receive important corrections from higher-order terms in the chiral Lagrangian.

To summarize, in the nonet-symmetry limit it is possible to make a reliable determination of the ratios $R(P \to l^+l^-)$, at lowest nontrivial order in the chiral expansion. A consistent picture of all measured $P \to l^+l^-$ decay modes is obtained, within the SM. The present data allow one to pin down the size of the relevant chiral counterterm and to get a constraint on the short-distance contribution to the $K_L \to \mu^+\mu^-$ amplitude. However, this constraint is found to be rather weak; more precise measurements of the $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$ branching ratios would be necessary in order to improve the bounds for $\delta\chi_{\rm SD}$ obtained through the SM box and penguin computations. In addition, a more detailed investigation of the theoretical uncertainties is needed to quantify how precisely the short-distance $K_L \to \mu^+\mu^-$ amplitude can be inferred from the data. Although it seems difficult to achieve a theoretical precision good enough for making useful tests of the SM flavor-mixing structure, it is worth a try.

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