

Rare Errors in a Well-Characterized Electron Pump: Comparison of Experiment and Theory

Mark W. Keller,* John M. Martinis, and R. L. Kautz

National Institute of Standards and Technology, Boulder, Colorado 80303

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By measuring each junction of an electron pump in a single electron box configuration, we determine all quantities needed to test the standard theory of pumping error and leakage. Background charges are determined with an imprecision of $\pm 0.01e$. Electron temperature is measured to 40 mK. Measured charging energies show all junctions have nearly the same capacitance. We find agreement with theory at 140 mK, but a disagreement of many orders of magnitude at 40 mK. We suggest that the excess error and leakage at 40 mK are due to photon-assisted cotunneling processes not included in the standard theory. [S0031-9007(98)06149-3]

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The electron pump consists of a chain of metal islands separated by ultrasmall tunnel junctions, with a gate voltage coupled to each island through a capacitor [1]. At a sufficiently low temperature, the energy required to charge the small island capacitance with one electron blocks electron tunneling. By pulsing the gate voltages in sequence, this blockade can be manipulated to transfer a single electron along the chain. Electron transfer through the pump can be extremely accurate [2] and may allow a new metrological standard of capacitance based on pumping a known number of electrons onto a capacitor and measuring the resulting voltage [3]. Devices similar to the pump have been proposed as components of a new type of digital circuitry in which information is represented and/or controlled by single electrons [4]. The pump and digital devices face two common technical challenges. The first challenge is fluctuating random background charges, due to microscopic defects located in the junctions or their immediate surroundings, that degrade pump accuracy and make digital device behavior unpredictable. The second challenge is reducing unwanted tunneling processes to extremely low levels. Standard theory predicts sufficiently low levels for realistic junction parameters, but it has been tested only by current vs voltage measurements where necessarily large tunneling rates obscure rare events, or in an electron trap where it was not possible to measure important parameters such as the background charges and charging energies of the individual junctions [5]. In this Letter, we describe measurements of the background charge, electron temperature, and charging energy for each junction in a 7-junction electron pump. We use these results to make a quantitative test of the standard theory in the regime where errors while pumping, and leakage events while not pumping, are rare.

Our measurement technique is illustrated in Fig. 1(a). We create a charge bias across a junction by applying voltages of opposite polarity to the gates on either side of the junction. The unbiased junctions can be treated as ordinary capacitors because they are set far from their tunneling thresholds using fixed gate voltages [not shown in Fig. 1(a)]. We can then use the Norton equivalence theorem [6] to redraw the circuit as shown in Fig. 1(b).

The bias charge Q and the background charge Q_b appear across the junction capacitance C_j in parallel with the effective shunt capacitance C_x [7]. This equivalent circuit is identical to that of the single electron box studied by Lafarge *et al.* [8]. The junction in Fig. 1(b) has a charging energy $E_c = e^2/2(C_j + C_x)$ and will allow tunneling when the charge on C_j itself reaches the critical value $(e/2)[C_j/(C_j + C_x)]$. We prefer to discuss the system in terms of the *total junction charge* $Q + Q_b$ on the total capacitance $C_j + C_x$, with the condition for tunneling $Q + Q_b = e/2$. As Q is swept, a transition in which the total charge changes by e will occur each time the junction allows tunneling. We can measure such a transition for any junction in the pump by applying a charge bias through the appropriate gates [9]. As we show below, these transitions can be used to determine Q_b , electron temperature T , and E_c for each junction.

Our 7-junction electron pump and the circuit used to measure it have been described previously [2]. A single electron transistor/electrometer, held at its most sensitive operating point by a feedback loop, monitors the charge at one end of the pump. For the electron box measurements

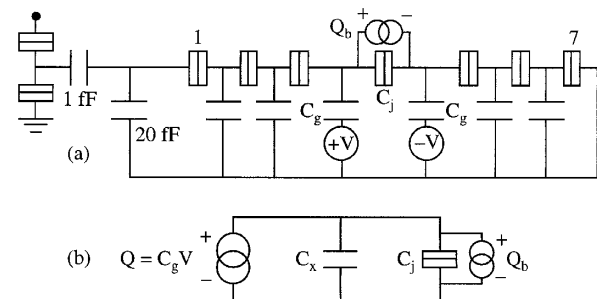


FIG. 1. (a) Schematic circuit for electron box measurement of the middle junction in a 7-junction electron pump. Double box symbols represent ultrasmall tunnel junctions. The middle junction is biased through a gate capacitor C_g on either side and has a background charge Q_b due to microscopic sources. The electrometer at the left monitors the voltage on the island at the end of the pump; the island has a stray capacitance of 20 fF. (b) Equivalent circuit showing the bias charge Q and the effective shunt capacitance C_x seen by the biased junction.

[8], we add a sinusoidal signal $\alpha \sin(\omega t)$ to the bias charge Q and detect the derivative of the electrometer feedback signal with a lock-in amplifier at 397 Hz. Details of pump design, fabrication, and operation are described elsewhere [10].

The measurement described above determines the total junction charge averaged over all possible values in thermal equilibrium [8]. The electrometer detects the fraction of this charge that couples to its 1 fF input capacitor. As Q is swept, the total junction charge increases linearly until it nears $e/2$, where the system can begin to access a new state corresponding to an electron having tunneled through the junction. As the total charge is swept past $e/2$, the probability for the system to occupy the new state increases from 0 to 1 and the total charge makes a transition from $Q + Q_b$ to $Q + Q_b - e$. Thus the derivative measured by the lock-in has a dip at each transition as shown in Fig. 2(a). We integrate this curve to obtain the average number of electrons $\langle n \rangle$ that have tunneled through the biased junction as a function of Q , shown in Fig. 2(b). In the limit $k_B T \ll E_c$, and for $Q + Q_b$ between 0 and e , the thermodynamic average of Lafarge *et al.* reduces to

$$\langle n \rangle_{0 \rightarrow 1} = \frac{1}{\exp\{[1 - 2(Q + Q_b)/e]/2\theta_0\} + 1}, \quad (1)$$

where $2\theta_0 \equiv k_B T/E_c$. We determine Q_b and the measured width 2θ by fitting Eq. (1) to the transition, as shown in the inset of Fig. 2(b).

In practice, we find Q_b directly from the dips in Fig. 2(a). The midpoint of the first transition occurs

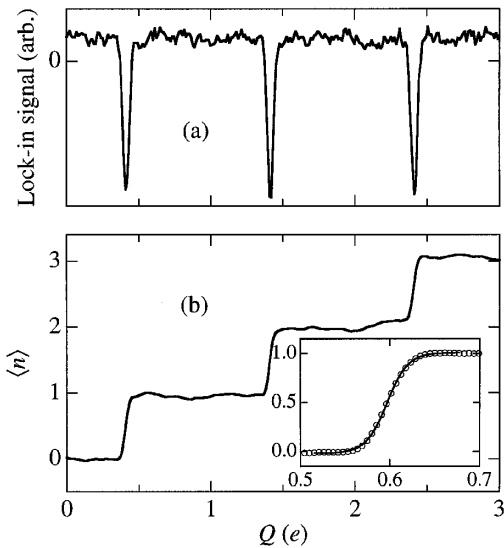


FIG. 2. (a) Lock-in signal vs Q for junction 3. The minima occur when $Q + Q_b$ is an odd-integer multiple of $e/2$. The constant signal between dips is due to direct coupling between the gates and the electrometer through stray capacitance. (b) Average number of electrons that have tunneled vs Q , obtained from (a) by subtracting the average value between dips, multiplying by -1 , integrating, and normalizing the step height to unity. Inset: Expanded view of transition showing a fit using Eq. (1).

when $Q + Q_b = 0.5e$, so if the first dip appears at $Q = 0.6e$, we infer $Q_b = -0.1e$. By measuring Q_b for each junction, we obtain the complete charge configuration of the pump. We can then change the fixed gate voltages to create a new configuration. If the fixed gate voltages are not changed, Q_b on each junction fluctuates over time due to microscopic charge sources. The fluctuation amplitude slowly decreases if the pump is kept cold. Immediately after cooling, typical fluctuations are $\geq 0.1e$ and occur several times per hour. A few weeks after cooling, typical fluctuations are much smaller and detectable changes ($\geq 0.01e$) occur about once per day.

We must remove broadening in 2θ due to the measurement in order to determine the intrinsic width $2\theta_0$ due to thermal broadening alone. Figure 3(a) shows 2θ as a function of sine wave amplitude α . Simulations of the lock-in measurement indicate that, when a hyperbola is fitted to these data, the intercept at $\alpha = 0$ is $2\theta_0$. Figure 3(b) shows $2\theta_0$ as a function of the temperature T_{mc} of the refrigerator mixing chamber, along with the best fit line through the origin representing the expected linear temperature dependence. The measured points do not deviate from the line, indicating that T is equal to T_{mc} down to 40 mK. This result also applies when the pump is operating because heating is negligible [11]. Thus our earlier observation [2] that the error and leakage rate are independent of T_{mc} below 100 mK cannot be explained by failure to cool the electrons below 100 mK.

Since T is known, we can determine E_c . For each junction, we measure $2\theta_0$ at $T_{mc} = 40, 60,$ and 80 mK, then fit $2\theta_0$ vs T_{mc} to a line constrained to include the origin. The slope of the line is k_B/E_c . The results, displayed in Fig. 4, show that E_c is slightly smaller for junctions at

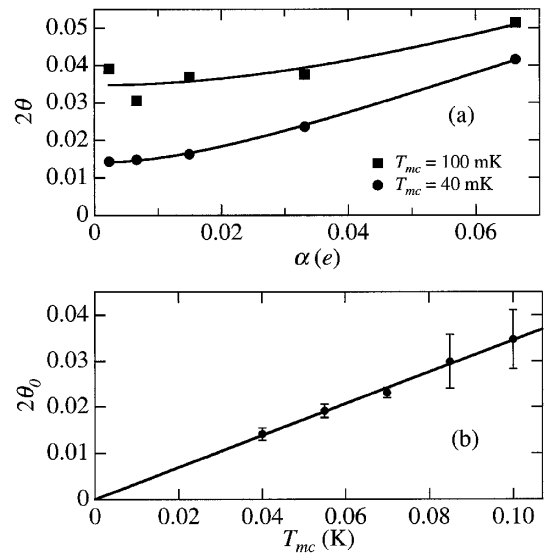


FIG. 3. (a) Transition width 2θ vs sine wave amplitude α for junction 3. The larger point scatter at 100 mK is caused by thermal smearing in the electrometer which reduces the signal-to-noise ratio. (b) Intrinsic transition width $2\theta_0$ as a function of T_{mc} for junction 3.

the ends of the pump than for those near the middle. We have calculated E_c using a circuit model in which all junctions have capacitance C_j and all other capacitance is represented by a capacitor C_{gnd} from each island to ground [12]. This model, also shown in Fig. 4, matches the data well for $C_j = 0.22$ fF and $C_{\text{gnd}} = 0.05$ fF. These values are consistent with other comparisons of the model with experiment, and also with estimates based on the geometry of the island and gate electrodes. The model reproduces the smaller values of E_c near the ends of the pump, showing that this effect is due to the different effective capacitance C_x seen by the end junctions. The comparison in Fig. 4 shows that only junctions 5 and 7 differ significantly ($\approx 10\%$) from the uniform value of C_j assumed in the model.

We now turn to the comparison between experiment and theory for error and leakage rate [13]. Previous calculations have determined the *net charge* through the pump after N pump cycles and compared it with the ideal behavior of $Q_{\text{net}} = Ne$ for error or $Q_{\text{net}} = 0$ for leakage. Significant cancellation of events in opposite directions occurs with this method. In our experiments, error and leakage events are counted *as they happen and regardless of direction*, and our calculations count events in the same way. We evaluate standard expressions for tunneling rates due to thermal activation and cotunneling [6] for two circuit models of the pump [14]. The “bare model” [6] neglects all capacitances except C_j . For the bare model, we take $C_j = 0.325$ fF, which is the average experimental value of $e^2/2E_c = C_j + C_x$ from Fig. 4. The “ C_{gnd} model” [12] is an extension of the bare model described above in the discussion of Fig. 4. For the C_{gnd} model, we take $C_j = 0.22$ fF and $C_{\text{gnd}} = 0.05$ fF from the comparison in Fig. 4. For both models, we use junction resistance $R_j = 0.47$ M Ω , voltage across the pump $V_p = 0$, $Q_b = 0$ on all junctions, and bias time of 40 ns per junction.

As described previously [2], we observe two regimes in error and leakage rate. Both quantities depend exponentially on T above 100 mK, but are independent of T below 100 mK. Thus we compare theory and experiment at 140 and 40 mK. The experimental values for error and leakage rate were measured after tuning the fixed gate voltages using the technique described previously [2,10]. For the error, we took measurements at both temperatures after tuning the fixed gate voltages for minimum error at 40 mK

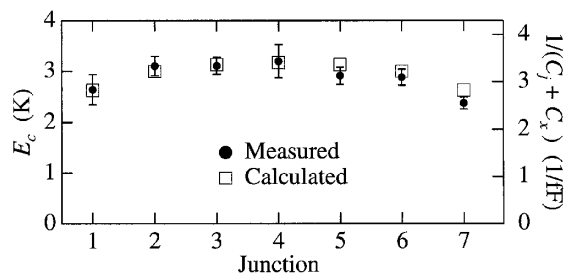


FIG. 4. Charging energy E_c for each junction. The right axis gives the corresponding total capacitance.

(retuning at 140 mK did not affect the results significantly). For the leakage rate at 140 mK, we tuned the gates for minimum leakage at 140 mK. For the leakage rate at 40 mK, the rate was too low to tune for minimum leakage, so we tuned for minimum error.

Table I shows the comparison between experiment and theory. At 140 mK, there is reasonable agreement, considering that the $\approx 15\%$ uncertainty in the values of E_c yields an order of magnitude uncertainty in the theory. Detailed analysis of the calculations shows that the dominant unwanted process in this regime is thermal activation over an energy barrier through multiple single-junction tunneling events. At 40 mK, theory predicts an error and leakage rate smaller than we observe by more than 10 orders of magnitude. The calculations predict that the dominant unwanted process in this regime is cotunneling (tunneling through multiple junctions at once).

We have considered three ways in which the experiment may not satisfy the assumptions made in the standard theory. (1) The amplitude, timing, and shape of the triangular gate pulses, as well as the cancellation of cross capacitance, have been carefully checked to verify that we are producing the optimal sequence of island charge polarizations prescribed by Jensen and Martinis [6]. (2) The actual voltage V_p on the island at the end of the pump is not known because the electrometer detects only *changes* in V_p . However, we observe that error and leakage events are equally likely in both directions, which implies V_p is near its optimal value. Since we pump $\pm e$ when measuring error, this optimal value will be near $V_p = 0$, and thus the comparison with calculations at $V_p = 0$ is valid. (3) Our procedure for tuning the fixed bias on each gate may not produce the optimal junction charge configuration. We expected to find a unique charge configuration for minimum error. As the background charges fluctuate, we expected that after each tuning we would find different fixed gate voltages but always the same charge configuration. To the contrary, we often find that repeated tunings give the same error but different charge configurations. This implies that there are several local minima in charge configuration space with equal error. The existence of a global minimum with much smaller error is unlikely, since we have

TABLE I. Comparison of experiment and theory. Repeated error measurements during several cooldowns gave values within a factor of 2 of the values shown. Leakage rate measurements were done fewer times because of the long time required, thus we show the range of measured values.

Temperature	140 mK	40 mK
Error per electron		
Experiment	100×10^{-8}	1×10^{-8}
C_{gnd} model	10×10^{-8}	0.5×10^{-20}
Bare model	4×10^{-8}	1×10^{-20}
Leakage rate (s^{-1})		
Experiment	$(60 \text{ to } 200) \times 10^{-4}$	$(3 \text{ to } 20) \times 10^{-4}$
C_{gnd} model	2×10^{-4}	0.2×10^{-20}
Bare model	0.2×10^{-4}	2×10^{-20}

repeated our tuning procedure dozens of times from a variety of initial conditions and have always found the same error within a factor of 2. Our conclusion is that we have found the optimal operating conditions, and the error we measure is close to the lowest possible for our pump [15].

The standard theory for error and leakage in the pump is based on principles that have been thoroughly tested in a variety of experiments, so it is unlikely to be simply wrong. However, the standard theory is incomplete in at least one important respect: It neglects assisted tunneling events in which energy from the environment is absorbed by the pump. Martinis and Nahum [16] have described a process of photon-assisted cotunneling in which photons with energy of order E_c ($f \sim 60$ GHz) can dramatically enhance the rate of unwanted tunneling events. The photons may come, for example, from a blackbody source with $T \sim 1$ K, and they may be transmitted to the pump by the coaxial electrical leads. We consider photon-assisted cotunneling to be a likely source for the excess error and leakage we observe at 40 mK. In our standard measurement configuration, we place Cu powder microwave filters [17] on each electrical lead outside the box in which the pump is mounted. We surround the box and the filters with a Cu shield at T_{mc} to block 4 K radiation from the vacuum can surrounding the refrigerator. If the photon source is outside the Cu box, the photons must enter the box through the electrical leads or by leaking directly through the connectors on the box, and thus removing the filters and shield should affect the photon-assisted error and leakage. We removed the filters and shield and found no effect on the measured error and leakage, so we conclude that if photon-assisted cotunneling causes the excess error and leakage, the photon source is inside the box. (Cosmic rays or radioactive decay products could cause errors, but they are inconsistent with the fact that the rate of error events is proportional to the pumping rate [2].) A possible photon source inside the box is the fluctuating charges in the substrate or in the pump itself, especially if these charges are relaxing from states occupied at higher temperatures during cooling [18]. Regardless of the source of the excess error and leakage, our results imply that the standard theory alone is not a reliable tool for estimating device parameters needed for applications that demand nearly perfect control over the tunneling of single electrons.

We have measured electron box transitions in an electron pump by applying charge biases to individual junctions. For each junction, the bias charge where the transition occurs gives the background charge, the linear dependence of the transition width on refrigerator temperature gives the electron temperature, and the transition width gives the charging energy. We have made a quantitative comparison with the standard theory for both error and leakage rate. We find agreement at high temperatures but a large discrepancy at low temperatures which we ascribe to photon-assisted cotunneling processes that are not included in the standard theory.

*Electronic address: mark.keller@boulder.nist.gov

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- [7] Q_b is defined as the total charge on the junction when the bias charge Q is zero. It may come from microscopic charged defects, from fixed voltages applied to any of the gates, or from a voltage across the whole pump.
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- [13] "Error" means the number of measured errors divided by the number of electrons pumped during a given time. "Leakage rate" means the number of times an electron leaks through the pump (while not pumping) divided by the time of the measurement.
- [14] Both models are approximations to the actual circuit, but the C_{gnd} model does not differ significantly from a model that includes all capacitances exactly, as discussed in Ref. [12].
- [15] Since we cannot tune the fixed gate voltages for minimum leakage at 40 mK, it is still possible that we have not found the optimal settings for this case.
- [16] J. M. Martinis and M. Nahum, *Phys. Rev. B* **48**, 18316 (1993).
- [17] The measured attenuation of our filters at frequencies above about 10 GHz is greater than the 80 dB limit of our instruments. Extrapolating from results between 1 and 10 GHz, we expect an attenuation >120 dB at 60 GHz.
- [18] The error and leakage rate we compare with theory were measured when the Q_b fluctuations were small. The error and leakage were often larger when the Q_b fluctuations were larger, but it is difficult to separate the *known* effect of shifts away from the optimal gate voltages from the *possible* effect of photons due to the fluctuating charges.