Quantum Interference and Electron-Electron Interactions at Strong Spin-Orbit Coupling in Disordered Systems

Yuli Lyanda-Geller
Department of Physics, Materials Research Laboratory and Beckman Institute, University of Illinois, Urbana, Illinois 61801

(Received 25 September 1997)

Transport and thermodynamic properties of disordered conductors are considerably modified when the angle through which the electron spin precesses due to spin-orbit interaction (SOI) during the mean free time becomes significant. Cooperon and diffusion equations are solved for the entire range of strength of SOI. The implications of SOI for the electron-electron interaction and interference effects in various experimental settings are discussed. [S0031-9007(98)06098-0]

PACS numbers: 73.20.Fz, 03.65.Bz, 71.70.Ej, 72.20.Fr

The effects of weak localization (WL) and electron-electron interaction on transport in disordered conductors are strongly influenced by interactions that affect electron phase coherence: by magnetic fields, magnetic impurities and spin-orbit interactions (SOI). The issue of the effects of SOI on WL [1,2] and electron-electron interaction corrections to conductivity [3] attracted considerable attention in early studies [4–8]. More recently [9–13], it was shown that, in addition, SOI can be regarded as generating an effective spin-dependent vector potential, which influences electron coherence rather like the electromagnetic vector potential does (via the Aharonov-Bohm, or AB, effect). To date, quantum corrections to conductivity have been conventionally studied under the assumption that the characteristic time scale which determines the SOI strength, τ_{so}, significantly exceeds the mean free time τ [4–15].

In the present Letter, I discuss quantum transport phenomena associated with SOI of arbitrary strength. Experiments and discussion in Refs. [16,17] suggest that SOI is strong, $\tau_{so} \sim \tau$, in Si metal-oxide-semiconductor field-effect transistors (MOSFETs) which are currently in the focus of attention due to observation of unusual temperature dependence of the conductivity in Refs. [18,19]. Of particular concern here will be implications of strong SOI for WL and electron-electron interaction effects.

It is important to recognize that two types of SOI can be identified. First, there is *random* SOI, due to impurity potentials. The scattering amplitude contains a spin-independent term and a much smaller spin-dependent term which, however, leads to SOI dephasing [4]. The SOI dephasing time due to this random SOI is always much larger than τ. The second type of SOI occurs in low-dimensional and low-symmetry systems, and owes its existence to the crystalline or confining potential. In this case, the electron Hamiltonian has the form

$$\mathcal{H} = p^2/2m^* + \hbar\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}(\boldsymbol{p}), \qquad (1)$$

where m^* is the effective electron mass, and Ω can be regarded as momentum-dependent spin-precession frequency. This type of SOI characterizes several recent experimental settings [14,15,18,19]. I consider here forms of $\boldsymbol{\Omega}(\mathbf{p})$ that transform like the Legendre polynomial P_1, which characterize two-dimensional (2D) systems (e.g., Si MOSFETs) and one-dimensional (1D) GaAs quantum wires and rings. Therefore $\Omega_i(\mathbf{p}) = \beta_{ij} p_j$ and the spin term in Eq. (1) can be written as $\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}(\mathbf{p}) = \mathbf{p} \cdot \tilde{\mathbf{A}}/m^*$, where $\tilde{\mathbf{A}}$ is the spin-dependent vector potential. It results in a number of interference phenomena [9–15] which can be regarded as manifestations of the Aharonov-Casher (AC) effect [20] in disordered electronic systems.

The strength of SOI in Eq. (1) can be characterized, in semiclassical terms, by the angle of spin precession during τ, $\Omega\tau$. When $\Omega\tau \ll 1$, the SOI dephasing time due to $\tilde{\mathbf{A}}$, is $1/\langle\Omega^2(\mathbf{p})\rangle\tau \gg \tau$, as for random SOI. For arbitrary $\Omega\tau$ [21] this is no longer the case.

The main results of this Letter are as follows: (i) At strong SOI positive magnetoresistance persists in 2D weakly disordered conductors in the whole range of magnetic fields. (ii) Because of electron-electron interactions, AC oscillations arise in the conductivity, the density of states, and thermodynamic quantities.

SOI and the interference correction to conductivity.— We now address the issue of how SOI of arbitrary strength influences the WL correction. We note, in passing, that the classical (i.e., Drude) expression for the conductivity σ_0 is left unchanged by SOI in Eq. (1), and $\sigma_0 = e^2 n\tau/m^*$. Interference corrections for disordered conductors in the diffusive regime have their origin in the increased amplitude for phase-coherent electron propagation along self-crossing trajectories. Addressing interference corrections to σ_0, one retains the maximally crossed diagrams in the quantity $G^R_\epsilon(\mathbf{p},\mathbf{p}')G^A_{\epsilon-\omega}(-\mathbf{p}' + \mathbf{q}, -\mathbf{p} + \mathbf{q})$, and thereby arrives at an equation for the Cooperon propagator (see, for instance, Ref. [22]). Here $G^{R(A)}$ are the single electron retarded (advanced) Green's functions, $\mathbf{q}$ is the total momentum of particles whose correlation is described, $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta for one of those particles. Similarly, ladder diagrams give rise to the Diffuson equation. For the physical system under consideration, the spin dependence in the Cooperon-Diffuson equations arises from propagation, i.e., results from G^R (G^A), and not from scattering. The Cooperon equation is given by

$$C = 1 + \int \frac{d\mathbf{o}/\Theta}{1 + i\omega\tau + \frac{i\mathbf{p}\tau}{m}(\mathbf{q} + \frac{2e}{c}\mathbf{A}_{\rm em} + \mathbf{A})} C\,, \tag{2}$$

where $\mathbf{o}$ denotes the orientation of momenta $\mathbf{p}$, Θ is the total solid angle in the momentum space, $\mathbf{A}_{\rm em}$ is the external electromagnetic vector potential, ω is the frequency, $\mathbf{A}$ is the spin-dependent vector potential, $A_j = 2\beta_{ij}\mathbf{S}_i$, and S is the total spin of particles. The conventional approach to Eq. (2) is the expansion of the integrand up to the second order in ql and $\mathbf{A}$, leading to a diffusionlike equation for the Cooperon-Diffuson propagators. In the present Letter, we calculate these propagators exactly, without such an expansion. Consider now 2D systems, and assume the that the tensor β has the form, appropriate for Si MOSFETs: $\beta_{xy} = -\beta yx = \beta$, where z is the direction normal to the 2D plane. Then, the solution for the Cooperon propagator reads:

$$C_0^0 = 1/(1 - f)\,, \tag{3}$$

$$C_1^0 = 1/(1 - f - 2g - 2h)\,, \tag{4}$$

$$C_1^{\pm 1} = \frac{1}{1 - f - 3g - h \pm \sqrt{t^2 + (g - h)^2}}\,. \tag{5}$$

Here the upper index in Eqs. (4) and (5) is the quantum number in the representation diagonalizing the Cooperon,

$$f = 1/\sqrt{1 + 2Dq^2\tau}\,, \tag{6}$$

$$g = \frac{1}{4}\left[\sum_{\pm} \frac{1}{b_\pm} - f\right], \tag{7}$$

$$h = \left[\frac{-2f^2}{(1 + f + fDq^2\tau)} + \sum_{\pm} \frac{1/b_\pm}{(a_\pm + b_\pm)^2}\right]\frac{Dq^2\tau}{2}\,, \tag{8}$$

$$t = \sum_{\pm} \frac{ilq(-1)^{(1\pm1)/2}}{2b_\pm[a_\pm + b_\pm]}\,, \tag{9}$$

where $a_\pm = 1 \pm 2i\beta m^* l = \pm 2i\Omega\tau$, $b_\pm = \sqrt{a_\pm^2 + 2Dq^2\tau}$.

We now consider the consequences of Eqs. (3)–(9) for interference corrections to the conductivity G, given by

$$\frac{e^2 D\tau}{\pi}\int_{1/L_\phi}^{Q_0}[dQ][-C_0^0 + C_1^0 + C_1^{+1} + C_1^{-1}], \tag{10}$$

where L_ϕ is the phase-breaking-length, and Q_0 is the upper cutoff, usually [23] regarded as being of order to $1/l$. The localization or antilocalization in weakly disordered conductors, the temperature (T), frequency, and magnetic field (H) dependence of the conductivity is determined by L_ϕ. In Fig. 1 I present the results for the conductivity dependence on L_ϕ at various magnitudes of SOI strength $\Omega\tau$. In the absence of SOI (curve 1) one observes the weak localization. At small $\Omega\tau$ (curves 2–3) conductivity exhibits antilocalization, if SOI dephasing length $L_{so} = 1/(\beta m^*) < L_\phi$, and weak localization in the opposite case. However, as $\Omega\tau$ approaches 1 (curves 4,5), the range of L_ϕ where electrons are localized diminishes. Finally, only antilocalization occurs at $\Omega\tau \geq 1$ (curves 6,7), because l cannot exceed L_ϕ. Therefore, in contrast to random SOI [24], as well as weak SOI in Eq. (1), all studied earlier, single-particle corrections always lead to an *increase* in the conductivity at strong SOI. As $L_\phi^{-2} \propto T$ in 2D case [22], Fig. 1 essentially represents dependence of the conductivity on $T^{1/2}$. Similarly, for such $H \parallel z$ that magnetic length $L_H = (\hbar c/2eH)^{1/2} < L_\phi$, or such ω that $D/\omega < L_\phi^2$, Fig. 1 adequately describes the anomalous MR (conductance versus $H^{1/2}$) or the interference correction dependence on $\omega^{1/2}$. I note that the exact account of all orders of expansion in ql in Eqs. (3)–(9) appears to be especially important at $\Omega\tau \sim 0.15$–0.5. For such strength of SOI, the conventional expansion up to the second order in ql would lead to substantial discrepancy with the conductance curves in Fig. 1 in the range $l/L_\phi \sim 0.1$–0.4, and cannot be applied. It is also noteworthy [25] that in this range of l/L_ϕ the interference corrections to conductivity are large and logarithmic-like.

Antilocalization characterizing interference corrections in 2D conductors in the whole range of temperatures, frequencies, and orbital magnetic fields occurs due to suppression by strong SOI of coherence of two electronic waves having total electron spin 1 and moving along time-reversed paths. Moreover, such a suppression leads to the following behavior of interference corrections to conductivity in magnetic field $H \perp z$. $H \perp z$ influences both the singlet (C_0^0) and triplet (C_1^j) sector of the Cooperon propagator due to Zeemann effect. At strong SOI $H \perp z$ leads to increasing antilocalization at small magnetic fields, when it influences only the singlet component and is negligible for triplet states entirely suppressed by SOI. However, at such magnetic fields that $g\nu H \sim 1/\tau$ [26], $H \perp z$ suppresses the triplet. Then, both singlet and triplet contributions, partially compensating each other, become

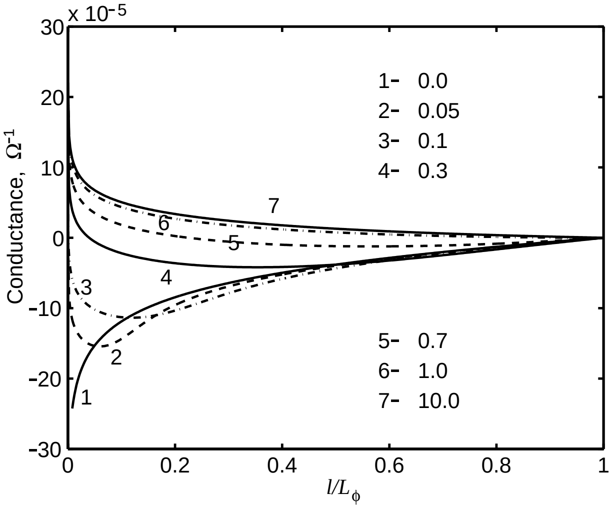

FIG. 1. The interference quantum correction to conductivity at various magnitudes of SOI strength $\Omega\tau$.

comparable in magnitude and magnetic field dependence of the conductivity weakens.

Interaction corrections to conductivity.—Quantum corrections to kinetic and thermodynamic quantities, due to electron-electron interactions in disordered conductors, have their origin in the enhancement of interactions between particles. The dominant contribution to this enhancement is due to electron diffusion leading to an increase of the interaction time and the effective interaction strength, for particles with a small difference in momenta and energies, this process being described by corrections in the Diffuson channel. These corrections are not affected by the AB phase, but are influenced by the Zeeman interaction, magnetic impurities, and SOI. As shown in Ref. [7], positive MR arises because interaction of an electron and a hole with total spin 1 and spin-projection ± 1 which enhances the conductivity at $H = 0$ is suppressed by Zeeman interaction. The suppression of the electron-electron corrections to conductivity in the Diffuson channel by the weak SOI was discussed in Ref. [22].

We now discuss the implications of the SOI in Eq. (1) for electron-electron interaction effects. The effect of SOI on the Diffuson propagator is determined (at $H = 0$) by Eq. (2) in which the net spin $\mathbf{S}$ and spin projection j correspond to the difference of electron spins ($\mathbf{S}$ is the total spin of electron and a hole). Diffuson in this case is given by Eqs. (3)–(9). Strong SOI, therefore, entirely suppresses contribution of the interaction of an electron and a hole with $S = 1$ (referred to below as the triplet Diffuson contribution) to the conductivity. Under these conditions magnetic field has no effect on the interaction contribution to MR in the Diffuson channel, as it does not affect the contribution of the interaction of an electron and a hole with $S = 0$ (referred to as the singlet Diffuson contribution). Therefore, at strong SOI, MR is determined by interference corrections. However, the temperature- and frequency-dependence of the conductivity are governed by the singlet Diffuson contribution [22]. This contribution is described by curve 1 in Fig. 1 if $l/L_\phi = l/L_T \le 0.2$ ($L_T \equiv l\sqrt{D/T}$) and the value of conductance is multiplied by the factor of 2. (In this temperature range corrections from processes neglected at $T\tau \ll 1$ are not essential.) At strong SOI and sufficiently low T, singlet Diffuson correction leads to the negative sign of the total quantum correction to conductivity which includes interaction and interference contributions.

Oscillatory electron-electron interaction effects due to SOI.—In the quasi-1D case, SOI in Eq. (1) leads to oscillations in ring-shaped samples of the interaction contributions to the conductance, and, in general, all quantities affected by electron-electron interaction corrections in the Diffuson channel. The oscillations with the variation of SOI strength arise in the triplet Diffuson correction to the conductivity (with dominant terms characterized by three diffusion poles in Hartree processes).

Let SOI in a ring with angular coordinate ϕ be described by the tensor β having nonzero component $\beta_{z\phi} = \beta_1$ which should characterize ring-shaped constriction created by asymmetric radial ($\boldsymbol{\rho}$) confinement in a GaAs rectangular quantum well grown along $z \parallel 110$ crystal axis [27]. Estimates of β_1 due to asymmetric confinement in GaAs (see, for example, [15]) show that for this setting $\Omega\tau \ll 1$. The Diffuson propagator in the coordinate $\mathbf{r} = (\rho, \phi)$representation in quasi-1D ring with radius R for this case reads

$$\mathcal{D}_j^S(\mathbf{r}, \mathbf{r}') = \int \frac{dQ_\rho}{2\pi L} \sum_{l=-\infty}^{l=\infty} \times \frac{e^{i\mathbf{Q}(\mathbf{r}-\mathbf{r}')}}{DQ_\rho^2 + D(Q_\phi^l + \eta_j)^2 - i\omega}, \quad (11)$$

where $Q_\phi = l/R$, $L = 2\pi R$, $\eta_j = 2jm^*\beta_1$. In a ring of width $a \ll L_\phi$ and circumference $L < L_T$, at $T\tau \ll 1$, the oscillating contribution to conductance has the form

$$\delta\sigma^{\text{osc}} = \frac{e^2 L_T \lambda_1}{2^{3/2}\pi\hbar a^2} \sum_{n=1}^{\infty} e^{-\delta}(\sin\delta - \cos\delta)\cos n\zeta, \quad (12)$$

where $\delta = nL/\sqrt{2}L_T$, $\zeta = 2\beta_1 m^* L$, and λ_1 (discussed in [22,28]) is the constant describing the interaction of an electron and a hole with total spin 1. Similar oscillations characterize the density of states and the thermodynamic potential. The period of oscillations coincides with the period of AC oscillations in the interference contribution to conductivity [9–11]. However, interference contribution is affected by the AB flux which leads to beatings, and, thus, oscillations in the interference and electron-electron interaction channels can be distinguished. Moreover, as strong $H \parallel z$ suppresses interference contributions, AC oscillations may serve as an experimental tool for investigating the triplet Diffuson corrections. The variation of β_1 is due to gate voltage [29].

Discussion of experimental settings.—The SOI effects considered in the present Letter can be observed in MR of 2D metallic samples at $E_F\tau \gg 1$. At strong SOI MR must be positive for all magnetic fields, and the total quantum correction to the conductivity must be negative. I now discuss the existing data of recent experiments [18]. One of the structures, Si-12b, with the electron concentration $n_s = 1.37 \times 10^{11}$ cm^{-2}, $E_F = 0.8$ meV (10 K), and the conductivity $G = 3.5e^2/2\pi\hbar$ at $T = 2$ K is close to the range of parameters where the present consideration can be applied [30]. This particular set of experimental data can be described as follows. The dimensionless conductivity $G \sim E_F\tau + G_i + G_{ee}$, where G_i is the interference contribution, and G_{ee} is the interaction contribution. G_{ee} at such high temperatures ($T = 2$ K) is not logarithmic, as we estimate $T\tau \sim 0.8$ (because $E_F\tau \sim 3.2$ and $\tau = 2.8 \times 10^{-12}$ s). Therefore, G_{ee}, which at $T\tau \ll 1$ would be responsible for an insulating behavior, is not essential in this range of temperatures and varies very slowly with T. G_i is determined by intermediate SOI, as $\beta = 2.0 \times 10^{-10}$ eV cm [31] and $\Omega\tau = 0.7$, and leads to an increase in the conductivity. Assuming that

$G_i \sim G - E_F\tau \sim \ln(L_\phi/l)/\pi$ we obtain $l/L_\phi \sim 0.4$ which is consistent with $G_i \sim 0.4$ given by curve 5 in Fig. 1. Considering the temperature dependence of G_i given by this curve, we find $G \sim 5.5$ at $T = 0.4$ K, whereas in the experiment $G \sim 9$. As τ in this temperature range possibly increases, this values of G are in reasonable agreement. Note that at $T = 0.3$ K the parameter $T\tau \sim 0.2$. That is close to the region in which G_{ee} becomes logarithmic and overcompensates G_i. Thus, if this model is correct, further decrease in T should reveal a decrease in G. Experimental study of MR and the temperatures dependence of the conductivity at higher n_s [30] would be an important test of the localization/antilocalization and the strength of SOI in Si MOSFETs.

Although this Letter is not aimed at the analysis of those experiments in Refs. [18,19] in which $G \sim 1$, I would like to discuss the SOI strength in such a case. Its decrease estimated using the Drude model is not meaningful, as neither the Drude model nor the WL theory can be applied to this case. However, the renormalization of SOI strength with G is possible and important for a study of the regime $G \sim 1$ using the scaling approach in Ref. [1].

In conclusion, (i) the experimental tests proposed in this Letter for Si MOSFETs studied in Ref. [18] may be helpful for elucidating the nature of the metallic state in Refs. [18,32]. (ii) The experimental discovery of the AC oscillations in ring-shaped samples would bring the opportunity to distinguish the interference and interaction oscillatory contributions and to determine the electron-electron interaction constant λ_1.

I present my sincere gratitude to P. M. Goldbart for his attention to this work and numerous helpful discussions, and to I. L. Aleiner for helpful interesting discussions. Support by the U.S. Department of Energy, Division of Materials Sciences under Grant No. DEFG02-96ER45439 is gratefully acknowledged.

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[25] Note, that Hikami-box diagrams (with impurity line aside the Cooperon, see [22]) may be essential only at l/L_ϕ closer to 1. This contribution to δG is small.

[26] In the case of relatively weak SOI [6] this condition reads $g\nu H \sim 1/\tau_{so}$ and characteristic H are much lower.

[27] One can construct a gate in 2D GaAs layer with a surrounding ring-shaped region from which GaAs is removed. When bias is applied between 2D GaAs region and the gate, a ring-shaped diffusive inversion channel should appear on the boundary between the region surrounding the gate and the 2D gas.

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[29] The configuration proposed here avoids decay of AC oscillations in quasdi-1D rings in quantum wells grown along $z \parallel 100$, in which $A_\rho = 2\beta m^* S_\rho$ is nonzero. In such a case $L_{so} = 1/m\beta$ substitutes for L_T in Eq. (12), and as β is tuned, oscillations decaying within a few periods could be observed. In the case of $z \parallel 110$ and asymmetric radial confinement, oscillations does not decay.

[30] The applicability of this consideration at $n_s = 1.37 \times 10^{11}$ cm^{-2} is restricted, as the Coulomb correlation energy $\epsilon_C = e^2 n_s^{1/2}/\kappa$, where κ is the dielectric constant, significantly exceeds E_F. Experiments with higher n_s would be better described by the present model, as $E_F\tau$ increases and E_F increases faster than the Coulomb correlation energy.

[31] This β corresponds to the SOI gap $\Delta = 3.6$ K in Ref. [17]. $\beta = 1.8 \times 10^{-9}$ eV cm in Ref. [17] is the overestimate. Intermediate strength of SOI in Si MOSFETs have to be confirmed by futher experiments, especially by magnetoresistance measurements at low $H \parallel z$.

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