Wave Packet Dynamics with Bose-Einstein Condensates

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We numerically study wave packet dynamics of a Bose-Einstein condensate in a periodically shaken trap. Dynamic splitting of the condensate, and dynamic stabilization against escape from the trap are analyzed in analogy with similar behavior of atoms in strong laser fields. [S0031-9007(98)06005-0]

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Recently it has become possible to prepare Bose-Einstein condensates of alkali gases [1] with up to 10^7 magnetically trapped atoms. The condensed state is a macroscopically populated quantum state well localized in the magnetic trap. It is, therefore, an ideal tool to study wave packet dynamics under experimentally feasible conditions. There are many interesting quantum phenomena resulting from the electronic wave packet dynamics: e.g., the phenomena of *wave packet splitting (dichotomy)* and *stabilization* exhibited by an electron bound by an atomic potential in the presence of a strong laser field [2]. We argue that similar phenomena occur in the dynamics of the condensate in a periodically shaken trap. The analogy is based on the fact that the effect of the laser is equivalent to a periodic shaking of the atomic potential along the polarization axis. A condensate in a periodically shaken trap could show, *a priori* a similar behavior.

Let us analyze the analogy between the electron and the condensate in more detail. An electron bound by an atomic potential $U(\vec{r})$ interacting with a laser of amplitude $\mathcal{E}\vec{e}_z$ is, for our purposes, best described in the Kramers-Henneberger frame of reference [2]. This frame is moving as a *free* electron oscillating in the laser field, so that the actual electron in this frame feels a time dependent potential:

$$\left[-i\hbar\partial_t - \frac{\hbar^2\vec{\nabla}^2}{2m_e} + U(\vec{r} + \alpha_L \sin(\omega_L t)\vec{e}_z)\right]\Psi_e(\vec{r},t) = 0\,, \quad (1)$$

where $\alpha_L = e\mathcal{E}/m_e\omega_L^2$ is the electron excursion amplitude, while ω_L is the laser frequency. For a hydrogen atom $U(r) \propto -1/r$, for more complex atoms $U(r)$ includes screening of the Coulomb potential by other electrons. As the intense laser field drives the electron, ionization occurs. By increasing the laser intensity one normally increases the ionization rate. However, for very intense fields of high frequency, this rate eventually starts to decrease with intensity—this is referred to as *adiabatic atomic stabilization* [2,3]. In this process the electronic wave packet remains bound, i.e., well localized in space (without spreading), although highly distorted. The *effective* (time-averaged) atom-laser potential exhibits a double well structure which splits the electronic wave packet into two spatially separate parts; this is called *dichotomy*.

Consider now a condensate with N atoms in a trap potential $V(\vec{r})$ which is periodically shaken along the z axis. In a Hartree-Fock treatment, the state of the condensate is described by the Gross-Pitaevskii equation (GPE) [4]. It accurately describes the wave function Ψ of the condensate in the presence of particle interactions in thermal equilibrium at temperatures well below the critical temperature. Furthermore, the time dependent GPE describes the dynamics of the condensate in more general time dependent conditions [5]. In particular for a periodically shaken trap the GPE reads

$$\left[-i\hbar\partial_t - \frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r} + \alpha(t)\vec{e}_z) + gN|\Psi(\vec{r},t)|^2\right]\Psi(\vec{r},t) = 0\,. \quad (2)$$

Here $\alpha(t) = \alpha_0 \sin(\omega t)$ is the shaking amplitude, ω the shaking frequency, and g describes the atomic interactions and is related to the s-wave scattering length $a_s > 0$ by $g = 4\pi\hbar^2 a_s/m$. The presence of atomic interactions as well as the mass difference between atoms and electrons are crucial differences between these two systems. Nevertheless, we find that their dynamical behavior under periodic perturbations is remarkably similar.

We model the trapping potential V by a harmonic potential with frequencies $\Omega_{x,y,z}$ which is cut at an energy V_c [see solid line in Fig. 1(a)]:

$$V(\vec{r}) = \begin{cases} \frac{m}{2}(\Omega_x^2 x^2 + \Omega_y^2 y^2 + \Omega_z^2 z^2), & V(\vec{r}) \le V_c\,, \\ V_c, & V(\vec{r}) > V_c\,. \end{cases} \quad (3)$$

A possible realization of this trapping potential would be a condensate in a dipole trap formed by a strong off-resonant laser field. The condensation might take place in such a trap, or the magnetically trapped condensate may

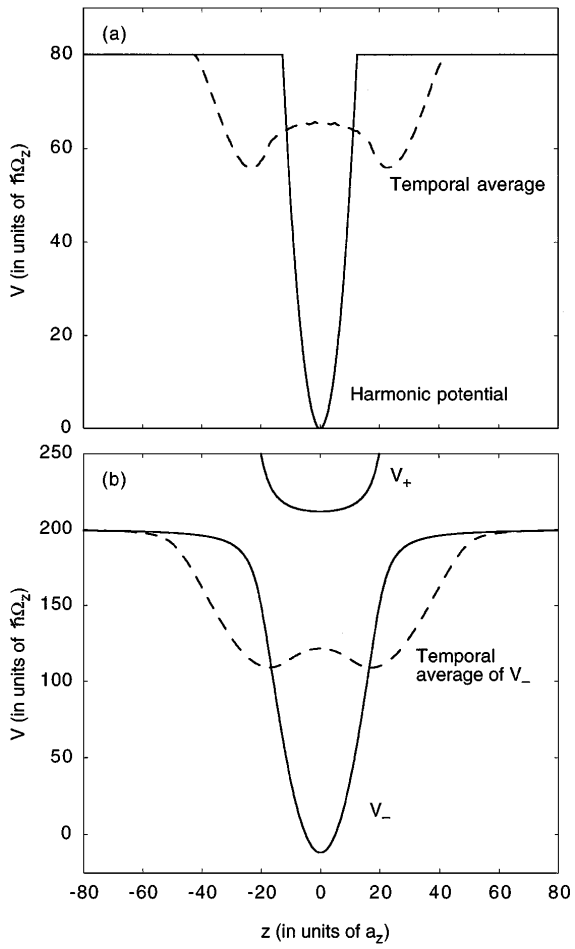

FIG. 1. (a) Cut harmonic potential (solid line) and time averaged potential (dashed line) for $\alpha_0 = 30a_z$, $V_c = 80\hbar\Omega_z$, and $gN = 100$; $a_z = (\hbar/2m\Omega_z)^{1/2}$. (b) Dressed state potentials V_+ and V_- and time averaged potential of V_- from Eq. (6) for $\alpha_0 = 30a_z$, $\omega_R = 100\Omega_z$, and $\Delta = 200\Omega_z$.

be loaded into it. In [6] it is realized as an effective trap potential "dressed" by the microwave coupling between a trapped and an untrapped state in the magnetic field; both models are discussed in detail later. The actual shapes of the electronic $U(\vec{r})$ and the atomic $V(\vec{r})$ potentials are different. However, both potentials tend to a finite value, 0 or V_c, respectively, as $|r| \to \infty$. This results in a threshold kinetic energy for particle escape from $V(\vec{r})$ analogous to the ionization threshold of $U(\vec{r})$.

We study first the time evolution of the condensate in the periodically shaken trap potential [Eq. (3)] by solving numerically the GPE [Eq. (2)] using a standard spit operator technique [5(a)]. We assume an adiabatic turn-on of the shaking modeled by

$$\alpha(t) = \begin{cases} \alpha_0 \sin^2(\frac{\pi}{2}\frac{t}{t_{\rm on}})\sin(\omega t), & \text{for } 0 \le t \le t_{\rm on}\,, \\ \alpha_0 \sin(\omega t), & \text{for } t \ge t_{\rm on}\,. \end{cases} \quad (4)$$

In order to achieve an adiabatic transition from the initial state of the condensate in Eq. (3) to the steady state in the presence of the perturbation (shaking), we require the turn-on time to be much larger than one shaking period, i.e., $t_{\rm on} \gg 2\pi/\omega$. In practice we take $t_{\rm on} \ge (50\text{–}150) \times 2\pi/\omega \simeq 2\pi\alpha_0/a_z\omega$, where $a_z = \sqrt{\hbar/2m\Omega_z}$.

Let us first assume that the perpendicular motion does not play a significant role. We can then reduce the time dependent GPE to a 1D GPE along the z axis. We solve the 1D GPE for a large range of parameters: $\alpha_0 \in [10\text{–}50]a_z$, $V_c \in [20\text{–}100]\hbar\Omega_z$; $gN \in [0\text{–}100]$ and $\omega \in [2\text{–}20]\Omega_z$. A typical result of the time evolution is displayed in Fig. 2(a). Initially at $t = 0$, the wave packet corresponds to the ground state of the condensate in the harmonic trap. As the shaking is slowly turned on the wave packet gradually splits into two separate wave packets. At $t \ge t_{\rm on}$, these two components are centered near the turning points of the oscillation $\pm\alpha_0$.

We consider now the model with two internal hyperfine levels: a trapped state $(F, m_F \neq 0)$ and an untrapped state $(F', m_{F'} = 0)$. They are coupled via a microwave field which allows coherent transitions between the states [6].

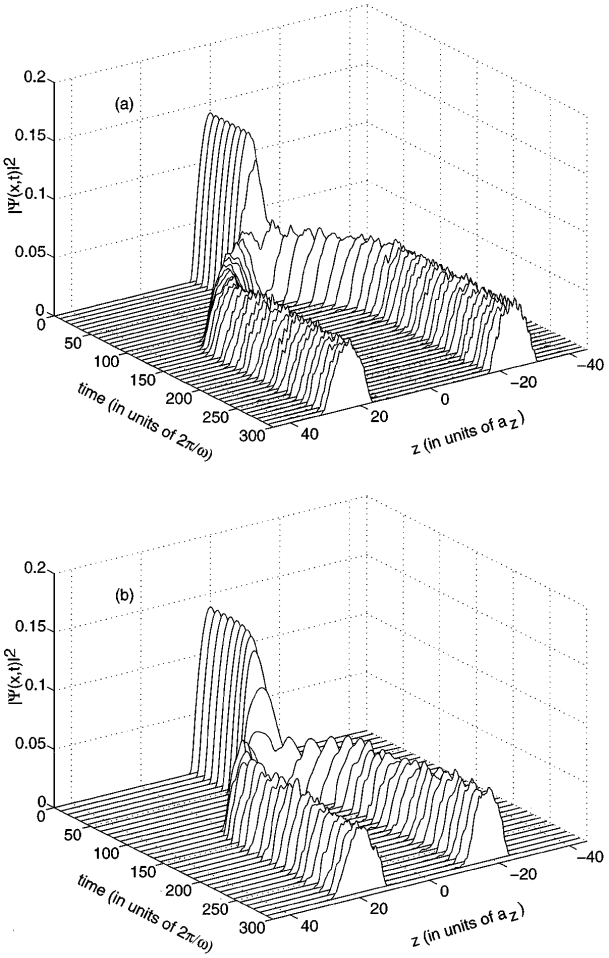

FIG. 2. (a) Time evolution of the condensate density $|\Psi(z,t)|^2$ undergoing 300 shaking cycles for $\alpha_0 = 30a_z$, $\omega = 10\Omega_z$, $t_{\rm on} = 150 \times (2\pi/\omega)$, and $V_c = 80\hbar\Omega_z$. The nonlinear coupling is $gN = 100$, that corresponds to $\hbar\mu = 14.13\Omega_z$. (b) Same as (a) for the total condensate density $|\Psi(z,t)|^2 = |\Psi_1(z,t)|^2 + |\Psi_0(z,t)|^2$, calculated from the 1D two state model for $\alpha_0 = 30a_z$, $\omega = 2.5\Omega_z$, $t_{\rm on} = 150 \times (2\pi/\omega)$, $\omega_R = 100\Omega_z$, and $\Delta = 200\Omega_z$.

In the rotating wave approximation the GPE is given by

$$\left[-i\hbar\partial_t - \frac{\hbar^2\nabla^2}{2m} + gN(|\Psi_0(\mathbf{r},t)|^2 + |\Psi_1(\mathbf{r},t)|^2) + \begin{pmatrix} m\Omega_z^2[z+\alpha(t)]^2/2 & \hbar\omega_R/2 \\ \hbar\omega_R/2 & \hbar\Delta \end{pmatrix}\right]\begin{pmatrix}\Psi_0(\mathbf{r},t)\\ \Psi_1(\mathbf{r},t)\end{pmatrix} = 0. \tag{5}$$

Here Ψ_0, Ψ_1 are the wave functions of atoms in the trapped and untrapped state normalized to the respective fraction of atoms in these states, Δ is the detuning of the microwave from the transition frequency, and ω_R is the Rabi frequency of the microwave transition. For simplicity we assume all coupling constants are equal to g. For $\omega_R \to \infty$, the coupled states can be replaced by uncoupled dressed states with potentials:

$$V_\pm(z,t) = \frac{1}{2}\left[m\Omega_z^2[z+\alpha(t)]^2/2 + \hbar\Delta \pm \sqrt{(m\Omega_z^2[z+\alpha(t)]^2/2 - \hbar\Delta)^2 + \hbar^2\omega_R^2}\right], \tag{6}$$

plotted for $\alpha(t) = 0$ as solid lines in Fig. 1(b).

Again, we solve numerically the 1D two-state model [Eq. (5)] for suitable parameters and compute the time evolution of the two-component condensate for an adiabatic turn on. The time evolution of the two-state condensate is displayed in Fig. 2(b). The evolution is remarkably similar to the previous case. The gradual splitting of the condensate is achieved during the turn on. This model includes nonadiabatic (Landau-Zener) transitions from the lower to the upper dressed state which might destroy the dichotomy. In order to avoid these Landau-Zener transitions ω has to be smaller than a certain critical value (see [7]).

The similarity between both cases is not surprising. In fact, the potential V_- from Fig. 1(b) closely resembles that of the model potential of Eq. (3) shown in Fig. 1(a). Obviously, Eq. (2), with V replaced by V_-, does not give exact solutions of the model of Eq. (5), since it entirely neglects the nonadiabatic transitions to an upper branch of the dressed potential. The trap potential model of Eq. (3) approximates nevetheless that of Eq. (5) very well, and leads to very similar results, as shown in Fig. 2.

The condensate splitting originates from the periodicity of shaking. The time-dependent potential can be Fourier decomposed:

$$V(\vec{r} + \alpha(t)\vec{e}_z) = \sum_{m=-\infty}^{\infty} V_m(\vec{r}, \alpha_0\vec{e}_z)e^{-im\omega t}. \tag{7}$$

If the time scale of the shaking, $1/\omega$, is shorter than the other relevant time scales of the system, the time-dependent potential can be replaced by the zero term in the Fourier expansion. This is just the time averaged potential over one shaking period T:

$$V_0(\vec{r}, \alpha_0\vec{e}_z) = \frac{1}{T}\int_{-T/2}^{T/2} V(\vec{r} + \alpha(t)\vec{e}_z)dt. \tag{8}$$

For sufficiently large shaking amplitude α_0, this time averaged potential V_0 exhibits a double well structure in the z direction, as shown in Figs. 1(a) and 1(b). Thus, for a sufficiently slow turn on, the condensate will evolve adiabatically from the ground state of the trap potential into the corresponding ground state of the time-averaged trap potential V_0. Note that for the pure harmonic potential (i.e., $V_c \to \infty$) the time-averaged potential will not exhibit the double well structure, and therefore the shaking of the trap would only lead to an undistorted oscillation of the condensate.

To test the validity of the above ideas, we can now replace the time dependent GPE [Eq. (2)] by a time independent GPE with the time-averaged potential V_0:

$$\left[-\mu_{\text{eff}} - \frac{\hbar^2\vec{\nabla}^2}{2m} + V_0(\vec{r}, \alpha_0\vec{e}_z) + Ng|\Psi_0(\vec{r})|^2\right]\Psi_0(\vec{r}) = 0, \tag{9}$$

where μ_{eff} is now the effective chemical potential. Indeed, the solution Ψ_0 of Eq. (9) agrees very well with the dynamically stable wave packets displayed in Fig. 2. We conclude that the splitting appears due to the trapping of the atomic wave function into the two potential wells of V_0. Notice that the atoms in such a state remain trapped due to the fact that the time scale for their motion is much larger than the time scale of shaking, in other words $\hbar/\mu_{\text{eff}} \geq 1/\omega$. This prevents them from reacting fast enough to being momentarily out of the trap.

We now discuss the phenomenon of stabilization. Because of the cut V_c in the trapping potential of Eq. (3) atoms may escape from the trap in analogy to ionization of electrons due to strong laser fields. By introducing standard absorbing boundary conditions at the edges of the numerical grid, the norm (and thereby the number of particles escaping from the trap) can be easily monitored. As can be seen from Fig. 2 no significant number of particles is lost during the shaking of the potential indicating thus that the particles remain trapped due to the high cutoff V_c. To favor the escape from the trap we lower the potential cutoff to $V_c = 50\hbar\Omega_z$ and we study the escaping rate as a function of the shaking amplitude α_0. For small values of α_0 the escaping rate increases as α_0 increases, but for high amplitudes we observe a *decrease* in the escape rate when the shaking amplitude *increases*. More specifically, when increasing α_0 from $15a_z$ to $20a_z$, the escape rate decreases by a factor $\simeq 2$. Because of the large atomic mass the escape rate is overall very small, i.e. $\leq 1\%$ of the trapped population per 100 shaking cycles. For the same reason, the condensate stabilization occurs already for relatively small shaking frequencies $\hbar\omega < V_c - \mu_{\text{eff}}$.

Up to now we have analyzed dynamical behavior of condensates closely related to similar phenomena of atoms in strong laser fields. However, in contrast to Eq. (1), the GPE accounts for atomic interactions, so that the condensate splitting depends on gN. Furthermore, the nonlinear term reduces the relative role of the kinetic

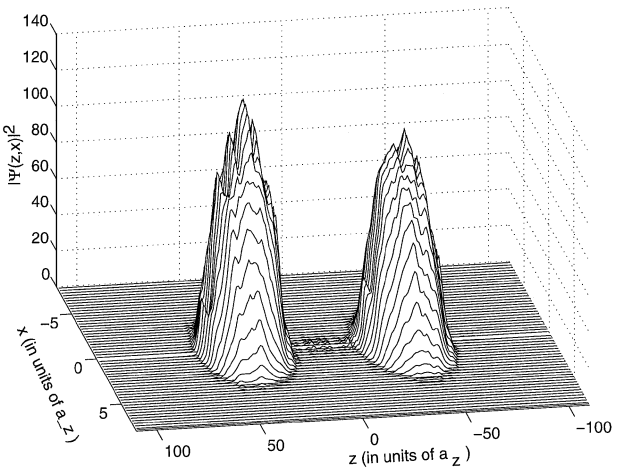

FIG. 3. 2D cross section of the condensate density $|\Psi(x,z,t)|^2$ for a cigar shaped trap at $t = 400 \times (2\pi/\omega)$ for $\alpha_0 = 60a_z$, $\omega = 10\Omega_z$, $t_{\rm on} = 250 \times (2\pi/\omega)$, $V_c = 30\hbar\Omega_z$, $gN = 100$, and $\Omega_x = \Omega_y = 5\Omega_z$.

energy of the condensate, which favors both splitting and stabilization. Our numerical simulations show that in fact, as gN increases the condensate splits more easily and attains a more regular shape: It approaches the Thomas-Fermi solution [5(b)] in the effective time averaged potential. However, beyond a critical value of N the wave function will overcome the potential barrier between the wells in V_0, and the splitting will disappear. This will happen when the effective chemical potential ($\mu_{\rm eff}$) exceeds the height of the double well.

Finally, to ensure that the presence of degrees of freedom perpendicular to z axis does not invalidate our results, we have generalized our study to a realistic 3D case. We chose parameters that resemble those of the MIT experiment [1], that is a cigar shaped trap with a small Ω_z and equal frequencies perpendicular to it. We shake the trap potential along the long z axis and we solve numerically the GPE in 3D. As shown in Fig. 3 the presence of perpendicular motion does not invalidate the conclusions from the 1D approximation: The splitting of the condensate is clearly visible.

In traps with time dependent trapping frequencies strong population of noncondensate modes and a significant condensate depletion might occur [8]. It is, therefore, a pertinent question to ask whether a similar depletion will occur in the case of shaking of the trap potential. As shown in [8] depletion is directly related to linear stability of the solution of the time dependent GPE. So far, we could prove only that the sloshing motion of the condensate in a purely harmonic trap (without cutoff) is linearly stable. Unfortunately, the stability of our *dichotomic* solutions of the GPE remains an open problem. They show, however, remarkable shape stability will respect to changes of the relevant physical parameters of the model. This observation excludes with high certitude the possibility of the most dangerous *exponential* instability, and allows us to conjecture that our solutions are in fact linearly stable.

Summarizing, we have shown that Bose-Einstein condensates are ideal tools for studying wave packet behavior of condensates, in particular, the splitting of the condensate wave function [3]. We believe that wave packet dynamics of condensates might lead to interesting possibilities of condensate state engineering. For instance, so far double peaked condensates have been created using laser "knives" that cut a single condensate into two parts. We offer here an alternative method to achieve a similar dichotomy in a more controlled way which opens new perspectives for condensate interference studies. Furthermore, by shaking the condensate in the z and y direction simultaneously one obtains a ring-shaped condensate. This might open possibilities to study the dynamics of vortices.

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[7] The Landau-Zener probability for a nonadiabatic transition is $\exp(-2\pi\Lambda)$; $\Lambda = \hbar(\omega_R/2)^2/v\lambda$, where $v = \alpha_0\omega$ is the atomic speed at the resonance point $z_0(m\Omega_z^2 z_0^2/2 = \hbar\Delta)$ and $\lambda = |\partial(m\Omega_z^2 z^2/2 - \hbar\Delta)/\partial z|_{z=z_0}$. The condition for adiabatic following is $\Lambda \gg 1$, which leads to $\alpha_0\omega \ll \omega_R^2(m\Omega_z^2\Delta/\hbar)^{-1/2}$.

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