First Experimental Study of the Darrieus-Landau Instability

C. Clanet and G. Searby
I.R.P.H.E., CNRS - Universités d'Aix-Marseille I & II, Service 252, Campus Universitaire de St. Jérôme, 13397 Marseille Cedex 20 France

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This paper presents the first direct experimental measurements of the growth rate of the Darrieus-Landau instability on a planar laminar premixed flame front. Prior to measurements, the intrinsically unstable flame is maintained stable by a novel technique based on the response of the flame to an acoustic parametric forcing. The growth rates of the instability, measured when the forcing is removed, are in agreement with the theoretically predicted values. [S0031-9007(98)05920-1]

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Historically, Darrieus [1] in 1938 first recognized that the gas expansion produced by heat release in a wrinkled premixed flame, considered as a thin interface propagating towards the fresh gas with the constant normal speed U_L, will deviate flow lines across the front towards the normal to the flame. Since the Mach number, $M \equiv U_L/c$ where c is the speed of sound, is very small, the flow is quasi-incompressible and the upstream flow lines are also deviated, creating flow divergence and velocity gradients that will increase the wrinkling of the flame (see Fig. 1).

This unconditional hydrodynamic instability, was predicted independently in 1944 by Landau [2]. Considering only gas expansion, he found that the growth rate of the instability σ should vary as $\sigma = k \cdot U_L \cdot f(E)$, where k is the wave number of the perturbation $E = \rho_u/\rho_b$, the gas expansion ratio defined as the ratio of unburned to burned gas density, and $f(E)$ is a positive function of order unity vanishing for $E = 1$. A more complete derivation of the dispersion relation, including the effects of preferential diffusion, temperature dependent diffusion coefficients, and acceleration of the flame front has been given by Clavin and Garcia [3]. Typical plots of the real part of the growth rate as a function of wave number are shown in Fig. 2. The wave number and the growth rate are, respectively, nondimensionalized by the thermal flame thickness $\delta \equiv D_{\text{th}}/U_L$ and the flame transit time $\tau \equiv \delta/U_L$, where D_{th} is the thermal diffusion coefficient. The effect of preferential diffusion is to restabilize flames at large wave numbers, leading to the existence of a wave number having the highest growth rate. For very slow flames propagating downwards, the effect of gravity can be sufficient to stabilize the flame at all wave numbers [4].

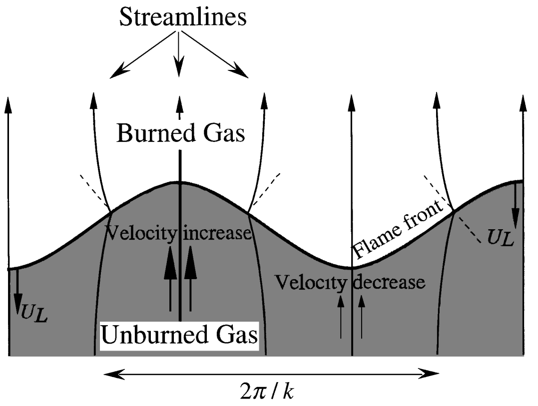

FIG. 1. Deviation of flow lines leading to the Darrieus-Landau instability.

This linear analysis is expected to be correct during the linear part of the growth of this instability. However, up to now, there has been no direct experimental verification, the difficulty lying in the production of the initial condition of a planar, freely propagating, unstable flame. The characteristic growth time of the instability is typically 20–50 ms, which is short compared to the time needed to establish a freely propagating flame of finite dimensions. It follows that the Darrieus-Landau instability has always been observed in the phase of nonlinear saturation. In this paper we use a novel technique of acoustic restabilization to produce a perfectly planar laminar flame which is otherwise intrinsically cellular. The imposed acoustic field is then removed on a short time scale, ≈1 ms, and the unconstrained growth of the Darrieus-Landau instability is observed.

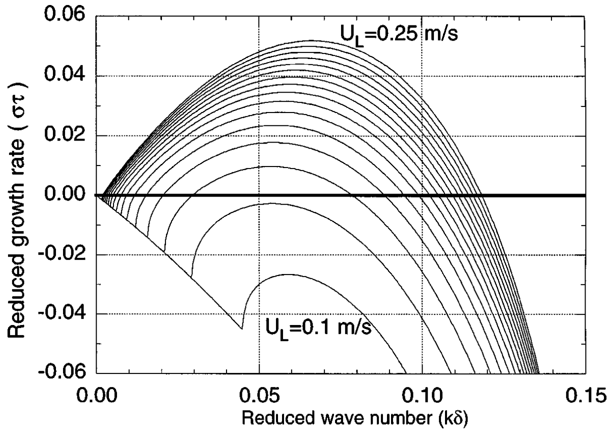

FIG. 2. Reduced growth rates plotted as a function of reduced wave number for various Froude numbers, ($4.89 < \text{Fr} \equiv U_L^2/g\delta < 76$), corresponding to flame speeds in the range $0.1 \leq U_L \leq 0.25$ m/s (lower to upper in steps of 0.01 m/s). Other parameters are appropriate for lean propane-air flames.

The effect of an imposed acoustic field on a freely propagating premixed laminar flame front has been given by Searby and Rochwerger [5] using an extension of the work presented in [3] and [6]. In the approximation $\alpha \cdot k \ll 1$, where α is the amplitude of the wrinkling, Searby and Rochwerger have shown that the dynamics of a flame front in an imposed acoustic field, characterized by the frequency ω_a, is dominated by the effect of the periodic acoustic acceleration acting on the flame, seen as a thin interface between two fluids of different density. The complete system is modeled as a parametrically driven harmonic oscillator in which the acoustic acceleration appears as the driving force:

$$A(k)\partial^2\alpha(k,t)/\partial t^2 + B(k)\partial\alpha(k,t)/\partial t + [C_o(k) - C_1(k)\cos(\omega_a t)]\alpha(k,t) = 0\,, \quad (1)$$

Detailed expressions of the coefficients A, B, C_0, and C_1 are given in [5]. These coefficients are functions of the wave number of the wrinkling and of flame-related parameters such as the gas expansion ratio E, the Froude number Fr, and the Markstein number Ma, characterizing the effect of weak curvature and stretch on local flame burning velocity [7]. Their values are known or can be easily calculated. C_1 is the forcing term containing the amplitude of the acoustic velocity u'_a at the flame front. The dispersion relation for a wrinkled flame can be obtained from Eq. (1) by putting $C_1 = 0$ and writing $\alpha(k,t) = a\exp(\sigma t + ky)$. The parametric oscillator, Eq. (1), can be reduced to the Mathieu equation [8] by a simple substitution of variables [6]. The solutions of this latter are known to present several regions of instability separated by a stable domain. Note that the flame front does not have a single characteristic frequency, but a continuum of frequencies associated with the continuum of wavelengths that can be excited on its surface.

In Fig. 3, we have plotted an example of the zones of stable and unstable solutions to Eq. (1). It can be seen that, as the acoustic excitation is increased from zero, the range of wave numbers unstable to the Darrieus-Landau mechanism decreases. With the values of parameters in this example, the flame front is stable at all wave numbers for acoustic levels in the range $3.1 < u'_a/U_L < 3.6$. At even higher acoustic levels, smaller cellular structures appear through parametric excitation. We have used this phenomenon of parametric restabilization to produce a perfectly planar laminar flame prior to measurement of the growth rate of the Darrieus-Landau instability.

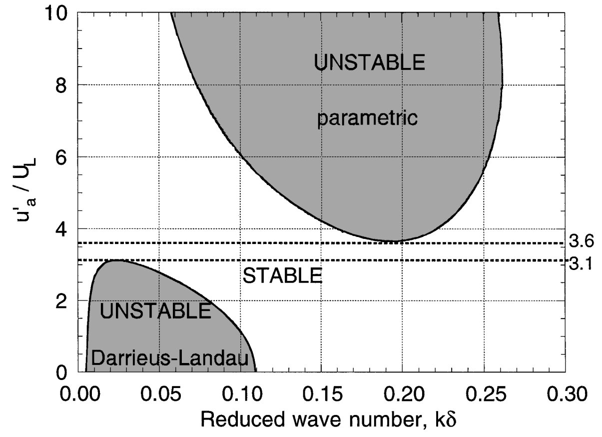

FIG. 3. Stability domain of acoustically forced flame from Eq. (1). Parameters appropriate for lean propane-air flame: burning speed 19 cm/s, acoustic frequency 230 Hz, reduced frequency $\omega_a\tau = 0.88$, Ma = 4.0.

The experimental apparatus is shown in Fig. 4. The premixed gases are fed into the bottom of a Pyrex glass tube 100 mm diameter and 400 mm long, just below a 50 μm porous plate whose role is to laminarize the flow. The flame was held stationary in the laboratory frame, about 50 mm below the tube exit, by careful adjustment of the gas flow rate. A 2 mm aluminum honeycomb structure 40 mm long was placed a few centimeters upstream from the flame to help maintain a laminar and homogeneous gas flow. A helical cooling tube was wound around the outside of the Pyrex tube, below the flame front, in order to prevent wall heating by heat conduction and radiation from the flame. Downstream cooling was not used. The Pyrex tube was closed at its lower extremity by a loudspeaker which was fed from a microcomputer used as a programmable signal generator. The acoustic impedance of the porous plate was high and the Pyrex tube behaved as an open-closed resonator. It was excited in the one-fourth wavelength longitudinal mode (230 Hz). Although the acoustic damping of this short wide tube was high, the natural acoustic damping time (≈12 ms) was not always short compared to the growth time of the Landau instability. To overcome this limitation, at the start of a measurement, the damping of the acoustic standing wave was increased artificially by injecting one cycle of a signal in phase opposition with the pressure in the tube (see Fig. 5). The amplitude and phase of the “antisignal” were adjusted for maximum efficiency. The luminous emission from the flame was recorded by a high speed cine camera, equipped with a Cinemax intensifier and synchronized with the acoustic signal generator.

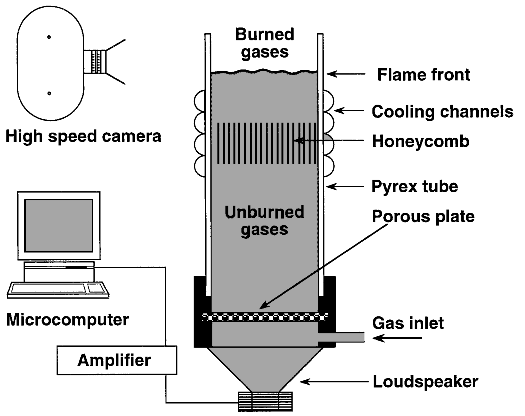

FIG. 4. Schematic diagram of experimental apparatus.

Experiments were performed with lean propane-air mixtures with equivalence ratios in the range $0.56 \leq \phi \leq 0.67$, corresponding to burning velocities in the range $11.5 \leq U_L \leq 20$ cm/s. For leaner flames, the front was intrinsically stable at all wavelengths. For faster flames, the two regions of instability overlapped and it was not possible to obtain a flat laminar flame by this method of stabilization. In order to control the wavelength and the spatial phase of the cellular structures that developed when the acoustic stabilization was removed, the upstream gas flow was perturbed by placing an array of parallel wires, 2 mm in diameter, on the downstream face of the honeycomb. The object of this scheme was to excite purely 2D cells at chosen wavelengths. The spacing between the wires was chosen to be an integral divisor of the tube diameter (i.e., $10/n$ cm). However, it was found that, if the spacing of the wires was not sufficiently close ($\approx \pm 30\%$) to the naturally most unstable wavelength, cells with this latter wavelength appeared, either compounded with the forced wavelength and/or in the direction perpendicular to the forcing. Because of these two limitations, it was not possible to measure the growth rate at a fixed flame speed over a significant range of wavelengths. However, it was possible to excite all flames in the range 11.5–20 cm/s at the same fixed wavelength of 2 cm. The luminous emission from the flame front was filmed edge on in a direction parallel to the axis of the cells. Figure 6 shows typical images taken from a high speed film of the flame during the growth of the 2D Darrieus-Landau instability.

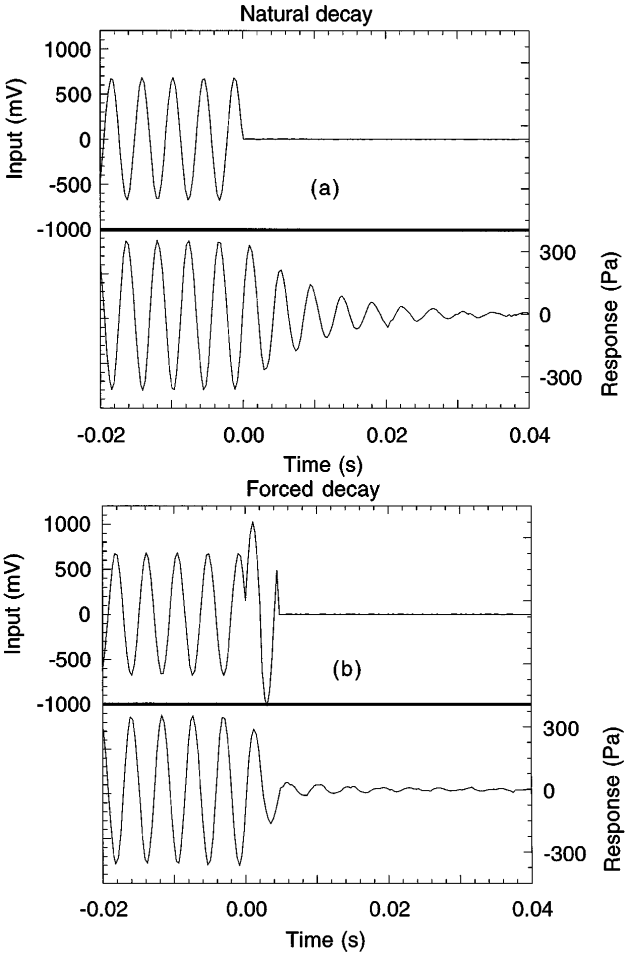

FIG. 5. Decay of acoustic level in tube: (a) Natural decay; (b) forced decay. Upper traces: Input to loudspeaker; lower traces: Acoustic pressure in tube.

The apparent thickening of the flame, particularly at high cell aspect ratios, indicates the presence of slight three dimensionality of the wrinkling. The peak-to-peak amplitude of the wrinkling was measured on digitized images and plotted in semilog coordinates as shown in Fig. 7.

The large scatter of the points in the early stages of the growth arise from the small amplitude of the cells, of the order of the apparent flame thickness. The nonlinearity at long times indicates the onset of saturation of the instability. The nonlinearity of the shape of the cells is clearly visible in Fig. 6 after 140 ms. These points were systematically eliminated before data reduction to obtain the growth rate. The points were fitted to an exponential function of the form

$$y(t) = \frac{1}{2}\left[\left(y_0 + \frac{\Delta v}{\sigma}\right)e^{\sigma t} + \left(y_0 - \frac{\Delta v}{\sigma}\right)e^{-\sigma t}\right], \quad (2)$$

which is the general solution of $\partial^2 y/\partial t^2 = \sigma^2 y$ with the initial conditions $y(0) = y_0$ and $\partial y(0)/\partial t = \Delta v$. Here, Δv is the rate of increase of the wrinkling at time $t = 0$, supposed equal to the measured peak-to-peak velocity modulation produced by the wires in the flow. The

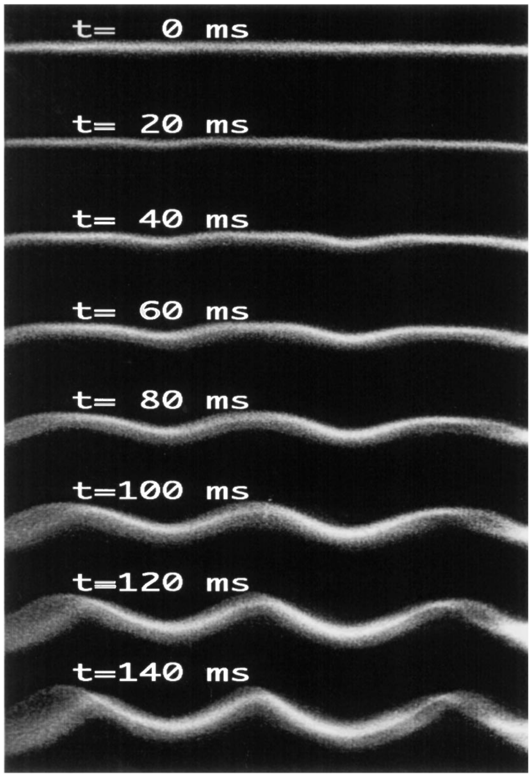

FIG. 6. Images taken from high speed film of growth of instability. Framing rate 500 images/s, wavelength 2 cm, flame speed 11.5 cm/s.

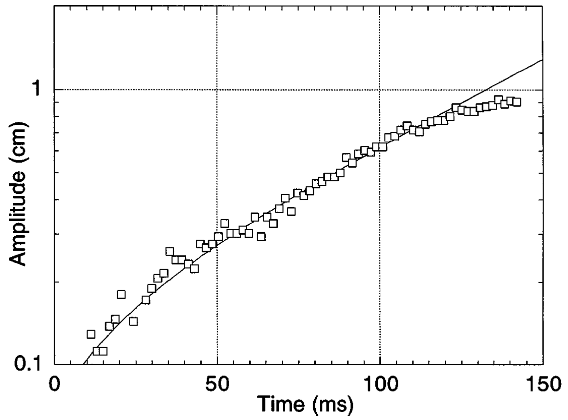

FIG. 7. Semi-log plot of the amplitude of the cellular structures as a function of time; fitted curve from Eq. (2).

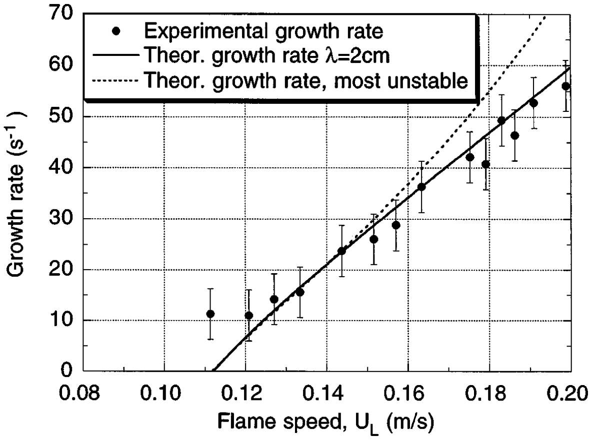

FIG. 8. Comparison of experimental and theoretical growth rates.

precision of the growth rate obtained by this procedure was $\pm 5\ \mathrm{s}^{-1}$. The experimentally measured growth rates are plotted as a function of laminar flame speed in Fig. 8. The error bars indicate $\pm 5\ \mathrm{s}^{-1}$. All measurements were made at a fixed (forced) wavelength of 2 cm. The solid line in Fig. 8 shows the theoretical growth rates obtained from the dispersion relation derived from Eq. (1). The values of flame speed U_L were taken from [9] and the gas expansion ratios were calculated using the CHEMKIN program [10]. A Markstein number of 4 was found to give the best agreement with the experimental data. This value of 4, for lean propane-air mixtures, may be compared with the value 5 found by Tseng *et al.* [11] and the value 4.3 found previously by Searby and Quinard [4]. For the purpose of comparison, the dotted line shows the growth rate calculated using the most unstable wavelength at each flame speed. The difference between the two theoretical curves is small, as expected, since the growth rate of the instability changes only slowly with wave number in the vicinity of the maximum. It can be seen that the experimental points agree with the theoretical curve at fixed wavelength to within experimental error, except for the measurement at the lowest flame velocity, where the experimentally measured value is higher than the theoretical value.

The reason for the discrepancy for very slow flames, close to the stability threshold, is not yet clear. One possibility investigated is that the time dependent increase in flame surface area could lead to an increase in total gas consumption and an acceleration of the flame towards the unburned gas. This downward acceleration would reduce the apparent acceleration of gravity seen by the flame and increase the growth rate of the instability. However, a numerical application, using the measured instantaneous flame area, shows that this second-order effect is stronger for faster flames and is never greater than a few percent. Thus this mechanism cannot explain the discrepancy observed for very slow flames.

In conclusion, we have used a novel technique of acoustic restabilization to obtain laminar planar flames of lean propane-air mixtures which are normally unstable with respect to the Darrieus-Landau instability. Phase inversion of the imposed acoustic signal is used to produce fast damping of the acoustic standing wave. The development of the Darrieus-Landau instability has then been observed, by a high speed camera, on initially planar freely propagating flame fronts. The wavelength at which the instability occurs is controlled by placing parallel rods in the upstream flow to produce a 2D perturbation. The effect of the initial perturbation on the development of the instability is included in the data reduction procedure. The measured values of the growth rate of the Darrieus-Landau instability agree well with the theoretical values, except for measurements made close to the theoretical threshold of stability.

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