

Determination of the CP Violation Angle γ Using $B \rightarrow D^*V$ Modes

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We propose a method to determine the angle $\gamma = \arg(V_{ub})$, using the $B \rightarrow D^*V$ ($V = K^*, \rho$) modes. The D^* is considered to decay to $D\pi$. An interference of the $B \rightarrow D^{*0}V$ and $B \rightarrow \overline{D}^{*0}V$ amplitudes is achieved by looking at a common final state f , in the subsequent decays of D^0/\overline{D}^0 . A detailed analysis of the angular distribution allows determination, not only of γ and $|V_{ub}|$ but also of all of the hadronic amplitudes and strong phases involved. No prior knowledge of doubly Cabibbo suppressed branching ratios of D are required. Large CP violating asymmetries ($\sim 30\%$ for $\gamma = 30^\circ$) are possible if $\overline{D}^0 \rightarrow f$ is doubly Cabibbo suppressed, while $D^0 \rightarrow f$ is Cabibbo allowed, for decays of B^+ or B^0 . [S0031-9007(98)05912-2]

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CP violation is one of the unsolved mysteries in particle physics. In the standard model, however, it is parametrized by including a phase in the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The aim of the several upcoming factories and detectors dedicated to studying B physics is to test this parametrization, by measuring the three angles of the unitarity triangle [2]. The angle γ , which is the phase of the element V_{ub} of the CKM matrix, is one of the most difficult to measure [2]. γ is also important, as its nonvanishing value is a signal of direct CP violation. Although CP violation was seen in K system more than 30 years ago, no signature of direct CP violation has yet been established.

One of the promising methods of measuring the angle γ is the so-called Gronau, London, and Wyler (GLW) method [3,4]. In this method, γ is obtained from an interference of the mode $B \rightarrow D^0K$ with $B \rightarrow \overline{D}^0K$, which occurs if and only if both D^0 and \overline{D}^0 decay to a common final state f ; in particular, f is taken to be a CP eigenstate. This technique of extracting γ requires a measurement of the branching ratio for $B^+ \rightarrow D^0K^+$ which is not experimentally feasible as pointed out in Ref. [5]. Moreover, the CP violating asymmetries tend to be small as the interfering amplitudes are not comparable. The use of non- CP eigenstates “ f ” has also been considered [6] in literature. Recently Atwood, Dunietz, and Soni (ADS) [5] extended this proposal by considering f to be non- CP eigenstates that are also doubly Cabibbo suppressed modes of D . The two interfering amplitudes then are of the same magnitude, resulting in large asymmetries. Their proposal is to use two final states f_1 and f_2 with at least one being a non- CP eigenstate. The use of more than one final state enables not only the determination of γ , but also of all of the strong phases involved and the difficult to measure branching ratio $\mathcal{B}(B^+ \rightarrow D^0K^+)$. However, an input into the determination of γ is the branching ratio of the doubly Cabibbo suppressed mode of D . Although D decays have been studied for a long time, only one doubly Cabibbo suppressed mode has been observed with an error that is currently as large as 50%.

In this Letter we extend these proposals to the corresponding decays of B into two vector mesons, by considering $B \rightarrow D^*V$, where V is either a K^* or ρ . The D^{*0}/\overline{D}^{*0} will decay into D^0/\overline{D}^0 which, if it subsequently decays to a final state f that is common to both D^0 and \overline{D}^0 , then the two decay channels $D^{*0}V$ and $\overline{D}^{*0}V$ can interfere, giving rise to the desired CP violating effects. The several amplitudes provided by the various partial waves of a single vector-vector final state enable us to extract γ , all the relevant hadronic amplitudes, and strong phases, thereby removing any hadronic uncertainties. Our approach does not require a prior knowledge of the poorly known doubly Cabibbo suppressed branching ratios of D , which in fact can be determined here, due to interference effects.

The most general covariant amplitude for a B meson decaying to a pair of vector mesons has the form [7,8],

$$A[B(p) \rightarrow V_1(k)V_2(q)] = \epsilon_1^{*\mu} \epsilon_2^{*\nu} \left(a g_{\mu\nu} + \frac{b}{m_1 m_2} p_\mu p_\nu + i \frac{c}{m_1 m_2} \epsilon_{\nu\alpha\beta} k^\alpha q^\beta \right), \quad (1)$$

where ϵ_1, ϵ_2 and m_1, m_2 represent the polarization vectors and the masses of the vector mesons V_1 and V_2 , respectively. The coefficients a, b , and c can be expressed in terms of the linear polarization basis A^\parallel, A^0 , and A^\perp [8]. If both of the vector mesons subsequently decay to two pseudoscalar mesons, i.e., $V_1 \rightarrow P_1 P'_1$ and $V_2 \rightarrow P_2 P'_2$, the amplitude can be expressed as

$$A(B \rightarrow V_1 V_2) = 4|\vec{k}_1| |\vec{q}_1| \times \left(-A^0 \cos \theta_1 \cos \theta_2 - \frac{A^\parallel}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \cos \phi + i \frac{A^\perp}{\sqrt{2}} \sin \theta_1 \sin \theta_2 \sin \phi \right), \quad (2)$$

where θ_1 (θ_2) is the angle between the P_1 (P_2) three-momentum vector \vec{k}_1 (\vec{q}_1) in the V_1 (V_2) rest frame and the direction of total V_1 (V_2) three-momentum vector defined in the B rest frame. ϕ is the angle between the normals to the planes defined by P_1P_1' and P_2P_2' in the B rest frame.

The differential decay rate is then given by [8,9]

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = N & \left(|A^0|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{|A^\perp|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \sin^2\phi + \frac{|A^\parallel|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \cos^2\phi \right. \\ & + \frac{\text{Re}(A^0 A^\parallel)^*}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos\phi - \frac{\text{Im}(A^\perp A^{0*})}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin\phi \\ & \left. - \frac{\text{Im}(A^\perp A^\parallel)^*}{2} \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi \right), \end{aligned} \quad (3)$$

where $N = \frac{|\vec{k}|}{16\pi^2 M^2} \frac{9}{4} \mathcal{B}(D^* \rightarrow D\pi)$. The rich kinematics of the vector-vector final state allows separation of each of the six combinations of the linear polarization amplitudes in the above. Using the Fourier transform in ϕ and the orthonormality of Legendry Polynomials in $\cos\theta_1$ ($\cos\theta_2$), it is possible to construct weight functions that project out each of these six combinations. An observable \mathcal{O}_i can then be determined from its weight factor \mathcal{W}_i , given in Table I, by using

$$\mathcal{O}_i = \int d\cos\theta_1 d\cos\theta_2 d\phi \frac{\mathcal{W}_i}{N} \frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi}.$$

The weight functions in Table I are not unique and they can be optimized through numerical simulations. No additional measurements are required in the determination of these observables, as the reconstruction of the vector-vector modes itself generates the angular distributions required.

We first focus our attention to the case of a charged B meson decaying to D^*V , $V \in \{K^*, \rho\}$. These final states involve only tree level amplitudes and no penguin contributions. The amplitude for the B^+ decays for a given linear polarization state “ λ ” can be written as

$$\begin{aligned} A^\lambda(B^+ \rightarrow D^{*0}V^+) &= V_{ub}^* V_{cq} \mathcal{A}_u^\lambda e^{i\delta_u^\lambda}, \\ A^\lambda(B^+ \rightarrow \overline{D}^{*0}V^+) &= V_{cb}^* V_{uq} \mathcal{A}_c^\lambda e^{i\delta_c^\lambda}, \end{aligned} \quad (4)$$

where $q = s$ for $V = K^*$ and $q = d$ for $V = \rho$; $\lambda = \{0, \parallel, \perp\}$. It may be noted that \mathcal{A}_u^λ and \mathcal{A}_c^λ are real. Since D^{*0} and \overline{D}^{*0} belong to different isodoublets, \mathcal{A}_u^λ and \mathcal{A}_c^λ , as well as the corresponding strong phases δ_u^λ and δ_c^λ , are not related. *No assumption is made regarding the explicit form of the amplitudes $A_{c,u}^\lambda$ or the strong phases $\delta_{c,u}^\lambda$.* For instance, the amplitudes $A_{c,u}^\lambda$ could include contributions from W -exchange and annihilation diagrams as well, since these involve the same CKM phases. Further, our approach does not require the use of factorization approximation. The amplitude for the antiparticle decay, $A^\lambda(B \rightarrow D^* \overline{V})$, has the same strong phases but opposite weak phases to that of $A^\lambda(B \rightarrow D^* V)$. In addition, using CPT invariance for the B^- decays, we get

$$\begin{aligned} A^\lambda(B^- \rightarrow D^{*0}V^-) &= \sigma^\lambda V_{cb} V_{uq}^* \mathcal{A}_c^\lambda e^{i\delta_c^\lambda}, \\ A^\lambda(B^- \rightarrow \overline{D}^{*0}V^-) &= \sigma^\lambda V_{ub} V_{cq}^* \mathcal{A}_u^\lambda e^{i\delta_u^\lambda}, \end{aligned} \quad (5)$$

where $\sigma^\perp = -1$, $\sigma^{0,\parallel} = 1$.

We consider D^{*0}/\overline{D}^{*0} decaying into $D^0\pi^0/\overline{D}^0\pi^0$, with the D^0/\overline{D}^0 meson further decaying to a final state f that is common to both D^0 and \overline{D}^0 . f is chosen to be a Cabibbo allowed mode of D^0 (hence, doubly suppressed mode of \overline{D}^0). To be specific, we may take $f = K^-\rho^+$, as this has the largest branching ratio among two-body hadronic decay modes, $\mathcal{B}(D^0 \rightarrow K^-\rho^+) \approx 10.8\%$ [10]. The accompanying V decays to $K\pi$ for $V = K^*$ and to $\pi\pi$ for $V = \rho$. In the $D^0 - \overline{D}^0$ system, CKM predicts negligible mixing effects, which we disregard. The amplitudes for the decays of B^+, B^- to a final state involving f and its CP conjugate will be a sum of the contributions from D^{*0} and \overline{D}^{*0} and can be written as

$$\begin{aligned} A_f^\lambda &= A^\lambda(B^+ \rightarrow [[f]_D \pi]_{D^*} V^+) \\ &= \sqrt{B} (V_{ub}^* V_{cq} \mathcal{A}_u^\lambda e^{i\delta_u^\lambda} + V_{cb}^* V_{uq} \mathcal{R} \mathcal{A}_c^\lambda e^{i\delta_c^\lambda} e^{i\Delta}), \\ \bar{A}_f^\lambda &= A^\lambda(B^- \rightarrow [[\bar{f}]_D \pi]_{D^*} V^-) \\ &= \sigma^\lambda \sqrt{B} (V_{ub} V_{cq}^* \mathcal{A}_u^\lambda e^{i\delta_u^\lambda} + V_{cb} V_{uq}^* \mathcal{R} \mathcal{A}_c^\lambda e^{i\delta_c^\lambda} e^{i\Delta}), \\ \bar{A}_f^\lambda &= A^\lambda(B^- \rightarrow [[f]_D \pi]_{D^*} V^-) \\ &= \sigma^\lambda \sqrt{B} (V_{ub} V_{cq}^* \mathcal{R} \mathcal{A}_u^\lambda e^{i\delta_u^\lambda} e^{i\Delta} + V_{cb} V_{uq}^* \mathcal{A}_c^\lambda e^{i\delta_c^\lambda}), \\ A_{\bar{f}}^\lambda &= A^\lambda(B^+ \rightarrow [[\bar{f}]_D \pi]_{D^*} V^+) \\ &= \sqrt{B} (V_{ub}^* V_{cq} \mathcal{R} \mathcal{A}_u^\lambda e^{i\delta_u^\lambda} e^{i\Delta} + V_{cb}^* V_{uq} \mathcal{A}_c^\lambda e^{i\delta_c^\lambda}), \end{aligned} \quad (6)$$

where $[X]_M$ indicates that the state X is reconstructed to have the invariant mass of M ; $B = \mathcal{B}(D^0 \rightarrow f)$, $\mathcal{R}^2 = \mathcal{B}(D^0 \rightarrow f)/\mathcal{B}(D^0 \rightarrow \bar{f})$, and Δ is the strong phase difference between $D^0 \rightarrow f$ and $D^0 \rightarrow \bar{f}$ (or that between $D^0 \rightarrow f$ and $\overline{D}^0 \rightarrow f$, since $D^0 \rightarrow \bar{f}$ and $\overline{D}^0 \rightarrow f$ have the same strong phase).

A measurement of the angular distribution given in Eq. (3), for each of the four modes noted above in (6) yield a total of twenty-four observables, six for each mode. These can be extracted experimentally using Table I. This is much larger than the sixteen unknowns: $R, \Delta, \gamma, |V_{ub}|$ and three variables for each $A_u^\lambda, A_c^\lambda, \delta_u^\lambda$, and δ_c^λ . Thus, γ would be overdetermined and sign ambiguities possibly resolved. Since $|V_{ub} V_{cq}^* \mathcal{R} \mathcal{A}_u^\lambda| \ll |V_{cb} V_{uq}^* \mathcal{A}_c^\lambda|$, the last two equations in Eq. (6) may not be distinguishable, i.e., $|\bar{A}_f^\lambda| \approx |A_{\bar{f}}^\lambda|$. This reduces the number of independent equations to eighteen, but still

allows γ to be determined. The conditions, $\mathcal{R}, \frac{\mathcal{A}_u^\lambda}{\mathcal{A}_c^\lambda} \ll 1$, can also help reduce the sign ambiguities.

It is well known that a study of the angular correlations can be used to extract CP violating asymmetries [11]. In addition to the usual signature of CP violation,

$$|A_f^\lambda|^2 - |\bar{A}_f^\lambda|^2 = 4|V_{ub}^* V_{cq} V_{cb} V_{uq}^*| \times \mathcal{R} B \mathcal{A}_u^\lambda \mathcal{A}_c^\lambda \sin(\delta_c^\lambda - \delta_u^\lambda + \Delta) \sin \gamma, \quad (7)$$

the complete study of the angular distribution of vector-vector final states provides the following alternative signatures for CP violation:

$$\begin{aligned} \text{Im}\{(A^\lambda A^{\rho*})_f + (\bar{A}^\lambda \bar{A}^{\rho*})_{\bar{f}}\} &= 2|V_{ub}^* V_{cq} V_{cb} V_{uq}^*| \mathcal{R} B \sin \gamma \\ &\times \{\mathcal{A}_u^\lambda \mathcal{A}_c^\rho \cos(\delta_u^\lambda - \delta_c^\rho - \Delta) - \mathcal{A}_c^\lambda \mathcal{A}_u^\rho \cos(\delta_c^\lambda - \delta_u^\rho + \Delta)\}, \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Im}\{(\bar{A}^\lambda \bar{A}^{\rho*})_{\bar{f}} + (A^\lambda A^{\rho*})_f\} &= 2|V_{ub}^* V_{cq} V_{cb} V_{uq}^*| \mathcal{R} B \sin \gamma \\ &\times \{\mathcal{A}_u^\lambda \mathcal{A}_c^\rho \cos(\delta_u^\lambda - \delta_c^\rho + \Delta) - \mathcal{A}_c^\lambda \mathcal{A}_u^\rho \cos(\delta_c^\lambda - \delta_u^\rho - \Delta)\}, \end{aligned} \quad (9)$$

$$\begin{aligned} \text{Im}\{(A^\lambda A^{\rho*})_f + (\bar{A}^\lambda \bar{A}^{\rho*})_{\bar{f}} + (\bar{A}^\lambda \bar{A}^{\rho*})_{\bar{f}} + (A^\lambda A^{\rho*})_f\} &= 4|V_{ub}^* V_{cq} V_{cb} V_{uq}^*| \mathcal{R} B \sin \gamma \cos \Delta \\ &\times \{\mathcal{A}_u^\lambda \mathcal{A}_c^\rho \cos(\delta_u^\lambda - \delta_c^\rho) - \mathcal{A}_c^\lambda \mathcal{A}_u^\rho \cos(\delta_c^\lambda - \delta_u^\rho)\}, \end{aligned} \quad (10)$$

where $\lambda = \perp$ and $\rho = \parallel$ or 0. The signals in Eqs. (8)–(10) are coefficients of $\sin \phi$ and $\sin 2\phi$ in the angular distribution in Eq. (3). The advantage here is that these signals of CP violation are not diluted by sine of strong phase as was the case in Eq. (7) and also that they are obtained by adding B and \bar{B} events. We wish to emphasize that the angle ϕ between the planes of the decay products of D^* and V plays a crucial role. If one measures only $|A_f^\lambda|^2$, $|\bar{A}_f^\lambda|^2$, $|\bar{A}_f^\lambda|^2$, and $|A_f^\lambda|^2$, and were to overlook the interference terms of the helicity amplitudes that appear in the complete angular distribution, one would have twelve observables by considering all three polarizations with a total of thirteen unknowns. Unless one of the variables is assumed to be measured elsewhere, γ cannot be extracted. The situation would be worse if $|\bar{A}_f^\lambda| \approx |A_f^\lambda|$, as there would be even fewer observables than unknowns, rendering γ truly unmeasurable if the $A^\lambda A^{\rho*} (\lambda \neq \rho)$ terms are ignored.

TABLE I. The weight factors corresponding to the observables in the angular distribution [Eq. (3)] for $B \rightarrow VV$ decays. Note that the weight factors would give identical results under $\theta_1 \leftrightarrow \theta_2$

Observable \mathcal{O}_i	Weight \mathcal{W}_i
$ A^0 ^2$	$\frac{3}{16\pi} (15 \cos^2 \theta_1 - 3)$
$ A^\parallel ^2$	$\frac{3}{16\pi} (-6 + 12 \cos^2 \phi + \frac{9-15 \cos^2 \theta_1}{2})$
$ A^\perp ^2$	$\frac{3}{16\pi} (6 - 12 \cos^2 \phi + \frac{9-15 \cos^2 \theta_1}{2})$
$\text{Re}(A^0 A^{\parallel*})$	$\frac{2^5 \sqrt{2}}{\pi^3} \cos \phi \cos \theta_1 \cos \theta_2$
$\text{Im}(A^\perp A^{0*})$	$-\frac{2^5 \sqrt{2}}{\pi^3} \sin \phi \cos \theta_1 \cos \theta_2$
$\text{Im}(A^\perp A^{\parallel*})$	$-\frac{9}{8\pi} \sin 2\phi$

Note that since all of the amplitudes and strong phases involved in the right-hand side of Eq. (6) are solved, using the observables constructed from these amplitudes, we need not disentangle the strong phases associated with each isospin state of the various partial waves.

In the case of neutral B mesons the $D^* K^*$ decay mode is self-tagging [4] if K^{*0}/\bar{K}^{*0} is seen in the $K^+ \pi^- / K^- \pi^+$ mode. Hence, no time dependent measurements are required, and the observables for the decays of B^0 and \bar{B}^0 to any final state f and its CP conjugate may be obtained by the replacement of the charged B decay amplitudes $A_{u,c}^\lambda$ by the corresponding neutral B amplitudes $a_{u,c}^\lambda$ in Eq. (6). Within factorization approximation, a_c differs from A_c due to the fact that the charged B decay amplitudes include contributions from both color allowed as well as color suppressed diagrams, whereas neutral B decay amplitudes come only from the color suppressed diagrams; a_u and A_u , however, are identical. The signatures of CP violation are similar to Eqs. (8)–(10), with $A_{c,u}$ replaced by $a_{c,u}$. Even in the case where tagging is not possible, B^0 and \bar{B}^0 observables can be added resulting in an asymmetry independent of the mixing parameters, $\Delta m/\Gamma$ and β , and again of the same form as in Eqs. (8)–(10). The addition of B^0 and \bar{B}^0 observables reduces the number of available equations and, hence, we need to consider D^0/\bar{D}^0 decaying not only to the final state f but also to an additional CP eigenstate. Further, all three linear polarization states will have to be analyzed. *This makes it possible to extract γ without any need for time or flavor tagging.*

Next, we construct CP violating asymmetries corresponding to the signals suggested in Eq. (8). As pointed out earlier, the coefficients of the $\sin \phi$ and $\sin 2\phi$ terms need to be isolated in order to obtain $\text{Im}(A^\perp A^{0*})$ and $\text{Im}(A^\perp A^{\parallel*})$ terms, respectively. The coefficient of the $\sin \phi$ term in Eq. (3) can be determined by defining the

following asymmetry:

$$A_1 = \frac{\left(\int_0^\pi - \int_{2\pi}^\pi\right) d\phi \int_D d\cos\theta_1 \int_D d\cos\theta_2 \frac{d\Gamma_{\text{sum}}}{d\Omega}}{\int_0^{2\pi} d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{\text{sum}}}{d\Omega}},$$

where $\int_{D(S)} \equiv \int_{-1}^0 + \int_0^1$, $d\Omega = d\cos\theta_1 d\cos\theta_2 d\phi$, and $\Gamma_{\text{sum}} = \{\Gamma(B \rightarrow [[f]_D \pi]_{D^*} V) + \Gamma(\bar{B} \rightarrow [[\bar{f}]_D \pi]_{D^*} \bar{V})\}$. On performing the angular integrals, this asymmetry is equivalent to

$$A_1 = \frac{-2\sqrt{2} \text{Im}\{(A^\perp A^{0*})_f + (\bar{A}^\perp \bar{A}^{0*})_{\bar{f}}\}}{\pi \sum_{\lambda=\perp, \parallel, 0} (|A_f^\lambda|^2 + |\bar{A}_{\bar{f}}^\lambda|^2)}. \quad (11)$$

Yet another symmetry comes from the coefficient of the $\sin 2\phi$ term in Eq. (3) and is defined as

$$A_2 = \frac{\int_Q d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{\text{sum}}}{d\Omega}}{\int_0^{2\pi} d\phi \int_S d\cos\theta_1 \int_S d\cos\theta_2 \frac{d\Gamma_{\text{sum}}}{d\Omega}},$$

$$= \frac{-4 \text{Im}\{(A^\perp A^{\parallel*})_f + (\bar{A}^\perp \bar{A}^{\parallel*})_{\bar{f}}\}}{\pi \sum_{\lambda=\perp, \parallel, 0} (|A_f^\lambda|^2 + |\bar{A}_{\bar{f}}^\lambda|^2)}, \quad (12)$$

where $\int_Q = (\int_0^{\pi/2} - \int_{\pi/2}^\pi + \int_{3\pi/2}^\pi - \int_{\pi/2}^{3\pi/2})$. The asymmetries A_1 and A_2 can be similarly constructed for the signals in Eqs. (9) and (10). However, these will be much smaller as they involve interference of amplitudes that are not comparable.

We now compute a rough estimate of the number of B 's required to observe the CP violating signal in our method. Exact numbers can of course only be obtained once the strong phases $\delta_{u,c}^\lambda$ and Δ , as well as the amplitudes $A_{u,c}^\lambda$, are determined from the observables measured experimentally. For our estimates, we set $\delta_{u,c}^\lambda = \Delta = 0$ and $|V_{ub}V_{cs}/(V_{cb}V_{us})| = 0.38$. The form factors and decay constants are chosen from Ref. [12] (which uses the factorization approximation), and the ratio of the coefficients of color suppressed ($\sim a_2$) to color allowed ($\sim a_1$) amplitudes (as defined in Ref. [12]) is taken to be $|a_2/a_1| \approx 0.26$ [13]. \mathcal{R} is estimated as [5,14]

$$\mathcal{R}^2 = \frac{\mathcal{B}(\bar{D}^0 \rightarrow K^- \pi^+)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)} = \frac{\mathcal{B}(\bar{D}^0 \rightarrow K^- \rho^+)}{\mathcal{B}(D^0 \rightarrow K^- \rho^+)}$$

$$= 0.0077.$$

The resulting asymmetries for $B^+ \rightarrow D^* K^{*+} (B^+ \rightarrow D^* \rho^+)$ at $\gamma = \frac{\pi}{6}$ are found to be $A_1 = -28\%$ (0.5%) and $A_2 = 9.1\%$ (-0.16%). The total number of charged B 's required to observe these asymmetries at 3σ significance are $N_1^{3\sigma} = 7.6 \times 10^8$ (4.2×10^{10}) and $N_2^{3\sigma} = 7.3 \times 10^9$ (4.0×10^{11}). The number of B 's required can easily be reduced by a factor of ~ 3 -4, if one sums over all the doubly Cabibbo suppressed modes of

\bar{D}^0 . The corresponding asymmetries for the neutral B 's vanish identically, under the factorization approximation, in the absence of strong phases. This is due to the fact that the factorization approximation implies $a_u^\lambda = a_c^\lambda$. A test of this relation would provide a unique model independent test of the factorization approximation.

In conclusion, we have extended the GLW and ADS proposals to measure γ using vector-vector final states. The rich kinematics of these modes provide a large number of observables that can be obtained using appropriate weight functions, if the angular distributions are available. The reconstruction of these modes generates the angular distributions required. One particular final state is enough to extract γ as well as all of the hadronic amplitudes and strong phases involved.

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