Cumulative Parity Violation in Supernovae

C. J. Horowitz* and Gang Li†

Nuclear Theory Center, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47405

(Received 16 May 1997)

Supernovae provide a unique opportunity for large scale parity violation because they are dominated by neutrinos. We calculate the parity violating asymmetry A of neutrino emission in a strong magnetic field. We assume the neutrinos elastically scatter many times from slightly polarized neutrons. Because of the multiple interactions, A grows with the optical thickness of the protoneutron star and may be much larger than previous estimates. As a result, the neutron star could recoil at a significant velocity. [S0031-9007(98)05940-7]

PACS numbers: 97.60.Bw, 11.30.Er, 95.30.Cq

Weakly interacting neutrinos dominate core collapse supernovae. This provides a unique opportunity for large scale or macroscopic parity violation [1,2]. Therefore, it is of fundamental importance to study parity violation in supernovae.

We focus on an asymmetry induced in the explosion because of parity violation in a strong magnetic field. This asymmetry could lead to a recoil of the newly formed neutron star. Indeed, neutron stars have large velocities $\approx$500 km/s [3]. An asymmetry of order one percent could produce these velocities [4] (see below). However, this asymmetry need not arise from parity violation (see, for example, [5]).

Strong magnetic fields are present in pulsars. Indeed, external dipole fields of 10^{12} to 10^{13} G are inferred in many cases [6]. In this paper, we estimate the magnitude of parity violating effects from *known* neutrino interactions in fields near 10^{13} G. Others have speculated on parity violating effects in much stronger fields [1,2,7–11] and with new neutrino interactions [12]. We find that repeated interactions as neutrinos diffuse through an optically thick medium may greatly enhance the asymmetry.

There is some controversy on observational correlations between recoil velocities and magnetic field strength. For example, Birkel and Toldra [13] argue that for rapidly spinning pulsars there is no correlation between the recoil velocity and the projection of B on the spin axis. We think their analyses may be over simplified because they do not consider possible effects of rapid rotation on the dynamics of the collapse and on the asymmetry. Unfortunately, observational tests involve incomplete information. The strength of the external dipole field is inferred. However, little is known about nondipole fields. We find that the most important variable may be the volume of the core occupied by the strong field (see below) rather than simply its strength. Thus we keep an open mind with respect to present observations.

Independent of observation, it is important to estimate parity violating effects. We see three possibilities: parity violating effects could be small and thus irrelevant, they could be large and observed, or they could be potentially large and not observed. If they are not observed, it may still be possible to set useful limits on the magnetic field configuration or on new weak interactions. In any case, we need accurate theoretical estimates.

Previous estimates of parity violation may be incomplete because they ignore possible enhancements from repeated interactions. Neutrino transport involves diffusion with neutrinos undergoing many parity violating interactions before they escape. We estimate these cumulative effects below.

Much previous work focused on electrons [1,11]. It is natural to think that neutrino electron scattering will dominate the asymmetry because the electron's magnetic moment is 1000 times that of a nucleon. However, the electron polarization is reduced because they are relativistic and degenerate. Furthermore, because of the small ν-e cross section this polarization may lead to an asymmetry that is *smaller* than that from nucleon reactions. It is important to examine other processes to identify the largest contribution to the asymmetry.

In this paper, we consider neutrino elastic scattering from slightly polarized neutrons. This may be important (even though the neutron polarization is small). We do not claim that it is the largest contribution. Instead we focus on elastic neutron scattering for simplicity. The differential cross section is (see, for example, [14]),

$$d\sigma/d\Omega = \frac{G^2E_\nu^2}{4\pi^2}\{c_v^2 + 3c_a^2 + (c_v^2 - c_a^2)\cos\theta + 2P_n c_a[(c_v - c_a)\cos\theta_{\rm in} + (c_v + c_a)\cos\theta_{\rm out}]\}. \tag{1}$$

Here G is the Fermi constant, E_ν the neutrino energy (assumed much smaller than the nucleon mass M), and $c_v = -1/2$, $c_a = -g_a/2$ with $g_a = 1.26$. The incident neutrino momentum makes an angle $\theta_{\rm in}$ with the polarization direction, scatters through an angle θ, and then the outgoing momentum is at an angle $\theta_{\rm out}$ with the polarization.

The polarization of the neutrons P_n depends on the magnetic field B and temperature T, $P_n \approx eB/MT$,

$$P_n \approx 6 \times 10^{-6}\left[\frac{B}{10^{13}\text{ G}}\right]\left[\frac{10\text{ MeV}}{T}\right]. \qquad (2)$$

In the limit $g_a = 1$, Eq. (1) becomes

$$d\sigma/d\Omega = \sigma_0(1 + P_n \cos\theta_{\text{out}})\,, \qquad (3)$$

with $\sigma_0 = G^2 E_\nu^2/4\pi^2$. This simple form provides insight and is good to 10 percent for the asymmetry. [However, we use the full result Eq. (1) in the Monte Carlo below.] Equation (3) does not depend on θ_{in}. Thus the mean free path is independent of direction. The asymmetry arises because the outgoing neutrino angular distribution is biased towards the polarization direction.

The total dipole asymmetry A in the neutrino angular distribution $I(\theta)$ is

$$A = \int_{-1}^{1} d\cos\theta I(\theta)\cos\theta \Bigg/ \int_{-1}^{1} d\cos\theta I(\theta)\,. \qquad (4)$$

If the angular distribution from the supernova is proportional to Eq. (3), then $A = P_n/3$. This asymmetry is related to the recoil velocity of the star.

The gravitational binding energy of a neutron star is of order 100 MeV per nucleon. This is radiated away in neutrinos of momentum 100 MeV$/n$ leaving a protoneutron star of mass about 839 MeV$/n$. Therefore the recoil velocity v of the star is

$$v/c \approx \frac{100}{839} A \approx 0.1A\,. \qquad (5)$$

Thus if A is only of order P_n the velocity will be small (around 10^{-6} of the speed of light c for B near 10^{13} G).

Neutrinos must diffuse through many mean free paths in order to escape the star so they interact repeatedly with the polarized neutrons. The crucial question is do these repeated interactions enhance the asymmetry? If the temperature distribution at the neutrino sphere is independent of direction then what happens inside may not be important. The asymmetry in the neutrino flux will simply arise from the last scattering and be of order P_n.

However, neutrinos dominate the energy transport. [Note, the effects of convection on A remain to be investigated.] Therefore we expect the temperature distribution to be asymmetric because of the asymmetric neutrino flux. This could lead to an asymmetry much larger than P_n. To investigate multiple interactions, we calculate the asymmetry of a "reference configuration" with a very simple Monte Carlo.

This reference configuration is not meant to be a realistic supernova simulation. Instead, it is the simplest system with slight nucleon polarization and repeated neutrino interactions. This may allow a simple exploration of the physics. We consider a uniform sphere of slightly polarized neutrons. The neutrinos start either at the center or uniformly throughout the volume and then interact only via elastic neutron scattering, Eq. (1). We discuss these assumptions below.

The model has two parameters: the polarization of the neutrons P_n and the optical depth of the sphere r/λ. This is the radius r measured in units of the neutrino mean free path λ. For $B = 1.7 \times 10^{13}$ G and $T = 10$ MeV the polarization is

$$P_n \approx 1 \times 10^{-5}. \qquad (6)$$

Near the center of the star the neutrons will become slightly degenerate. This will decrease P_n somewhat. Also at high densities, strong interactions may modify P_n. It is even possible that there is a ferromagnetic phase of dense neutron rich matter [15]. (While this is unlikely at the high temperatures of a supernova, there could still be an enhancement in P_n.) There could also be some cancellation from scattering off of protons. Perhaps most importantly, we are assuming that the strong B field penetrates the central region. If B is excluded, the average polarization will be lower. Alternatively, there could be regions of very high internal fields. For simplicity, we adopt Eq. (6) for P_n and assume that it is uniform throughout the sphere. Of course, the final asymmetry is proportional to P_n so it is easy to consider other values.

We chose the optical depth so that the time scale for neutrinos to diffuse out of the sphere is approximately correct. Neutrinos were detected from SN 1987A over about 10 sec [16]. The escape time t is of order the light travel time r/c ($\approx$0.1 msec) multiplied by the optical depth r/λ. For t to be of order 1 sec requires

$$r/\lambda \approx 1 \times 10^4. \qquad (7)$$

This value is consistent with theoretical simulations [17].

We have calculated the overall asymmetry, Eq. (4) using a very simple Monte Carlo code; see Fig. 1. To save computer time we use a larger $P_n = 0.01$ than

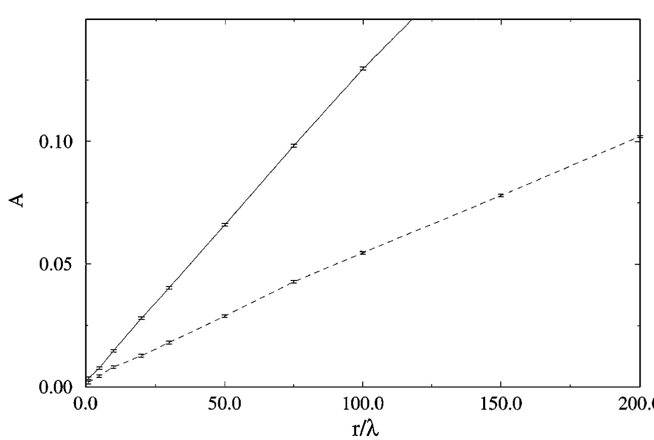

FIG. 1. Asymmetry A, Eq. (4), of neutrinos emitted from a neutron sphere of optical depth r/λ. The polarization of the neutrons is $P_n = 0.01$. The solid curve assumes the neutrinos start at $r = 0$, while the dotted curve is for neutrinos starting uniformly throughout the volume.

Eq. (6) and smaller r/λ than Eq. (7). The results can be scaled to the desired values. We find that *the asymmetry grows with* r/λ,

$$A \approx \alpha P_n\left(\frac{r}{\lambda}\right) + O\left(P_n \frac{r}{\lambda}\right)^2. \tag{8}$$

This is our most important result. Note, the second term in Eq. (8) is needed since A saturates for very large r/λ. The coefficient α arises primarily from the angle averaging in the Monte Carlo. For an $r = 0$ source $\alpha \approx 0.14$. For uniformly distributed sources α is somewhat smaller $\alpha \approx 0.057$. This reflects the shorter path length for neutrinos starting close to the surface. However, A still grows strongly with r/λ.

The linear dependence of Eq. (8) on r/λ can be understood with a simple one-dimensional biased random walk. At each time step the probability to hop left is $0.5 + P_n/2$ (and right is $0.5 - P_n/2$). Because of this bias, the mean value of the neutrino's position $\langle x\rangle$ is not zero but drifts left with a velocity of order P_n. Therefore $\langle x\rangle$ is proportional to t. In contrast, the width of the neutrino distribution grows because of diffusion but only with $t^{1/2}$. The macroscopic asymmetry A depends on the ratio of the mean value to the width. Therefore A grows with $t/t^{1/2} = t^{1/2}$. Finally, the time to escape t is proportional to $(r/\lambda)^2$ so A grows linearly with r/λ.

For the parameters of Eqs. (6),(7) the reference configuration asymmetry is

$$A \approx 0.014 \tag{9}$$

(assuming an $r = 0$ source). This value is interesting and much larger than previous estimates. However, it is only a ballpark estimate, and we must discuss a number of our assumptions.

First, we considered only neutrino-neutron elastic scattering. For mu and tau neutrinos this is a reasonable first approximation to the opacity. However, pair production and annihilation can change the number of neutrinos. Multiple interactions will tend to bring neutrinos into thermodynamic equilibrium with the other matter. The number of neutrinos then depends on the temperature. Thus, the asymmetric neutrino flux will lead to an asymmetric temperature. One side of the star will be warmer than the other side. We expect the asymmetry in the temperature to be small near $r = 0$ and grow as one moves out nearer the neutrino sphere. Microscopic simulations to study the temperature distribution would be very useful.

For electron neutrinos one should also consider neutrino capture followed by electron capture. These reactions also have asymmetries. For electron capture the asymmetry depends on the electron polarization P_e, which can be somewhat higher than for neutrons. However, P_e is multiplied by the small coefficient $(c_v^2 - c_a^2)/(c_v^2 + 3c_a^2) \approx 0.1$ [7,9] so its contribution may be similar to Eq. (1).

It is important to discuss antineutrinos. The antineutrino-neutron elastic cross section is obtained from Eq. (1) by switching $\cos\theta_{\text{in}}$ with $\cos\theta_{\text{out}}$. This will lead to an opposite sign for the asymmetry. Thus there will be some cancellation between neutrinos and antineutrinos. If one assumes that all ν and $\overline{\nu}$ cancel except for the extra ν which carry away the net lepton number then Eq. (8) will be reduced by a factor $(N_{\nu_e} - N_{\overline{\nu}_e})/N_{\text{tot}} \approx 0.1$ so that (for an $r = 0$ source),

$$A_{\text{tot}} \approx 0.14P_n\left(\frac{r}{\lambda}\right)0.1 \approx 0.008B_{14}\,, \tag{10}$$

where $B_{14} = B/10^{14}$ G. This will produce a recoil velocity,

$$v \approx 250B_{14}\ \text{km/s}\,, \tag{11}$$

for the neutron star.

Neutrino and antineutrino asymmetries will *add* in calculating the net lepton number flux. This could lead to significant asymmetries in the trapped lepton fraction and in the chemical composition. This may have an important impact on the explosion. For example, on one side of the star the larger neutrino flux may enhance the proton fraction, while on the other side the antineutrino flux may increase the number of neutrons. One should investigate asymmetries in the chemical composition.

In this paper we examined the asymmetry induced in a supernova from repeated parity violating neutrino-neutron elastic scatterings in a strong magnetic field. As a model, we calculated the asymmetry of neutrinos emerging from an optically thick sphere of slightly polarized neutrons. We find that the asymmetry *grows* with the optical thickness through which the neutrinos diffuse. This is because the neutrinos interact very many times and leads to parity violating effects that are much larger than in previous estimates.

Future work should examine asymmetries induced in the temperature and chemical composition of the protoneutron star. One should also examine asymmetries in other reactions and the magnetic properties of very dense neutron rich matter. We note that parity violation may be sensitive to conditions deep inside the protoneutron star in addition to near the neutrino sphere. In particular, the volume of the core occupied by a strong magnetic field may influence the asymmetry.

This work was supported in part by DOE Grant No. DE-FG02-87ER40365.

Note added.—We recently became aware of two other works based on this paper. H. Th. Janka has performed some further Monte Carlo simulations which support our conclusions [18]. Lai and Qian present a diffusion calculation also with similar conclusions [19].

*Electronic address: charlie@iucf.indiana.edu
†Electronic address: ganli@indiana.edu

[1] A. Vilenkin, report, 1979 (unpublished); Astrophys. J. **451**, 700 (1995).

[2] N. N. Chugai, Pis'ma Astron. Zh. **10**, 210 (1984) [Sov. Astron. Lett. **10**, 87 (1984)].
[3] A. G. Lyne and D. R. Lorimer, Nature (London) **369**, 127 (1994).
[4] S. E. Woosley, in "The Origin and Evolution of Neutron Stars," edited by D. J. Helfand and J. H. Huang (Reidel, Dordrecht, 1987), p. 255.
[5] A. Burrows and J. Hayes, Phys. Rev. Lett. **76**, 352 (1996).
[6] V. S. Beskin, A. V. Gurevich, and Ya. N. Istomin, *Physics of the Pulsar Magnetosphere* (Cambridge University Press, Cambridge, 1993).
[7] O. F. Dorafeev, V. N. Rodionov, and I. M. Ternov, Sov. Astron. Lett. **11**, 123 (1985).
[8] R. C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992).
[9] C. J. Horowitz and J. Piekarewicz, Los Alamos Report No. hep-ph/9701214.
[10] A. V. Kuznetsov and N. V. Mikheev, Phys. Lett. B **394**, 123 (1997).
[11] V. G. Bezchastnov and P. Haensel, Phys. Rev. D **54**, 3706 (1996).
[12] A. Kusenko and G. Segré, Phys. Rev. Lett. **77**, 4872 (1996); E. Kh. Akhiredov, A. Lanza, and D. W. Sciama, hep-ph/9702436.
[13] Michael Birkel and Ramon Toldra, Los Alamos Report No. astro-ph/9704138.
[14] C. J. Horowitz and K. Wehrberger, Nucl. Phys. **A531**, 665 (1991).
[15] M. Kutschera and W. Wojcik, Phys. Lett. **B223**, 11 (1989).
[16] K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987); R. M. Bionta *et al.*, Phys. Rev. Lett. **58**, 1494 (1987).
[17] Adam S. Burrows, in *Supernovae*, edited by A. G. Petschek (Springer-Verlag, New York, 1990), p. 143.
[18] H.-Th. Janka, in *Proceedings "Neutrino Astrophysics,"* edited by M. Altmann *et al.* (Sonderforschungsbereich, Technische Universität München, Garching, 1998) (Report No. astro-ph/9801320), p. 60.
[19] Dong Lai and Yong-Zhong Qian, Astrophys. J. Lett. **495**, L103 (1998); erratum (to be published).