Theory of ac Josephson Effect in Superconducting Constrictions

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We have developed a microscopic theory of the ac Josephson effect within the general model of a high-transparency Josephson junction: short superconducting constriction with either ballistic or diffusive electron transport. Applications of the theory were studied, including smearing of the subgap current singularities by pair-breaking effects in the superconducting electrodes of the constriction, and the structure of these singularities in constrictions between the composite superconducting-normal electrodes with the proximity-induced gap in the normal layer. [S0031-9007(98)05901-8]

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The mechanism of electron transport in high-transparency Josephson junctions at finite bias voltages V is known to be the process of multiple Andreev reflections (MAR) [1]. Considerable progress has been made recently in quantitative understanding of this process. Development of the controllable break junction technique [2] made it possible to study the current-voltage (I-V) characteristics of atomic-size Josephson junctions with transparency D varying from the tunnel-junction limit $D \ll 1$ to almost ballistic contacts with $D \to 1$. Junctions of other types that exhibit MAR-related features in the I-V characteristics include semiconductor/superconductor heterostructures [3] with different mechanisms of electron transport, and tunnel junctions with high critical current density [4]. Most of these junctions have complex structure, with superconductivity in the junction electrodes complicated by the proximity effects and elastic and inelastic scattering. The aim of this work was to develop the theory of MAR in high-transparency Josephson junctions with arbitrary microscopic structure, including inelastic scattering and pair-breaking processes. All these effects cannot be accounted for within the existing approaches to MAR [5–8] based on the Bogolyubov–de Gennes (BdG) equations [5–7] or Hamiltonian methods [8], and require the Green's function technique. Despite the long history of applications of this technique to ac Josephson effect [6(b),9–12], there is still no tractable theory of this effect in junctions with arbitrary transparency. In this work we present such a theory and apply it to study the smearing of the MAR-related current singularities by pair-breaking scattering, and the ac Josephson effect between the normal conductors with the proximity-induced superconducting order parameter.

The basic model of a high-transparency Josephson junction is a short superconducting constriction (shorter than the coherence length ξ and elastic and inelastic scattering lengths of the constriction) with a transparency D. In the Green's function technique, the constriction is described with the nonequilibrium quasiclassical Green's function $\check{G}$ which is a 4×4 matrix consisting of 2×2 retarded, advanced, and Keldysh matrixes $\hat{G}^{R,A}$ and $\hat{G}$ [13]. To calculate the current we need to know the asymmetric part $\check{J}$ of the Green's function $\check{J} = \check{G}(p_{zj}) - \check{G}(-p_{zj})$, where p_z is the momentum in transport direction, and $j = 1, 2$ numbers the constriction electrodes. Solving the quasiclassical equations for $\check{G}$ inside the two electrodes and matching the solution across the constriction with the boundary conditions [10(b)] one can show that the matrix $\check{J}(t, t')$ is continuous inside the constriction and is given by the expression—see Eq. (35) of [10(b)]:

$$\begin{aligned}\check{J} &= 2D\check{G}_- * \check{G}_+ * (\check{\mathbf{1}} - D\check{G}_- * \check{G}_-)^{-1} \\ &= \check{G}_- * (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} - (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} * \check{G}_- . \qquad (1)\end{aligned}$$

Here the product denoted by $*$ means the convolution with respect to the internal time variable, i.e., $\check{G}_\mu * \check{G}_\eta = \int dt_1 \check{G}_\mu(t, t_1)\check{G}_\eta(t_1, t')$, and we used the following notations: $\lambda = \sqrt{(1 - D)/D}$, $\check{\mathbf{1}} = \check{1}\delta(t - t')$, $\check{G}_\pm = (\check{G}_1 \pm \check{G}_2)/2$, $\check{G}_j(t, t') = \check{S}_j(t)\check{g}_j(t - t')\check{S}_j^*(t')$. In the last equation, $\check{g}_j(t) = \int \check{g}_j(\epsilon) \exp(-i\epsilon t)\, d\epsilon/(2\pi)$ is the equilibrium Green's function of jth superconductor: $\hat{g}_j(\epsilon) = [\hat{g}_j^R(\epsilon) - \hat{g}_j^A(\epsilon)] \tanh(\epsilon/2T)$, $\hat{g}_j^R(\epsilon) = g_j^R(\epsilon)\hat{\tau}_z + f_j^R(\epsilon) i\hat{\tau}_y = -[\hat{g}_j^A(\epsilon)]^*$, and $\check{S}_j(t) = \exp[i\varphi_j(t)\check{\tau}_z/2]$, where $\varphi_j(t)$ is the phase of the order parameter of the jth electrode. The current $I(t)$ is determined by the Keldysh component of $\check{J}$ as $\mathrm{Tr}\, \hat{\tau}_z \hat{J}(t, t)$ [13], and using Eq. (1) we can write it in following symmetric form:

$$I(t) = \frac{V(t)}{R_N} + \delta I_{12}(t) + \delta I_{21}(t),$$

$$\delta I_{jk}(t) = \frac{\pi}{4eR_0} \mathrm{Tr}\, \hat{\tau}_z (\hat{G}_j^R * \hat{q}_{jk} - \hat{q}_{jk} * \hat{G}_j^A)(t, t). \qquad (2)$$

In these equations, we have separated the normal-state current V/R_N, where $R_N = R_0/D$, and $R_0 = \pi\hbar/e^2$ for a single-mode constriction. Other notations in Eq. (2) are as follows: $\hat{q}_{jk} = \hat{q}_{jk}^R * (\hat{g}_k^R * f - f * \hat{g}_k^A) * \hat{q}_{jk}^A$, $\hat{q}_{jk}^\mu = 2/(\hat{G}_j^\mu + \hat{g}_k^\mu + 2i\lambda\hat{\mathbf{1}})^{-1}$, $f(t, t') = \int f(\epsilon) \exp[-i\epsilon(t - t')] d\epsilon/2\pi$, and $f(\epsilon) = \tanh(\epsilon/2T)$. For a constriction

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with a cross-section area $\mathcal{A}$ and a large number of propagating electron modes, angular averaging over directions of momentum should be carried out in (2), and $R_0 = (2\pi/ep_{F1})^2\hbar^3/\mathcal{A}$, where $p_{F1} = \min p_{Fj}$. The functions $\hat{G}_1^\mu$ and $\hat{G}_2^\mu$ in Eq. (2) are $\hat{G}_j^\mu = \hat{S}(t)\hat{g}_j^\mu(t - t')\hat{S}^*(t')$, where $\hat{S}(t) = \exp[i\varphi(t)\hat{\tau}_z/2]$, and the phase difference $\varphi = \varphi_2 - \varphi_1$ is determined by the applied voltage $V(t)$: $\dot{\varphi}(t) = (2e/\hbar)V(t)$. It is convenient to express $\hat{q}_{jk}^R$ and $\hat{q}_{jk}^A$ through the matrices $(\hat{G}_j^R + \hat{\tau}_z) * \hat{q}_{jk}^R * (\hat{g}_k^R + \hat{\tau}_z)$ and $(\hat{g}_k^A - \hat{\tau}_z) * \hat{q}_{jk}^A * (\hat{G}_j^A - \tilde{\tau}_z)$, where $\check{\tau}_z = \hat{\tau}_z\delta(t - t')$. After this transformation, Eq. (2) is simplified and can be written in terms of the functions $\hat{\alpha}_{jk}^{R,A}$ obeying the following equations:

$$(\hat{\rho}_k^R - \hat{\Gamma}_j^R * \hat{\eta}_k^R) * \hat{\alpha}_{jk}^R = r\hat{\mathbf{1}} + \hat{\Gamma}_j^R, \tag{3}$$

and $\hat{\alpha}_{jk}^A(\epsilon, \epsilon') = [\hat{\alpha}_{jk}^R(\epsilon', \epsilon)]^\dagger$. Here $r = \sqrt{1 - D}$ is the reflection amplitude of the constriction, $\hat{\Gamma}_j^R(t, t') = \Gamma_j^R(t - t')\hat{S}(t)\hat{S}(t')\hat{\tau}_x$, $\Gamma_j^R(t) = \int \gamma_j^R(\epsilon)\exp(-i\epsilon t)d\epsilon/(2\pi)$, $\gamma_j^R(\epsilon) = f_j^R(\epsilon)/[g_j^R(\epsilon) + 1]$, $\gamma_j^A(\epsilon) = \gamma_j^{R*}(\epsilon)$, $\rho_k^R(t, t') = \hat{\mathbf{1}} - r\Gamma_k^R(t - t')\hat{\tau}_x$, and $\hat{\eta}_k^R(t, t') = -r\hat{\mathbf{1}} + \Gamma_k^R(t - t')\hat{\tau}_x$. In terms of $\hat{\alpha}_{jk}^{R(A)}$, Eq. (2) reduces to the following form:

$$\delta I_{jk}(t) = \frac{1}{8eR_0}\int \frac{d\omega}{2\pi}\int d\epsilon J_{jk}(\epsilon, \epsilon - \omega)e^{-i\omega t}, \tag{4}$$

$$\begin{aligned} J_{jk}(\epsilon, \epsilon') = {} & \mathrm{Tr}\, i\hat{\tau}_y[\gamma_k^R(\epsilon)W_k(\epsilon')\hat{\alpha}_{jk}^R(\epsilon, \epsilon') - \gamma_k^A(\epsilon')W_k(\epsilon)\hat{\alpha}_{jk}^A(\epsilon, \epsilon')] \\ & + [1 + \gamma_k^R(\epsilon)\gamma_k^A(\epsilon')]\mathrm{Tr}\,\hat{\tau}_z \int \frac{d\epsilon_1}{2\pi} W_k(\epsilon_1)\hat{\alpha}_{jk}^R(\epsilon, \epsilon_1)\hat{\alpha}_{jk}^A(\epsilon_1, \epsilon'), \end{aligned}$$

where $W_k(\epsilon) = [1 - |\gamma_k^R(\epsilon)|^2]f(\epsilon)$. Equations (3) and (4) show that the problem of finding the current in a short ballistic superconducting constriction with arbitrary time-dependent bias voltage reduces to the problem of solving Eq. (3), which is a Fredholm integral equation (see, e.g., [14]).

Equations (2)–(4) can also be used to find the current in short diffusive constrictions, i.e., constrictions with a large number of propagating modes and a length d that satisfies the condition $l \ll d \ll \xi$, where l is elastic scattering length. Indeed, in this case, solution of the quasiclassical equations gives the expression $\check{J}_{\rm dif} = (2lv_{Fz}/dv_F)\ln(\check{G}_2 * \check{G}_1)$ for the asymmetric part of the Green's function [9]. This expression reduces to [15] $\check{J}_{\rm dif} = (2lv_{Fz}/dv_F)\int_0^1 dD\check{J}(D)/D\sqrt{1 - D}$, where $\check{J}(D)$ coincides with Eq. (1). Thus, we find that in diffusive constrictions

$$I(t) = \frac{\pi\hbar}{2e^2R_N}\int_0^1 \frac{dD}{D\sqrt{1 - D}} I(t; D), \tag{5}$$

where $I(t; D)$ is given by Eqs. (2) for a single-mode constriction and R_N is the normal-state resistance of the diffusive constriction. Equation (5) shows that similarly to the approach based on the BdG equations [7], in the general Green's function approach, the current in the diffusive superconducting constriction can be written as a sum of independent contributions from an infinite number of ballistic propagating modes with the distribution of transparencies given by the Dorokhov's [16] density function $(\pi\hbar/2e^2R_N)/D\sqrt{1 - D}$. It should be noted that this approach assumes that all frequencies (frequency of Josephson oscillations and typical frequency of voltage variations) are much smaller than the inverse time of electron motion through the constriction.

Equation (3) can be solved easily for the dc bias voltage V, when the phase difference is $\varphi(t) = \omega_J t + \varphi_0$, and $\omega_J = 2eV/\hbar$ is the Josephson oscillation frequency. In this case, the solution of Eq. (3) can be written as a series, $\hat{\alpha}_{jk}(\epsilon, \epsilon') = 2\pi\sum_n \hat{\alpha}_{n(jk)}(\epsilon')\delta(\epsilon - \epsilon' - n\hbar\omega_J)$ (we omit the superscript R), in which the amplitudes $\hat{\alpha}_{n(jk)}$ are determined by a system of recurrence relations. To obtain these relations explicitly it is convenient to write the matrix $\hat{\alpha}_{jk}$ in the form $\hat{\alpha}_{jk} = \frac{1}{2}\sum_{s=\pm}(\hat{\tau}_x a_{jk}^s + b_{jk}^s)(1 + s\hat{\tau}_z)$. Equation (3) shows that the pairs of functions a_{jk}^s, b_{jk}^s with $s = \pm$ satisfy the same equations with the different polarity of the bias voltage. For the dc bias, we get the following recurrence relations for these functions:

$$\begin{aligned} & a_{n+1} - \gamma_j(\epsilon_{2n+1})\gamma_k(\epsilon_{2n})a_n \\ & \quad = -r[\gamma_j(\epsilon_{2n+1})b_n - \gamma_k(\epsilon_{2n+2})b_{n+1}] + \gamma_j(\epsilon_1)\delta_{n0}, \\ & c(\epsilon_{2n+1})b_{n+1} - d(\epsilon_{2n})b_n + c(\epsilon_{2n-1})b_{n-1} = -r\delta_{n0}, \end{aligned} \tag{6}$$

where $a_n \equiv a_{n(jk)}^+(\epsilon)$, $b \equiv b_{n(jk)}^+(\epsilon)$, and $\epsilon_n = \epsilon + neV$. The coefficients in these recurrence relations are $c(\epsilon) = D\gamma_j(\epsilon)\gamma_k(\epsilon + eV)/[1 - \gamma_j^2(\epsilon)]$ and $d(\epsilon) = 1 - \gamma_k^2(\epsilon) + D\gamma_j^2(\epsilon + eV)/[1 - \gamma_j^2(\epsilon + eV)] + D\gamma_k^2(\epsilon)/[1 - \gamma_j^2(\epsilon - eV)]$. The amplitudes $a_{n(jk)}$ and $b_{n(jk)}$ determine Fourier components of the current $I(t) = \sum_n I_n\exp(in\omega_J t)$, which according to Eqs. (2) and (4) are given by the expression $I_n = V\delta_{n,0}/R_N + I_{n(12)} + I_{n(21)}$, where

$$\begin{aligned} I_{n(jk)} = \frac{1}{2eR_0}\int_{-\infty}^{\infty} d\epsilon W_k(\epsilon)\cdot\Big\{ & -\gamma_k(\epsilon_{2n})a_{n(jk)}(\epsilon) - \gamma_k^*(\epsilon_{-2n})a_{-n(jk)}^*(\epsilon) + \sum_m [1 + \gamma_k(\epsilon_{2m+2n})\gamma_k^*(\epsilon_{2m})] \\ & \times [b_{n+m}b_m^* - a_{n+m}a_m^*]_{(jk)}(\epsilon)\Big\}. \end{aligned} \tag{7}$$

For a constriction between two BCS superconductors the recurrence relations (6) and Eq. (7) for the current reproduce the corresponding expressions that can be obtained from the BdG equations—see [6(a)], where these expressions were derived for a symmetric constriction. This means that for the purpose of description of the ac Josephson effect in a short constriction all information about the microscopic structure of its electrodes is contained in the function $\gamma^R(\epsilon)$ [introduced after Eq. (3)] which has the meaning of the amplitude of Andreev reflection from the fully transparent superconducting-normal (S/N) interface.

Equations (6) can be solved by standard methods [14]. Namely, it follows from (6) that $b_n(\epsilon) = b_0(\epsilon)\prod_{m=\pm 1}^{n} p_\pm(\epsilon_{2m})$, where the functions $p_\pm(\epsilon)$ are solutions of the equations $p_\pm(\epsilon) = c(\epsilon_{\mp 1})/[d(\epsilon) - c(\epsilon_{\pm 1})p_\pm(\epsilon_{\pm 2})]$; the two signs $(\pm)$ here correspond to $n \geq 1$ $(n \leq -1)$, and $b_0(\epsilon) = r[d(\epsilon) - c(\epsilon_1)p_+(\epsilon) - c(\epsilon_{-1})p_-(\epsilon)]^{-1}$. These relations provide the basis for convenient numerical evaluation of the current, and therefore allow us to find the current in constrictions between the superconductors with arbitrary quasiparticle spectrum. As we will see below, all deviations of the quasiparticle spectrum from its "ideal" BCS form affect strongly MAR-related current singularities and especially the low-voltage behavior of the current.

The quasiparticle spectrum of a superconductor can differ significantly from the BCS form due to pair-breaking (PB) effects which can be caused by several factors: scattering on paramagnetic impurities, magnetic field, supercurrent flow—see, e.g., [17]. For disordered superconductors, various PB processes can be described in a unified manner, the difference between various mechanisms contained only in the microscopic expressions for the pair-breaking parameter ζ that gives the strength of the PB. For instance, in the case of paramagnetic impurities, $\zeta = \hbar/\tau_s\Delta$ with τ_s being the spin-flip scattering time. For a thin superconducting film of thickness $d \ll \xi$ in a magnetic field H parallel to the film, $\zeta = l v_F (eHd)^2/18\hbar\Delta$. As the first application of our general theory we consider the ac Josephson effect in a constriction between two disordered superconductors with some PB mechanism. The retarded Green's functions of such superconductors are [17]:

$$g^R = \frac{u}{\sqrt{u^2 - 1}} = u f^R, \qquad \frac{\epsilon}{\Delta} = u\left[1 - \frac{\zeta}{\sqrt{1 - u^2}}\right]. \tag{8}$$

Weak PB effects result in smearing of the BCS singularity in the quasiparticle spectrum and suppression of the superconducting energy gap to a reduced value $\Delta_g = \Delta(1 - \zeta^{2/3})^{3/2}$. The gap disappears completely at $\zeta \geq 1$. Even weak PB affects strongly the current-carrying states of the constriction. For example, symmetric constrictions are known to support two discrete states in the subgap range with energies $\epsilon_\pm = \pm\epsilon_\varphi$. In the case of BCS superconductors, $\epsilon_\varphi/\Delta = \sqrt{1 - D\sin^2(\varphi/2)} \equiv u_\varphi$, and the dc supercurrent is carried entirely by the discrete states [18]. At finite PB, these states exist when $|\sin(\varphi/2)| > \zeta^{1/3}/\sqrt{D}$ and their position is determined by the expression $\epsilon_\varphi = \Delta u_\varphi(1 - \zeta/\sqrt{D}\,|\sin(\varphi/2)|)$. In constrictions with transparencies $D < \zeta^{2/3} \equiv D_\zeta$ the discrete states disappear, and the current is carried only by the continuum of states above the gap. We note that D_ζ corresponds to rather high transparencies even for small pair-breaking parameter on the order of 0.1. For example, for the PB caused by magnetic field parallel to a thin Al film with $d \sim 100$ nm, $\zeta = 0.1$ implies H in the range of 100 G, the field that is much smaller than the parallel critical field $H_{c\parallel}$. Figure 1 shows how the changes in the quasiparticle spectrum caused by the PB are reflected in the current-voltage $(I\text{-}V)$ characteristics. All gap-related features are rapidly broadened at small ζ, and the curves become practically linear in the gapless regime $\zeta \geq 1$.

The changes in the spectrum are reflected also in the ac current. This can be seen explicitly in the case of the fully transparent constriction, $D = 1$, and low voltages, $eV \ll \Delta_g f(\Delta_g)$, when the recurrence relations (6) can be solved directly and we get the current:

$$I(t) = \frac{1}{2eR_N}\int de \times \mathrm{Re}\,\frac{i\sin\varphi F_+(\epsilon, V) + 2u\sqrt{u^2 - 1}\,F_-(\epsilon, V)}{u^2(\epsilon) - \cos^2(\varphi/2)}.$$

Here $F_\pm(\epsilon, V) = [F(\epsilon, V) \pm F(\epsilon, -V)]/2$, and

$$F(\epsilon, \pm V) = f(\epsilon) - \int_{\pm\Delta_g}^{\epsilon} d\epsilon' \frac{\partial f(\epsilon')}{\partial\epsilon'} \times \exp\left(-\int_{\epsilon}^{\epsilon'} \frac{dE}{eV}\ln|\gamma^2(E)|\right).$$

The current I in the constriction as a function of the (time-dependent) phase difference φ calculated from these equations is shown in the inset in Fig. 1. We see that even relatively small ζ has a strong effect on the dynamic current-phase relation, suppressing the dc component of the current and making it more similar to the stationary current-phase relation.

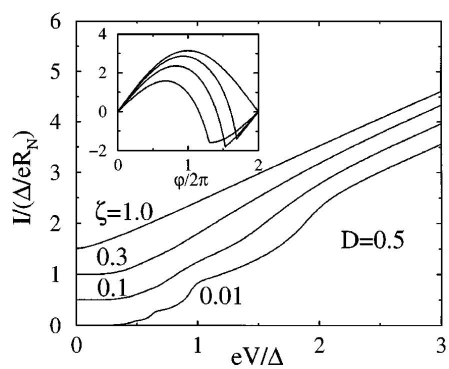

FIG. 1. dc I-V characteristics of a short ballistic constriction between the two superconductors with pair-breaking (PB) effects. The curves are shifted for clarity along the current axis and illustrate the smearing of the subharmonic gap structure with increasing strength of the PB. The upper curve with $\zeta = 1.0$ corresponds to the regime of gapless superconductivity. The inset shows the dynamic current-phase relation at low bias voltages and $D = 1$ for $\zeta = 0$, 0.1, 0.3, and 0.7 (from top to bottom).

We consider next a constriction between two normal conductors in which superconductivity is induced by the proximity effect, i.e., an $S/NcN/S$ junction. Besides general interest to the proximity effect, the importance of this model is due to its relevance for realistic description of the high-critical-current tunnel junctions [19]. We study a particular case of a thin dirty N layer of thickness $d_n \ll \xi_n$ with the S/N interface that has low transparency, $\langle D' \rangle \ll 1$, but resistance still negligible in comparison to the constriction resistance [20]. The Green's functions of the N layer are given then by the first equation in (8) with $u = (\epsilon + i\gamma_b g_S^R)/i\gamma_b f_S^R$; see, e.g., [19,21], and references therein. Here g_S^R and f_S^R are the Green's functions of the superconductor, and $\gamma_b/\hbar = \langle D' \rangle v_{Fn}/4d_n$ is the characteristic tunneling rate across the S/N interface which is assumed to be larger than electron-phonon inelastic scattering rate in the N layer. The energy gap Δ_g is induced in the N layer due to the proximity effect. If the S electrode of the structure is the BCS superconductor with energy gap Δ, the induced gap Δ_g is determined by the equation $\Delta_g = \Delta\gamma_b/[\sqrt{\Delta^2 - \Delta_g^2} + \gamma_b]$. Existence of the induced gap implies that there are two peaks in the density of states of the N layer, at energies Δ_g and Δ. This structure of the density of states results in a complex structure of the subharmonic gap singularities in the I-V characteristics of the constriction. An example of such a structure is shown in Fig. 2 for $\gamma_b/\Delta = 1$, when $\Delta_g \simeq 0.54\Delta$. We see that the most pronounced current singularities in this case are the subharmonic singularities at $eV = 2\Delta_g/n$ associated with the induced gap. Also visible are the singularities at "combination" energies $\Delta + \Delta_g$ and $(\Delta + \Delta_g)/2$.

In conclusion, we have developed the microscopic approach to MAR in short ballistic and diffusive constrictions between the superconductors with arbitrary quasiparticle spectrum. We used the developed approach to study the dc and ac current in constrictions between superconductors with pair-breaking effects, and also between normal conductors with the proximity-induced superconductivity. The effects studied in this work should be observable in high-transparency Josephson junctions either as a complex structure of the subharmonic singularities (similar to the one shown in Fig. 2) or as smearing of these singularities.

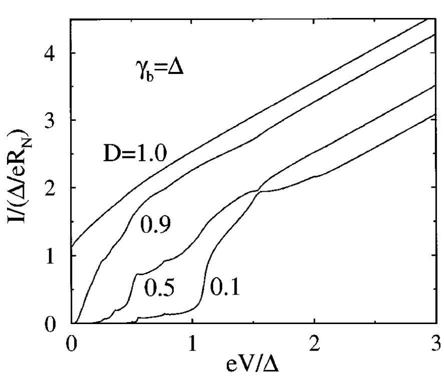

FIG. 2. dc current-voltage characteristics of a short symmetric $S/NcN/S$ constriction for different values of the constriction transparency D. The curves show the complex subharmonic gap structure associated with the two energy gaps: Δ in the S region, and proximity-induced gap Δ_g in the N region. For discussion, see text.

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[20] A similar model has been considered in [19], but the calculations of the current there were based on the phenomenological OBTK approach [1]. Although this approach is valid for fully transparent constrictions [6(b)] it breaks down when the reflection coefficient is finite because of the quantum mechanical interference between different quasiparticle trajectories in the energy space.

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