Acoustic Localization in Bubbly Liquid Media

Zhen Ye and Alberto Alvarez
Department of Physics and Center for Complex Systems, National Central University, Chung-li, Taiwan, Republic of China
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Acoustic wave propagation in liquid media containing many air-filled bubbles is *ab initio* considered. A self-consistent method is used to derive a set of coupled equations describing rigorously the multiple scattering of waves in such media. The wave transmission and backscattering are then solved exactly. The numerical results indicate localization of acoustic waves in a range of frequency. [S0031-9007(98)05787-1]

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The propagation of waves through random media has been and continues to be a subject of vivid research [1–6]. By analogy with the well-known localization of the electron transport in disordered condensed matters arising from the interference effects in multiple scattering, it has been discussed much in the literature that similar localization may also exist in the transmission of the classical waves in random media when the multiple scattering becomes strong [7–17]. Such a localization phenomenon may be characterized by two levels. One is the weak localization, associated with the enhanced backscattering. The second is termed as the strong localization, in which significant inhibition of transmission surfaces, indicating that the energy mostly remains in a region of space in the neighborhood of the emission. In other words, under certain conditions, the wave can be trapped in a finite spatial domain. While the weak localization, regarded as a precursor to the strong localization, has been well studied and observed experimentally [18], the phenomenon of strong localization in three dimensions still lacks experimental confirmation and remains a subject of much debate [19]. Rusek *et al.* [13] pointed out that "... despite remarkable efforts no deeper insight into localization ... can be found in the literature."

In this Letter, we report a simulation study of low-frequency acoustic propagation in liquid media containing many air-filled bubbles. The approach is based on a genuine self-consistent scheme bearing similarities to the procedure of Twersky [4] reviewed in Ref. [20], and has been detailed in Refs. [21]. In the approach, the wave propagation is represented by a set of coupled equations, and is solved rigorously. The wave transmission and backscattering are thus obtained. In particular, we consider the bubbly water. Moreover, all bubbles are taken as the single size for simplicity, yet without losing generality. The numerical results indicate the existence of both weak and strong localization in the range of frequency considered.

There are several advantages of studying sound in bubbly water. (1) The air-filled bubbles are strong acoustic scatterer. At low frequencies, about $ka \sim 0.0136$, it appears the resonant scattering and the scattering strength is greatly enhanced at resonance, making it an ideal system to study strong scattering. Here k is the acoustic wave number in water, and a is the radius of the bubbles. (2) The scattering function of a single bubble has been well studied and has a simple form. The scattering function of a spherical bubble can be found in many textbooks [22], whereas the scattering function of a deformed bubble, such as an ellipsoidal bubble, has also been analytically derived recently [23]. (3) It is found that the scattering of bubbles is isotropic in the range of frequency considered, permitting many simplifications. Furthermore, perhaps more important, such an isotropic scattering feature remains valid even as the bubbles are subject to significant deformation commonly seen in actual experiments [23]. (4) The propagation of waves in bubbly water may serve as a toy model for many more complicated situations. There are also many practical reasons for studying the sound propagation through bubbly liquids, because of the prominent role played by bubbles in a variety of situations of great interest. The bubbles generated by breaking waves at ocean surfaces are a good agent for probing upper-ocean dynamics, including shear turbulence, Langmuir circulation, fronts, and internal waves [24]. They are also useful in medical echocardiography [25].

Indeed, considerable efforts from both theoretical and experimental points of view have been devoted to propagation of acoustic waves in bubbly liquids. A review on general aspects of the subject may be found in the monography [26]. Strong localization of acoustic waves in bubbly liquids was first suggested by Sornette and Souillard [27]. But no detailed results were given. Later, Ye and Ding studied acoustic wave propagation in bubbly waters using the perturbative diagrammatic method [28]. Including a higher order correction representing the mutual interaction between two bubbles, their results show that when the concentration of the bubbles reaches a certain value, the wave phase speed can become negative; one of their suggestions is the appearance of wave trapping. An obvious shortcoming of the diagrammatic approach, however, is that it cannot exclude the possibility of the breakdown of the perturbative calculation, and how many and what type

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of perturbation terms should be included are unknown. In order to gain a definite insight into the problem, it is therefore highly desirable that one can study the problem in a rigorous or an effectively exact manner. Although this is virtually impossible for systems containing an infinite number of scatterers, the rigorous results can be obtained for systems consisting of a finite number of scatterers. This Letter presents one of such studies.

Different from the common approach which derives approximately a diffusion equation for the ensemble-averaged energy, our method is to solve rigorously the wave propagation from the fundamental wave equation. Consider a unit point source in bubbly water, emitting a monochromatic acoustic wave of angular frequency ω. For simplicity while not losing generality, we consider a simple model. The source is assumed to be at the origin. There are N spherical air bubbles randomly located at $\vec{r}_i$ ($i = 1, 2, \dots, N$), surrounding the emission point. For simplicity, all the bubbles are assumed to be of the same size and randomly distributed within a spatial domain, which is taken as the spherical shape. No bubble is located at the source, and no two bubbles can occupy the same spot, i.e., the hard sphere approximation. In other words, the point source is placed at the center of a spherical bubble cloud; such a model describes the acoustic noise naturally generated inside the bubble clouds in the upper-ocean processes. The radius of the bubbles is a. The void fraction, the fraction of volume occupied by the bubbles per unit volume, is β. Therefore the numerical density of the bubble is $n = 3\beta/4\pi a^3$, and the radius of the bubble cloud is $R = (N/\beta)^{1/3}a$. The radiated wave from the source is subject to multiple scattering by the surrounding bubbles. The wave equation can be generically written as

$$(\nabla^2 + k^2)p(\vec{r}) = -4\pi\delta(\vec{r}) + J[N, p], \qquad (1)$$

where k is the wave number of the pure water, the source is represented by the delta function, and the second term on the right-hand side refers to the interaction between the waves and the N scatterers. Without the interaction, the propagating wave is $p_0(\vec{r}) = e^{ikr}/r$. When the scatterers are present, the solution can be composed as

$$p(\vec{r}) = p_0(\vec{r}) + \sum_{i=1}^{N} p_s(\vec{r}; i), \qquad (2)$$

where $p_s(\vec{r}; i)$ is the scattered wave from the ith bubble in response to the direct incident wave and also all the scattered waves from other bubbles. Owing to the simplicity in sound scattering by air bubbles, the scattered wave can be written as

$$p_s(\vec{r}; i) = f\left(p_0(\vec{r}_i) + \sum_{j=1, j\neq i}^{N} p_s(\vec{r}_i; j)\right)\frac{e^{ik|\vec{r}-\vec{r}_i|}}{|\vec{r} - \vec{r}_i|}, \qquad (3)$$

where f is the isotropic scattering function of the single bubble, and in the resonant scattering regime it can be written as [22]

$$f = \frac{a}{\omega_0^2/\omega^2 - 1 - i\delta}, \qquad (4)$$

in which ω_0 is the resonant frequency, and δ is the damping factor. Although the damping factor can incorporate the radiation, thermal, and viscosity effects, it can be adjusted manually to check various concepts. For example, when the thermal and viscosity damping are turned off, the acoustic absorption diminishes, allowing study of the situation without acoustic absorption. The formula in Eq. (4) is valid for $ka < 0.4$ [29].

When setting $\vec{r}$ in Eq. (3) at another scatterer, we arrive at the following self-consistent equations:

$$p_s(\vec{r}_l; i) = f\left(p_0(\vec{r}_i) + \sum_{j=1, j\neq i}^{N} p_s(\vec{r}_i; j)\right)\frac{e^{ik|\vec{r}_l-\vec{r}_i|}}{|\vec{r}_l - \vec{r}_i|},$$
$$(i, j = 1, 2 \dots, N, \text{ but } i \neq l). \qquad (5)$$

These equations can be rewritten in the form of $N \times N$ matrices, and then solved exactly for an arbitrary configuration of the bubble distribution. The result can then be averaged over many configurations to obtain proper quantities. For example, for the quantity A, the averaged value is $\langle A\rangle = \frac{1}{M}\sum_i^M A_i$, where A_i is the quantity value for the ith realization, and the total realization number is M. This ensemble average is consistent with what has been described by Foldy [1] and Ishimaru [20]. The number of realization M is taken such that the results become stable. In this Letter, we consider the averaged wave referring to the coherent portion, as well as the averaged squared modulus of the wave function which includes both the coherent and diffusive portions. We define $I = \langle|p|^2\rangle$ to represent the squared modulus of the total wave corresponding to the total energy, $I_C = |\langle p\rangle|^2$ to the coherent portion, and the diffusive part is therefore $I_D = I - I_C$. The backscattering situation is simply obtained by replacing p with $\sum_{i=1}^N p_s(0; i)$. Here we note that it is the ensemble-averaged total intensity in long propagation range rather than mere coherent or diffusive portion which determines the localization phenomenon. The approach described here corresponds to taking the modulation frequency towards zero in the previous theories [17].

A set of numerical experiments has been carried out for various bubble sizes, numbers, and concentrations. The locations of the bubbles are randomly generated in the hard-sphere approximation within the prescribed sphere by the computer, to simulate the random configurations of the bubble cloud. More details have been documented in Ref. [21]. Figure 1 presents one of the typical results of the forward and backscattering as a function of frequency in terms of ka. Also plotted are the results of the coherent portion. The void fraction is taken as 10^{-3}, and the hydrophone is placed at a distance away from the bubble cloud to receive the forward transmission. Here it is shown in Fig. 1(a) that the transmission is greatly reduced

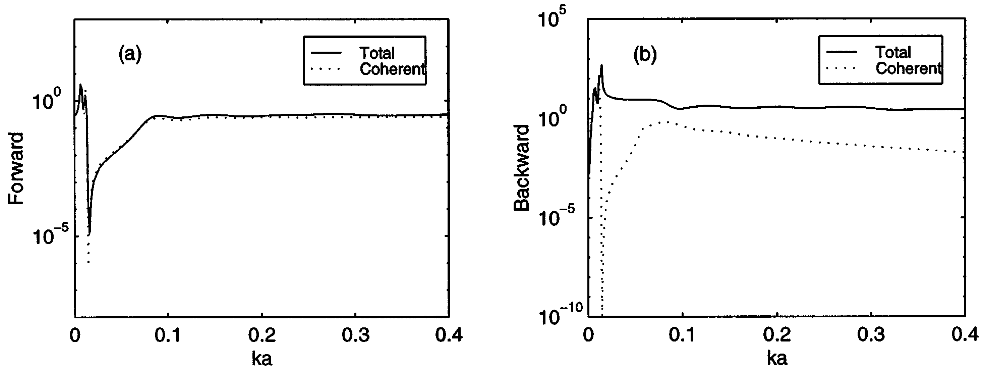

FIG. 1. Transmission and backscattering as a function of ka.

between 0.01 and 0.02 in ka, indicating the acoustic localization. At these frequencies, the Ioffe-Regel criterion [17] is satisfied. The greatest inhibition occurs at about $ka = 0.0176$. Since the sample size is finite, the transmission is not completely diminished, as expected. The result also shows that the coherent part dominates the transmission for most frequencies. We also notice that, before the localization region, there appear two resonant transmission peaks. The first peak at about $ka = 0.007$ may be attributed to the collective mode of the bubble cloud, whereas the second peak corresponds to the resonant peak of a single bubble. Because of the multiple scattering, the resonance peak of a single bubble is shifted slightly towards a lower frequency, in agreement with the previous results [21,30]. Figure 1(b) shows the backscattering situation, in which a hydrophone is put at the origin to receive the backscattered wave. The coherent contribution is represented by the dotted line. Here we observe the enhancement of backscattering, as compared to the transmission, particularly in the localization regime, where the diffusive wave dominates and is considered being trapped. The error analysis has also been conducted with respect to the number of ensemble averaging. We find that deviations are uniformly less than 0.5% for the number of realizations exceeding 1000. More detailed analysis is presented in Ref. [31].

Figure 2 plots the transmission as a function of the distance from the acoustic source for three frequencies in terms of ka, denoted by the numbers in the plots. The results are scaled by the geometric factors. This figure shows the following. (1) Outside the bubble cloud, the

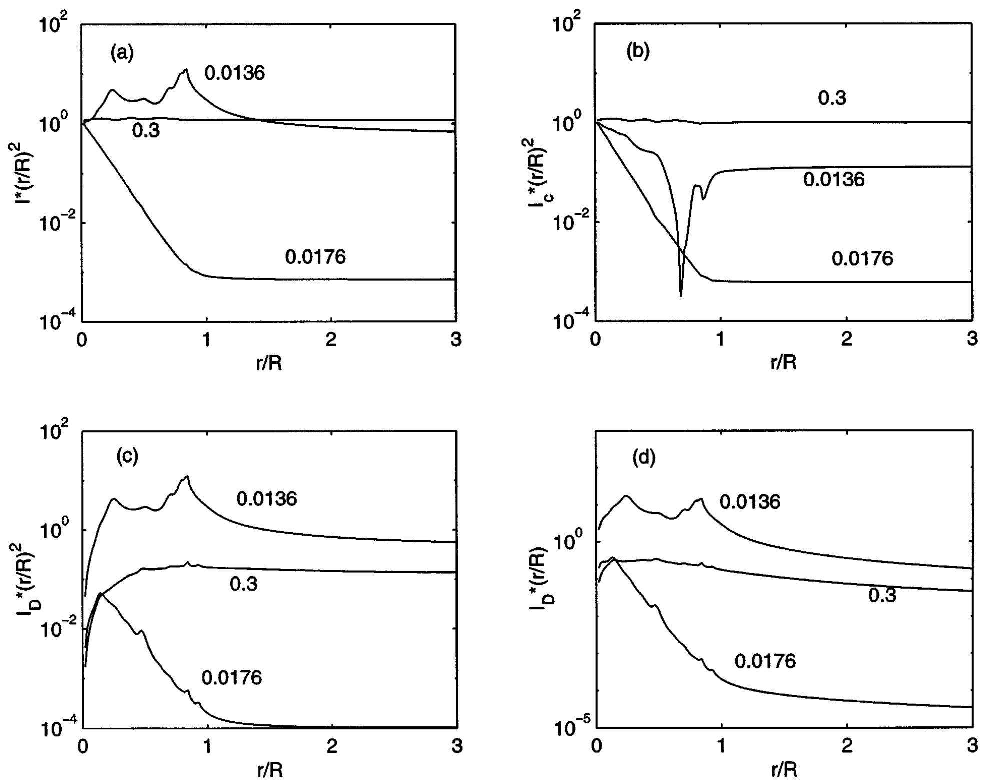

FIG. 2. Transmission as a function of the distance away from the source. (a) The total intensity, (b) the coherent portion, (c) the diffusive portion with the scaling of distance squared, and (d) the diffusive portion with the linear scaling of distance.

transmission falls off as $1/r^2$ for large r, as expected. (2) Inside the cloud, the energy is localized in certain domains for $ka = 0.0136$, and 0.0176; only a small portion of energy will be transmitted out of the cloud. The localization is mostly clear in the case of $ka = 0.0176$. At the higher frequency $ka = 0.3$, the localization disappears, though at this frequency the enhanced backscattering still remains according to Fig. 1. Therefore in this situation, only weak localization can be observed. (3) Both coherent and diffusive portions are localized spatially, when the localization occurs. The diffusive part in $ka = 0.3$ appears to follow the prediction of a diffusion equation. It, however, remains an open question how the observation for the strong localization can be fitted by the existing diffusion theory.

The localization may be understood as follows. When the source starts to transmit, the coherent energy will be lost due to multiple scattering by the random scatterers, partially transferring into diffusion energy. As a result, the diffusion energy will start to build up, and perhaps will reach a peak at a certain point [20]. Without localization, according to the transport theory the diffusion energy will propagate and reach a certain value (e.g., Sect. 14-3 in Ref. [20]). This is exactly the situation for $ka = 0.3$. Here we note that the geometry effect has been factorized out. When the localization occurs, although the diffusion energy can still be built up near the source, both the coherent and the diffusion energy transports will be blocked and remain in the neighborhood of the emission. Little energy is transmitted out of the cloud, indicated by the curtailment in the cases of $ka = 0.0176$ and $ka = 0.0136$.

The above localization phenomena are supported by further investigations [31]. (1) The localization is not due to the dissipation. This is verified by turning off or adjusting the dissipation terms involved in the scattering by bubbles, as aforementioned. (2) By increasing the bubble number while keeping the void fraction constant, it is shown that the localization is not due to the boundary of the bubble cloud either. (3) The localization is also insensitive to the size of bubbles. (4) However, it is shown that the localization disappears when the bubble concentration reaches a sufficiently low level. (5) When the bubble concentration is decreased, the window for the localization is narrowed, tending to disappear.

In summary, we have studied wave propagation through liquid media containing air-filled bubbles. A self-consistent method is employed to calculate the wave propagation in an exact manner. The results from the numerical experiments are reported. A narrow regime of strong localization is predicted. The results show that the weak localization can occur in a wider range of frequency.

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