Quantum Noise of an Atomic Spin Polarization Measurement

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We explore the fundamental noise of the atomic spin measurement performed via polarization analysis of the probe light. The noise is shown to consist of the quantum noise of the probe and the quantum noise of atomic spins. In the experiment with cold atoms in a magneto-optical trap we demonstrate the reduction of the former by 2.5 dB below the standard quantum limit. For the latter we reach the quantum limit set by fluctuations of uncorrelated individual atomic spins. We outline the way to overcome this limit using a recent theoretical proposal on spin squeezing. [S0031-9007(98)05843-8]

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A standard atomic spin polarization measurement is performed via polarization analysis of the probe light transmitted through the atomic ensemble. Such a measurement is widely used in atomic physics with experiments on parity nonconservation being just one notable example [1]. In this Letter we address the issue of the fundamental quantum noise of such a measurement which sets the ultimate sensitivity limits on the measurement of the atomic spin. The results reported here can be also viewed from a perspective different from their relevance to precision measurements. We essentially demonstrate in this Letter that quantum fluctuations of cold atomic spins in a magneto-optical trap (MOT) can be read out in a probe polarization measurement. This observation leads to the conclusion that an ensemble of a large number of cold atoms can be used for storage of quantum correlated states of light.

In a polarization interferometer with balanced detection a linearly polarized probe is analyzed with a polarizing beam splitter rendering the photocurrents proportional to the intensities of the probe components polarized at 45° and −45° (Fig. 1). The mean photocurrents depend on the degree of the polarization change of the probe caused by the atomic spin polarization. Throughout the Letter we refer to the spin polarization as to any deviation of the ensemble spin state from spherical symmetry. For a resonant probe $I(\pm 45) = \mp \frac{\sigma}{A} N P_t \frac{i}{2}[T^2_{-2} - T^2_2] + P_t(\pm 45)$ with $P_t(\pm 45) = P\exp(-\alpha)$ as the transmitted probe power in the absence of polarization changes, P the input probe power, $\frac{\sigma}{A}N$ the optical depth, N the number of atoms interacting with the probe, σ the atomic cross section for unpolarized light, A the cross section of the probe beam, and $T^2_{\pm 2}$ the components of the atomic alignment tensor [2,3]. $I_- = I(45) - I(-45)$ in this case is sensitive to the transverse alignment of the atomic medium. We express the photocurrent in units of elementary charge $e = 1$, the probe power in units of the photon number, and assume perfect detection efficiency. To elucidate the role of various atomic spin components in this expression consider, e.g., atoms in $F = 1, m_F = -1, 0, 1$ states for which $T^2_{-2} - T^2_2 = \rho_{1,-1} - \rho_{-1,1}$ [3] with $\rho_{k,l}$ as the atomic density matrix components in m_F representation.

To explore quantum fluctuations of I_- we must write it as a quantum variable: $\hat{I}_- = \frac{\sigma}{A} P_t i \sum_\mu (\hat{\sigma}^\mu_{1,-1} - \hat{\sigma}^\mu_{-1,1}) + [\hat{P}_t(45) - \hat{P}_t(-45)]$. The first term represents the effect of the collective atomic spin on the transmitted probe where the atomic operators for the μth atom $\hat{\sigma}^\mu_{k,l} = |k\rangle\langle l|_\mu$ are summed over all the atoms interacting with the probe [4]. This term can be expressed through the collective pseudospin operator $F_y = \frac{i}{2}\sum_\mu (\hat{\sigma}^\mu_{1,-1} - \hat{\sigma}^\mu_{-1,1})$ introduced in a conventional way [4,5]. The second term corresponds to the transmitted probe as if there were no polarization changes. Its mean value is zero, and the fluctuations depend on the quantum state of the probe. For the probe in the coherent state it represents the shot noise of the probe, whereas for the squeezed probe it can be below the shot-noise limit. Note that the transmitted probe power enters as an operator in the second term but not in the first one due to the

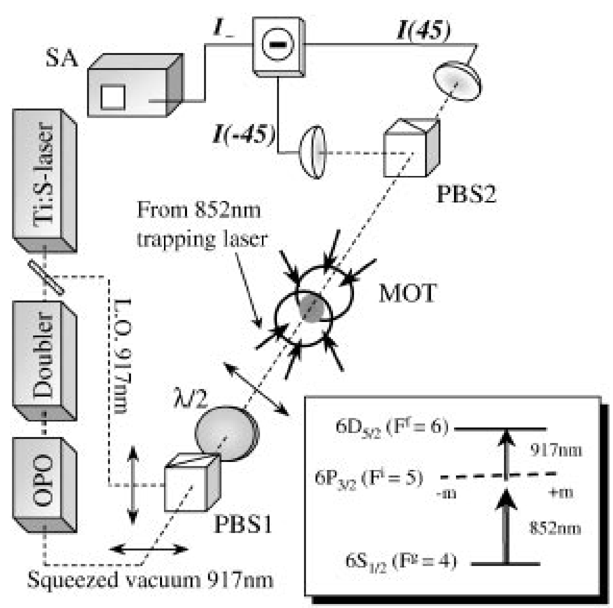

FIG. 1. Experimental setup.

assumption that the fluctuations in the probe power are small compared to the mean power. Assuming the probe with the degree of squeezing ξ we now obtain for the spectral density of I_-

$$\Phi_-^2(\Omega) = 4(\sigma/A)^2 \delta F_y^2(\Omega) P^2 \exp(-2\alpha) + 2P[\exp(-\alpha) - \xi \exp(-2\alpha)], \quad (1)$$

with $\delta F_y^2(\Omega)$ the spectral density of the spin noise, where the standard approach [6] is used to calculate the effect of absorption on the degradation of squeezing (second term). For an optically thin passive absorber (1) simplifies to $\Phi_-^2(\Omega) = 2P(1 - \xi)$ which is just the shot noise "squeezed" by the factor of $(1 - \xi)$ as first demonstrated for the Mach-Zehnder interferometer in [7], and for the polarization interferometer in [8]. Similar probe noise reduction can be achieved for the absorption measurement in an optically thin atomic sample [9] when the effect of the atomic noise similar to the first term in (1) can be neglected. We assume that the effect of dephasing and mixing in of the antisqueezed component of the probe [6] can be compensated by introducing a suitable phase change in the probe.

To further analyze the role of the atomic spin noise [first term in (1)] let us assume the exponential decay of the spin state with the rate $\gamma_{\rm spin}$. We obtain $\delta F_y^2(\Omega \ll \gamma_{\rm spin}) = (\pi\gamma_{\rm spin})^{-1}\langle \delta F_y^2\rangle$ with $\langle \delta F_y^2\rangle$ as the spin noise variance. For an ensemble of uncorrelated individual spin 1 particles, e.g., for a coherent collective spin state [10] $\langle \delta F_y^2\rangle = \frac{1}{2}N = \alpha A/2\sigma$. Hence (1) normalized to the shot noise of the incident probe $2P$ can be expressed as

$$\sqrt{\phi_-^2(\Omega \ll \gamma_{\rm spin})} = \{k\alpha \exp(-2\alpha)s + [\exp(-\alpha) - \xi \exp(-2\alpha)]\}^{1/2}, \quad (2)$$

with $k = \pi^{-1}\gamma_{\rm probe}/\gamma_{\rm spin}$, $s = P/P_{\rm sat}$ as the saturation parameter and $\gamma_{\rm probe} = \sigma P_{\rm sat}/A$ as the width for the probe transition. It is clear from (2) that for not very weak probes, moderately thick medium and broad probe transitions the first, spin noise contribution should become significant. For example, for $\xi = 0$ (coherent probe), $\alpha = 1$, $s = 1$, and $\gamma_{\rm probe} = 10\gamma_{\rm spin}$ spin noise constitutes around 50% of the overall quantum noise of the measurement (Fig, 2 curve 1a). Since $\gamma_{\rm probe} = \frac{1}{2}(\gamma_{\rm upper} + \gamma_{\rm lower})$, and $\gamma_{\rm lower} \approx \gamma_{\rm spin}$, we conclude that for probing the spin on the transitions with $\gamma_{\rm upper} \gg \gamma_{\rm lower}$ (ground state, and other long living states) the contribution of the fundamental spin noise dominates over the quantum noise of the probe within the frequency range bounded by $\gamma_{\rm spin}$.

To explore the result (2) experimentally we employ our frequency tunable source of polarization squeezed light [11] and the ensemble of cold Cs atoms trapped in a MOT. We study the atomic spin fluctuations in $6P_{3/2}, F = 5$ state (inset in Fig. 1) excited via the 852 nm transition and probed via the 917 nm transition. The experimental setup for measuring $\Phi_-^2(\Omega)$ (Fig. 1) consists of two polarizing beam splitters PBS1, PBS2, and a balanced detection scheme connected to the spectrum analyzer (SA). The SA is tuned to $\Omega = 3$ MHz, the frequency high enough to have a shot-noise limited probe and yet low enough to observe the spin noise in $6P_{3/2}$ state ($\gamma_{\rm spin} \approx 5$ MHz). A large (5 mm in diameter) Cs MOT [12] sustained by six 60 mW beams from a Ti:S laser (not shown in the Fig. 1) allows one to obtain $\alpha \approx 1$ at the probe transition. The probe is generated by another Ti:S laser and is injected into one port of the PBS1. For squeezed probe experiments frequency tunable squeezed vacuum in orthogonal polarization generated by the sub-threshold OPO [11] is injected into the other port of the PBS1. As first demonstrated for a thin passive absorber in [8], mixing a local oscillator and squeezed vacuum on a polarizing beam splitter allows for sub-shot-noise performance of the polarization interferometer. According to (2) for optically thin atomic medium and not very large ratio of $\gamma_{\rm probe}/\gamma_{\rm spin}$ frequency tunable polarization squeezed light employed in this Letter allows for the sub-shot-noise atomic spin polarization measurements. The observed 40% reduction of the quantum noise is shown in Fig. 2 (diamond) with the actual experimental scan reproduced in Fig. 3. To observe the polarization rotation signal shown in the Fig. 3 the trapping beams are amplitude modulated at 3 MHz [13] producing the modulation of the atomic orientation (double-peak signals in the figure) [13]. Without squeezed vacuum the signal is shot-noise limited for optically thin atomic medium as proved by various checks (trace 1). With squeezed vacuum injected and the phase of it locked to the value minimizing the quantum noise [11] the probe noise is reduced by 2.5 dB below the standard quantum limit with the corresponding improvement in the signal to noise ratio (trace 2).

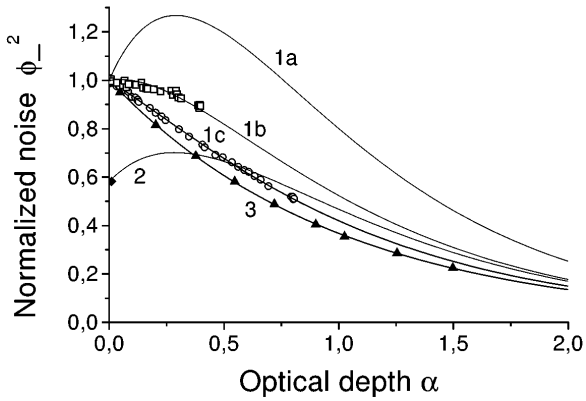

FIG. 2. Noise of the spin polarization measurement in a MOT. Curve 1a, Eq. (2) spin noise + shot noise, $\gamma_{\rm probe}/\gamma_{\rm spin} = 10$, $s = 1$; Curve 1b, same as 1a, but $\gamma_{\rm probe}/\gamma_{\rm spin} = 1.8$, $s = 2$; Curve 1c same as 1b but $s = 0.7$; curve 2 spin noise as in 1b+squeezed probe noise; curve 3 probe shot noise. For comments on experimental data see the text.

This improvement of the S/N (signal to noise ratio) due to the reduced quantum noise of the probe is

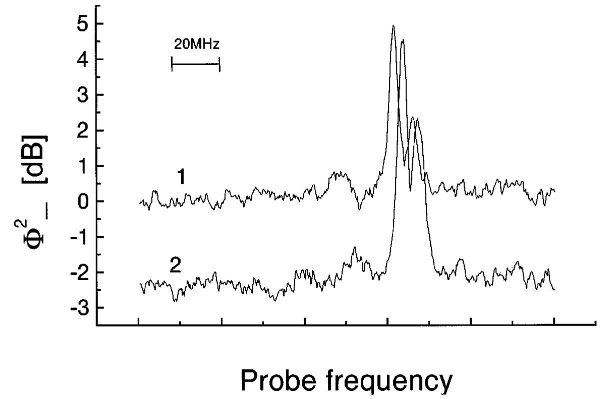

FIG. 3. Sub-shot-noise spin polarization measurement Curve 1, coherent probe. Shot level at 0 dB. Curve 2 squeezed probe.

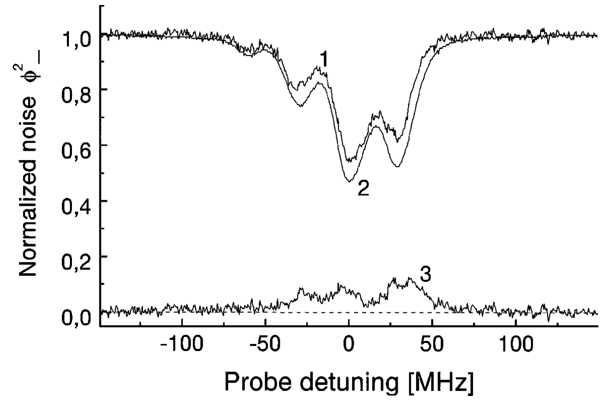

FIG. 4. Normalized noise around atomic resonance. Curve 1, Measured noise. Curve 2, dc absorption (shot noise only). Curve 3, spin noise (difference between curves 1 and 2). SA frequency 3 MHz.

achievable only for optically thin medium as shown in Fig. 2 The curves plotted in the figure as a function of α represent the noise (2) for coherent probes 1a, 1b, 1c and squeezed probe ($\xi = 0.4$)—curve 2, and the shot noise of the coherent probe attenuated by a passive absorber according to $\exp(-\alpha)$—curve 3. As seen from Fig. 2 the ratio of curve 2 to curve 1b (same spin noise as in curve 2) which reflects the effect of squeezing the probe on the S/N improvement is approaching unity already for $\alpha \gtrsim 0.5$. One reason for that has been known since the first theoretical studies of squeezed states of light [6]. It is reflected in the second term of (2), and is due to the degradation of the degree of squeezing in the process of attenuation. Another reason, though, is much less obvious and is due to the atomic spin noise [first term in (2)]. This additional quantum noise represented by the difference between curves 1 and 3 depends, as shown in (2), on parameter k. For the three level coherent spin state ensemble assumed in the derivation of (2) $k = 0.25$ from $\gamma_{\mathrm{probe}} \approx \frac{1}{2}(\gamma_{6D} + \gamma_{6P}) = 4$ MHz and $\gamma_{\mathrm{spin}} \approx \gamma_{6P} = 5$ MHz. The experimental data (circles and squares) are well fitted with (2) yielding $k_{\mathrm{exp}} = 0.55$ proving that (2) is useful for quantitative estimates of the spin noise. The numerical discrepancy is caused by the fact that the spin state in the MOT is not coherent (see below), the state has $F = 5$, and the 852 nm field is not weak, and therefore the dynamics of the spin is not described by an exponential decay. The way the measurements of the quantum noise shown in Fig. 2 have been taken is illustrated in Fig. 4. The spectral density $\phi^2_-(\Omega = 3\ \mathrm{MHz})$ has been measured with the coherent probe scanned around the atomic resonance (curve 1). Since atoms are excited with 852 and 917 nm fields in a ladder configuration the signal shown in the figure consists of the coherent (central minimum) and the incoherent (minimum on the right) parts (smaller structures on the left belong to other hf components). If the atoms were replaced by a passive absorber, we would expect the noise level to follow curve 2 which is the shot noise level of the probe attenuated according to the dc absorption profile. However, the actual noise level 1 definitely contains excess noise above the shot noise of the probe. This excess noise calculated as the difference between the two curves is shown as curve 3. Such excess noise can have different origins, the most trivial one being the presence of the classical phase noise/modulation at 3 MHz in the probe. Another reason can be the presence of such classical noise in the 852 nm light which would lead to the classical spin noise signal [11,13]. The profound difference between this classical noise and the fundamental noise of Eq. (2), is in that the latter depends on the square root of the number of atoms while the former depends on the number of atoms linearly (after normalization to the transmitted probe power). Various kinds of atomic and nuclear noise have been reported previously [14]. However, to the best of our knowledge only in [5] the fundamental quasispin projection noise for the case of ions in a trap has been explicitly studied. In [5] the variance of the projection noise for the quasispin associated with the two level hyperfine system is shown to be close to the square root function of the ion number, but the deviation from the $\sqrt{N_{\mathrm{atoms}}}$ due to the technical noise is significant already for hundreds of ions. In our experiment we demonstrate the dominance of this fundamental spin noise in the spectral domain for a large (up to 10^8) number of neutral magnetic spins. To study this noise in detail we lock the frequency of the probe laser to the center of the incoherent resonance where the relative size of the excess noise is large. To improve the quality of the measurement we chop the probe at 1 kHz and feed the output of the SA tuned to 3 MHz into the lock-in amplifier. We do it first with the atoms present obtaining according to (2) $\phi^2_- = k_{\mathrm{exp}}\alpha \exp(-2\alpha)s + \exp(-\alpha)$ (circles in Fig. 2), and then without the atoms for the same detected probe power obtaining $(\phi^2_-)_{\mathrm{shot}} = \exp(-\alpha)$ (triangles in Fig. 2). The difference between the two readings is the excess noise introduced into the probe by the atoms $(\phi_-^2)_{\rm spin} = k_{\rm exp}\alpha \exp(-2\alpha)s$. This simple expression allows for straightforward checks of our model of the spin noise. First, when normalized to the exponent $(\phi_-)_{\rm spin}$ should reveal the square root dependence on α, i.e., on the number of atoms. The experimental evidence for that is presented in Fig. 5 (curve 1). The $\sqrt{N_{\rm atoms}}$ dependence characteristic of the coherent spin state or any state composed of uncorrelated individual spins is clearly shown for our case of atoms trapped in the MOT. The actual spin state of atoms in the MOT is close to the spherically symmetric state due to the symmetry of the excitation with six trapping beams. The difference between the spin noise for such a state and that for the coherent spin state is in a numerical factor of the order of unity according to the following qualitative argument. For the spherically symmetric collective spin state formed by uncorrelated individual spins $\langle \delta \hat{F}_{x,y,z}^2 \rangle = \frac{1}{3}\langle \delta \hat{F}^2 \rangle = \frac{1}{3} N_{\rm atoms} F(F+1)$. Comparison with $\langle \delta \hat{F}_{x,y}^2 \rangle = \frac{1}{2} N_{\rm atoms} F$ for the coherent spin state yields the ratio of $\sqrt{\frac{2}{3}(F+1)}$ for the spin uncertainties in the two states. To further prove that we can also reproduce the classical spin noise linear dependence on $N_{\rm atoms}$ we apply frequency modulation to the 852 nm laser which leads to the (classical) modulation of the $6P_{3/2}$ spin state. Under those conditions the linear asymptotic behavior of the spin noise for large $N_{\rm atoms}$ shown as curve 2 in Fig. 5 is evident. Another parametric dependence of the spin noise that we have tested and confirmed experimentally is the proportionality to the saturation parameter s, i.e., to the probe intensity for weak probes. Finally, we have performed preliminary measurements of the spectrum of the spin noise and have found that, as expected, it is mainly concentrated within the bandwidth below 10 MHz.

To summarize, we have performed the study of the fundamental quantum noise of the atomic spin polarization measurement. We analyze quantum limits for this noise as set by the shot noise of the probe and the fundamental noise of the uncorrelated atomic spins. The reduction of the probe quantum noise by 2.5 dB beyond the standard quantum limit with the polarization squeezed probe has been demonstrated. We have predicted theoretically and have confirmed experimentally the significance of the $\sqrt{N_{\rm atoms}}$ spin noise in the probe polarization measurement. The fact that we have reached this limit in the spin noise puts us in a position to implement the recent proposal for generation of spin squeezed states via complete absorption of the nonclassical light [4]. Towards this end the collective atomic spin in, e.g., $6P$ state should be excited with σ_--polarized coherent light and σ_+-polarized squeezed vacuum. If complete absorption of the squeezed vacuum is achieved the collective $6P$ state pseudospin becomes squeezed [4] which will manifest itself in the reduction of the $\sqrt{N_{\rm atoms}}$ spin noise in the probe polarization measurement similar to the one described in the present paper.

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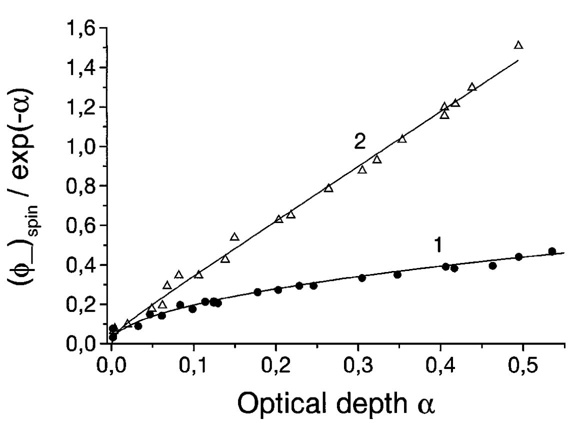

FIG. 5. Curve 1, Quantum spin noise and the fit with square root dependence on $\alpha \propto N_{\rm atom}$. Curve 2, classical spin noise with asymptotically linear dependence on α.

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