Interaction of Sound with Fast Crack Propagation

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The influence of ultrasounds on crack dynamics in brittle materials is experimentally studied by using both the natural sound emitted by the propagating crack and an artificially generated ultrasound burst. We show that, although the acoustic energy is only 5% of the energy needed to propagate the crack, the presence of sound waves in the specimen cannot be neglected because sound interacts with the crack tip and strongly modifies the fracture dynamics. [S0031-9007(97)04990-9]

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Fracture dynamics in brittle materials is a nonlinear pattern forming phenomenon which is not yet very well understood and has been widely studied, both theoretically [1–7] and experimentally, during the last few years. Recent experiments in Plexiglass [8–14] and glass [10] have carefully characterized the crack propagation properties. An important open problem is the influence of the sound, emitted by the crack, on the fracture dynamics [7]. In general, the amount of sound energy emitted by the crack is much smaller than the surface energy that one needs to supply to let the crack propagate. For example, in polymethylmethacrylate (PMMA), the sound energy is only 5% of the surface energy [10–12]. However we know from dynamical theory that a small perturbation may produce big changes on the dynamics of an unstable system. Thus the interaction of crack with sound cannot in general be neglected [7].

The purpose of this Letter is to prove on the basis of some experimental results, that, in spite of its weak energy, sound strongly modifies the crack dynamics. To show this we took advantage of the strong frequency dependence of the PMMA Young modulus and of sound speeds. This frequency dependence allows us to experimentally study the influence of sound on the final velocity of the crack.

Experiments have been done using cell-cast Plexiglass (PMMA). The samples had a rectangular shape with a length L of 29 cm and a height H equal to either 10 or 20 cm. The sample thickness d is varied from 1 to 10 mm. A schematic picture of the sample is shown in Fig. 1. To induce the fracture just in the center of the sample, an initial cut was made at $L/2$. Fracture is performed by applying a force in the plane of the sample and parallel to the longest side of the plate (mode I crack [1]). It is important to point out that the Young modulus of PMMA depends on the frequency at which it is measured whereas the Poisson ratio σ is constant and equal to 0.33. At low frequency, the typical value of the PMMA Young modulus is $E_0 = 3.2\ \mathrm{GN\,m^{-2}}$ and the corresponding Rayleigh wave speed is $V_R^0 = 989\ \mathrm{m/s}$. Instead a high-frequency acoustic wave (with a frequency $f > 150$ kHz), propagating in the material, induces a different elastic response. The longitudinal and transverse wave speeds are, respectively, 2800 m/s and 1450 m/s at $f > 150$ kHz. The Young modulus at high frequency is $E_1 = 6.0\ \mathrm{GN\,m^{-2}}$ and the corresponding Rayleigh wave speed is $V_R^1 = 1354\ \mathrm{m/s}$. We measured that, for frequencies larger than 150 kHz, E_1 is constant within experimental errors. Notice that the ratio of the low and high frequency Rayleigh wave speeds, V_R^0 and V_R^1, is $V_R^1/V_R^0 = \sqrt{E_1/E_0} = 1.37$. This is a very large ratio compared to that of other materials (in metals, for example, it is only 1.05 or less). This frequency dependence of the PMMA elastic constant is very useful for the interpretation of the results.

With our experimental apparatus described in Ref. [12], we measure the strain ΔL and the stress P applied to the sample, the length of the crack $l(t)$ as a function of time, and the sound emitted during the fracture. The maximum sensitivity in the measurement of $l(t)$ is better than 0.5 mm and the absolute accuracy about 5%. The velocity $V(t)$ of the crack is always obtained by taking the derivative of $l(t)$ and the maximum absolute error is between 1 and 10 m/s depending on the bandpass which is always larger than 1 MHz. The sound emission, produced by the moving crack, is measured in the range (10 kHz, 1 MHz) with two wide band microphones located as in Fig. 1. Experiments are done by increasing the sample strain, till the fracture point, by fixed increment steps of less than 50 μm. The strain rate is less than 10 μm/s. Between two consecutive steps we wait about 10 s, and crack usually occurs during this waiting time.

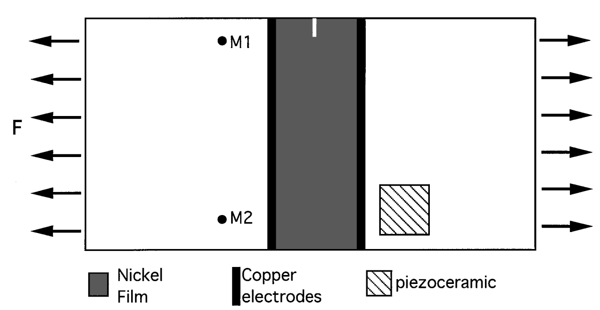

FIG. 1. Typical specimen used in our experiment. M1 and M2 indicate the microphone positions. The nickel film is used to measure $l(t)$.

Let us summarize the main experimental results established in mode I crack in PMMA [9,11,12]. Once started, crack accelerates till a steady state speed V_f, which is always smaller than the sound Rayleigh wave speed V_R^0, specifically $V_f < 0.7V_R^0$. The steady state velocity V_f is a function of the strain $\Delta L/L$ of the sample; that is, $V_f = V_R^0 g(\Delta L/L)$, where g is an increasing function. The final velocity normalized to V_R^0 is plotted in Fig. 2(a) as a function of the relative strain. It is worth recalling that, during crack propagation, several instabilities have been observed. These instabilities are controlled by the local velocity $V(t)$ of the crack [9,11,12]. The first one has a threshold at V_{osc} and it is characterized by the appearance of velocity oscillations and a continuous high frequency sound emission ($f \geq 150$ kHz). When $V(t)$ is higher than a second threshold V_c the surface of the crack develops a strong roughness which increases with $V(t)$. Finally for $V(t) > V_b$ the crack branches in several different paths. All of these thresholds are well defined fractions of V_R^0, specifically $V_{\text{osc}} = 0.33V_R^0$, $V_c = 0.45V_R^0$, and $V_b = 0.67V_R^0$. The velocities of a slow crack ($V_f = 200$ m/s) and a fast one ($V_f = 600$ m/s) are plotted as a function of l, in Figs. 3(a) and 3(c), respectively. In Fig. 3(a) no high frequency oscillations are present. In Fig. 3(c), we clearly see the appearance of the velocity oscillations as soon as $V(l) > V_{\text{osc}}$ at $l_1 = l(t_1) \simeq 2$ cm. These velocity oscillations go with a continuous acoustic emission from the crack tip [12]. After having been reflected by the sample edges, the emitted sound comes back to the crack tip at $l_2 = l(t_2) \simeq 4$ cm with a time delay $(t_2 - t_1) = 80$ μs. The time delay $(t_2 - t_1)$ is just the time needed by the sound emitted at l_1 to go to the sample edges and come back from the edges to l_2. In Fig. 3(b) we see that at l_2 the crack tip accelerates to reach a new velocity plateau. We will show that this acceleration is just produced by the crack-sound interaction. It is useful to trace these data in a different way to show more clearly the change of the mean velocity produced by sound. In order to do that we use the Freund formula that can be expressed in the following way:

$$V(l) = V_f(1 - l_c/l), \tag{1}$$

where l_c is the initial crack length. This Freund description is in general inaccurate for several reasons. We mention only two of them. One is that the time dependency of l, obtained from Eq. (1), presents a divergency for $l = l_c$. Another of these reasons is that the Freund equation [Eq. (1)] predicts that V_f is a fixed fraction of V_R, whereas, experimentally, a set of final velocities is found with 200 m/s $< V_f <$ 700 m/s in PMMA [Fig. 2(a)]. However it turns out that, for $l > l_c$, $V(l)$ has the same functional behavior of Eq. (1) in samples with $V_f < V_{\text{osc}}$. This can be easily checked, noticing that from Eq. (1) we immediately find

$$l(t)V(t) = V_f[l(t) - l_c]; \tag{2}$$

i.e., the quantity $[l(t)V(t)]$ has a linear dependence on $l(t)$. In Figs. 3(b) and 3(d) we plot $[l(t)V(t)]$, obtained, respectively, from the data shown in Figs. 3(a) and 3(c), as a function of l. We clearly see that the velocity of the slow crack [Fig. 3(b)] oscillates around a straight line, whose slope is the final velocity $V_f = 260$ m/s. The data of Fig. 3(b) indicate that, at least for slow plates, Eq. (2) is a good way to estimate the asymptotic mean velocity for $l > 1.1l_c$. In Fig. 3(d) we can observe that the instantaneous velocity of the fast crack is instead aligned on two lines one of slope $V_1 = 420$ m/s and a second of

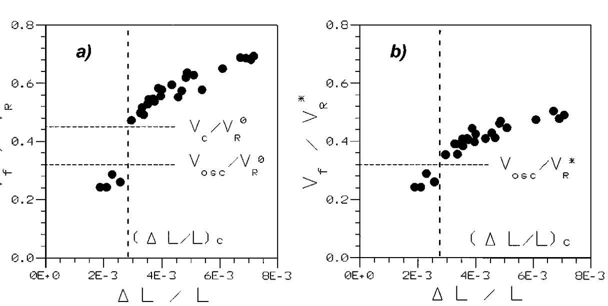

FIG. 2. (a) Final velocity V_f of the crack normalized to V_R^0 as a function of the applied strain $(\Delta L/L)$. V_{osc} is the acoustic emission threshold and V_c the critical velocity for roughness formation. $(\Delta L/L)_c$ is the strain value for V_{osc} and V_c. (b) Final velocity V_f of the crack normalized to V_R^* as a function of the applied strain $(\Delta L/L)$.

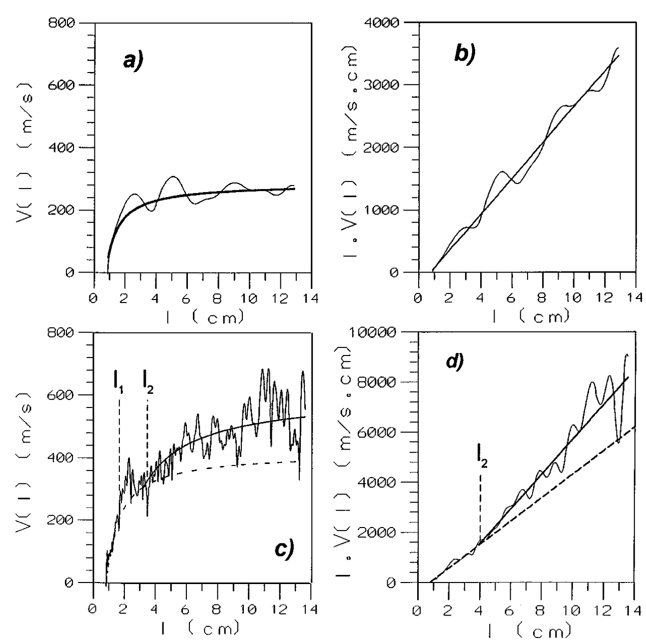

FIG. 3. (a) $V(l)$ as a function of l for a slow crack ($V_f < V_{\text{osc}}$) in a sample broken with an imposed strain $\Delta L/L = 2.3 \times 10^{-3}$. The continuous line is a fit done with Eq. (1). (b) $[lV(l)]$ as a function of l for the data of (a). The straight line is the best fit done using Eq. (2). (c) $V(l)$ as a function of l for a fast crack ($V_f > V_{\text{osc}}$) in a sample broken with an imposed strain $\Delta L/L = 5 \times 10^{-3}$. The dashed line is a fit done with Eq. (1) before the sound generation. The continuous line is a fit done with Eq. (1) after the sound reaches the tip of the crack. (d) $[lV(l)]$ as a function of l for the data of (c).

slope $V_2 = V_f = 580$ m/s. The ratio V_f/V_1 is constant for all the fast cracks and equal to 1.35 ± 0.05. The change of slope sharply occurs at l_2. This hints that an important link between acoustic emission and mean crack speed behavior exists. On the other hand, in slow cracks, which do not present acoustic emission, $V(l)$ does not change its behavior and only one asymptotic velocity is reached.

Another experimental observation can be linked up with this problem: the final velocity domain presents a forbidden gap between approximately $0.33V_R^0$ and $0.45V_R^0$. Notice that in Fig. 2(a) there are no points in this interval indicated by the two horizontal dashed lines. The interesting thing of this domain is that its limits are, respectively, V_{osc} and V_c. The ratio of these speeds is also equal to 1.37 as the ratio V_2/V_1.

To show the link between acoustic emission and the change of the mean crack speed, we have carried out the following experiment. The principle is to perturb the crack growth with ultrasounds emitted by a piezoceramic that was mounted on the sample as indicated in Fig. 1. The amplitude and the frequency of this artificially generated ultrasound burst (AGUB) are comparable with those of the natural acoustic emission of crack. This perturbation experiment is performed in the following way. When the crack, propagating with its constant velocity (which always occurs for $l > 4$ cm), arrives at $l \simeq 5$ cm, a burst generator is triggered. The generator signal is amplified and sent to the piezoceramic, which emits the AGUB. This sound burst reaches the crack tip 40 μs later, when the crack tip is already at $l \simeq 7$ cm. The AGUB is a burst of 25 cycles at frequency between 100–200 kHz, but because of the reflections of this signal on the edges of the samples, the crack is in interaction with sound till the plate is totally broken. In Ref. [12], we have already shown that the length and the shape of the initial cut determines the final crack speed. Thus we may chose to send the AGUB either on a slow crack, with $V_f < V_{\mathrm{osc}}$, or on a fast crack with $V_f > V_{\mathrm{osc}}$. We recall that for $V_f < V_{\mathrm{osc}}$ crack does not produce a continuous sound emission, so the only sound present in the plate is that generated by the piezoceramic.

In Fig. 4 we show the typical behavior of $V(l)$ for two cracks that have been perturbed by an AGUB of frequency 180 kHz. One of the two cracks [Figs. 4(a) and 4(b)] belongs to a specimen that has been broken at $(\Delta L/L) = 2.3 \times 10^{-3}$. The final velocity of this crack, without the AGUB, would be 250 m/s $= 0.25V_R^0 < V_{\mathrm{osc}}$, as one can easily check in Fig. 2(a). The second specimen was broken [Figs. 4(c) and 4(d)] at $(\Delta L/L) = 4 \times 10^{-3}$ and the final velocity of the crack, without AGUB, would be 500 m/s $= 0.55V_R^0 > V_{\mathrm{osc}}$. In Figs. 4(a) and 4(b) one can see that the mean speed of the slow crack changes from $V_1 = 250$ m/s to $V_2 = 342$ m/s, with $V_2/V_1 = 1.37$. This occurs just at the onset of the interaction between the AGUB and the crack at $l \simeq 7$ cm. Notice that in Fig. 4(b) the final velocity V_2 is just inside the

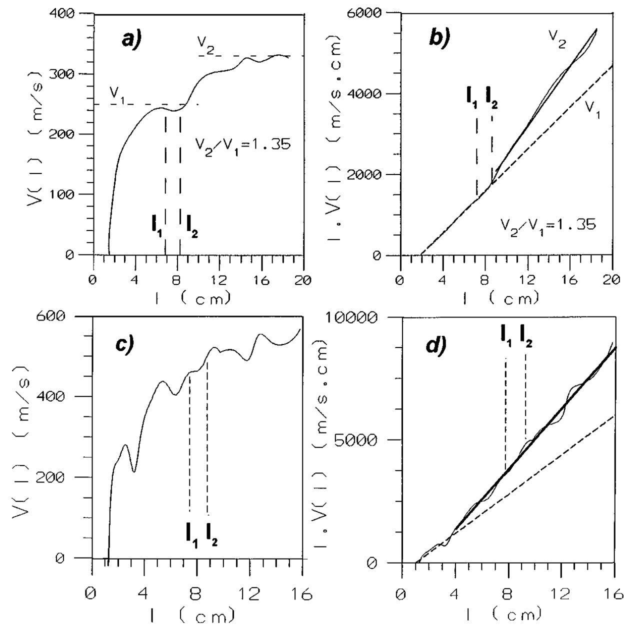

FIG. 4. Perturbation experiment. The lengths $l_1 = l(t_1)$ and $l_2 = l(t_2)$ are, respectively, the points where the AGUB is generated by the piezoceramic and where it reaches the crack tip. (a) $V(l)$ as a function of l for a slow crack in a sample broken with an imposed strain $\Delta L/L = 2.3 \times 10^{-3}$. (b) $[lV(l)]$ as a function of l for the data of (a). (c) $V(l)$ as a function of l for a fast crack in a sample broken with an imposed strain $\Delta L/L = 4 \times 10^{-3}$. (d) $[lV(l)]$ as a function of l for the data of (c).

forbidden gap of Fig. 2. Instead in Figs. 4(c) and 4(d) one sees that when the AGUB is generated in a fast crack, when $V(l) > V_{\mathrm{osc}}$ and the natural sound emission is already present (data have been filtered just to show the behavior of the mean velocity without oscillations), the crack is not further perturbed by AGUB and the mean crack speed does not change. In order to check the properties of the sound-crack interaction we changed the amplitude A and the frequency of the acoustic burst. The main results of these experiments are the following:

(i) For slow cracks with $V_f < V_{\mathrm{osc}}$ [Figs. 4(a) and 4(b)], the AGUB allows us to reproduce the change of mean crack speed behavior described before for fast crack perturbed by the natural sound emission (see Fig. 3). The ratio of speeds V_2/V_1 is reproducible for a constant frequency of the sound burst. This has been checked on more than 50 samples.

(ii) With the AGUB we are able to produce final velocities in the forbidden gap [$V_f = 360$ m/s $= 0.37V_r^0$; Fig. 4(b)].

(iii) This phenomenon does not depend on the acoustic emission frequency for $f > 150$ kHz. The frequency of the natural acoustic emission is always larger than 150 kHz.

(iv) A threshold exists. A minimal amplitude of the acoustic signal A_c is needed to change crack velocity. This threshold depends on the sound frequency. But as soon as the signal amplitude A is bigger than the threshold A_c ($10 \pm 5\ \mathrm{W\,cm^{-2}}$ at location $M1$ in Fig. 1). This amplitude is twice smaller than the natural acoustic emission emitted at V_{osc} and estimated at the crack tip after its reflexion on the sample edges, taking into account the sound attenuation in PMMA. The final speed does not depend on A.

(v) The previous items (i)–(iv) do not apply to fast cracks [Figs. 4(c) and 4(d)] where the AGUB is sent on a velocity plateau bigger than V_{osc}. In this case the AGUB does not trigger any mean speed modifications.

Items (iv) and (v) seem to exclude the possibility of energetic transfer between the acoustic signal and the crack tip. To summarize, the acoustic emission produced by the crack or generated by the piezoceramic changes the mean crack speed. This change of mean velocity is not affected by acoustic amplitude (if $A > A_c$) and it does not depend on the sound frequency when $f > 150$ kHz.

These results lead us to consider an interpretation of the interaction of crack with sound. As noticed before the Young modulus of PMMA depends on the frequency at which is measured whereas the Poisson ratio is constant. The ratio of the low and high frequency Rayleigh wave speeds, V_R^0 and V_R^1, is $V_R^1/V_R^0 = \sqrt{E_1/E_0} = 1.37$, which is equal to the ratio V_2/V_1 of the two velocity plateaus observed in our experiment. This dependence of PMMA elastic properties as a function of frequency suggests the following process: when the crack speed is lower than V_{osc}, the crack propagates in a elastic material of constant E_0. But if the crack is fast, the speed reaches V_{osc}; therefore there is a durable acoustic emission at high frequency, which reflects on the sample edges and comes back on the crack tip. Because of the presence of high-frequency oscillations the crack tip "sees" a more rigid medium and accelerates in order to reach a higher velocity V_2 that is compatible with the applied strain and the higher Rayleigh speed V_R^1. One can define an effective Rayleigh wave speed "seen" by the crack:

$$V_R^* = \begin{cases} V_R^0 & \text{when } V(t) < V_{\mathrm{osc}}\,, \\ V_R^1 & \text{when } V(t) > V_{\mathrm{osc}}\,. \end{cases} \qquad (3)$$

With this effective Rayleigh wave speed, we can give a new interpretation of the data.

(a) If we plot the final velocity V_f normalized at V_R^* as a function of $\Delta L/L$ [Fig. 2(b)], the forbidden gap disappears in favor of an increasing and continuous function. Moreover the final speeds domain becomes $0.2V_R^* < V_f < 0.5V_R^*$.

(b) The different thresholds of velocities change too: $V_{\mathrm{osc}} = 0.33V_R^*$, $V_c = 0.5V_R^0 = 0.33V_R^*$, $V_b = 0.7V_R^0 = 0.46V_R^*$. Both first thresholds are now identical, $V_{\mathrm{osc}}/V_R^* = V_c/V_R^* = 0.33$ through they are at different speeds: $V_c = 1.37V_{\mathrm{osc}}$.

(c) We may understand the existence of the forbidden gap and the differences between slow and fast plates in Freund's representation. A crack with a final velocity smaller than V_{osc} does not emit acoustic waves and the final velocity is not modified. In contrast a crack with a final speed V_1 equal to V_{osc} emits an acoustic wave for which the elastic properties of the PMMA are different. So the crack tip seems to propagate in a more rigid material. The new final velocity V_2 is now $V_2/V_R^* = V_1/V_R^*$; that is, $V_2 = (V_R^1/V_R^0)V_1 = 1.37V_1$ [15].

In conclusion we have shown that, in spite of the weak energy of the natural sound emitted by the crack, the interaction of sound with the crack tip cannot be neglected. The crack velocity in PMMA is indeed strongly modified by a sound wave. This result has been obtained using the large frequency dependence of the PMMA Young modulus. The interaction of sound with the crack tip allows us to explain the existence of a forbidden gap, of the two speed plateaus for fast cracks, and of only one velocity plateau for slow cracks. This paper opens the problem of the physical mechanisms which make the sound-tip interaction possible.

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[15] It has to be noticed that the forbidden gap will disappear in an infinite sample, where emitted sound cannot be reflected back to the tip. In this sense, the existence of the forbidden gap between V_{osc} and V_c may depend on the geometry.