Collective Dynamics of Intrinsic Josephson Junctions in High-T_c Superconductors

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The dynamics of one-dimensional arrays (mesostacks) of intrinsic Josephson junctions in high-T_c superconductors is theoretically investigated with both the quasineutrality breakdown effect and the quasiparticle charge imbalance effect taken into account. The current-voltage characteristics are obtained at various parameters and their peculiarities are discussed. The results of recent experiments on the intrinsic Josephson effect in high-T_c superconductors are compared with the presented theory. Nonequilibrium effects inside superconducting layers are proposed to be the origin of the collective dynamics. [S0031-9007(98)05845-1]

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Strong anisotropic high temperature superconductors (e.g., Bi-2212, Bi-2223, and Tl-2223 compounds) are generally considered as a set of superconducting layers of atomic thickness connected by intrinsic Josephson junctions. Direct measurement of dc and ac Josephson effects with the external current perpendicular to the layers have been carried out recently [1–13]. Four different one-dimensional (without Josephson vorticies) dynamical regimes have been observed.

(i) First of all, *multiple branches* at the current-voltage characteristics have been observed in ordinary $Bi_2Sr_2CaCu_2O_{8+\delta}$ [1,3,8–10,12], $Tl_2Ba_2Ca_2Cu_3O_{10+\delta}$ [1,7,8], and voltage-based $(Pb_yBi_{1-y})_2Sr_2Ca_2Cu_3O_{10+\delta}$ [11,13] structures. It is obviously a result of step by step switching of intrinsic junctions into a resistive state with a current increase and hysteretic behavior of individual S-I-S junctions [1]. Microwave emission measurements at rather high currents show that there are several groups of many coherently oscillating junctions. The current-voltage characteristics can be explained, if one assumes some difference in the junction parameters or interaction between them. In any case, we have to introduce interaction to explain coherent emission because mutual phase locking is necessary (see, e.g., [14]). (ii) In other compounds (e.g., $(Pb_yBi_{1-y})_2Sr_2CaCu_2O_8$ [1,3], oxygen-annealed Bi-2212 [1], and current-based $(Pb_yBi_{1-y})_2Sr_2Ca_2Cu_3O_{10+\delta}$ [11,13]), which are less anisotropic, a single hysteresis has been observed instead of multiple branching due to *collective switching* of many junctions into a resistive state, coherent emission has also been measured. These structures are considerably less anisotropic than lead free Bi-2212, thus interaction between the junctions is strong. (iii) In some other samples multibranching and hysteresis are absent; this behavior can be explained, if one assumes that normal resistance is mainly determined by parasitic shunts [1]. (iv) Finally, in some nonhysteretic samples microwave measurements show that junctions are unlocked and a broadband radiation frequency spectrum is observed. Summing up all of these experimental results we can conclude that some coupling mechanism determines collective dynamics of intrinsic Josephson junctions in one-dimensional geometry.

A theory of electromagnetic coupling in layered structures is developed in Refs. [15,16] and applied to the problem of the synchronization of the Josephson vortex motion in [17]. In [17] it was also suggested that in the case of thin superconducting layers some nonequilibrium mechanisms are to be taken into account. In recent papers [18] a time dependent Ginzburg-Landau equation in the framework of a Lawrence-Doniach model was used to account the nonequilibrium effects and a similar approach was used in [19]. The microscopic mechanisms of disequilibrium inside layers are discussed in [20–22].

The key point of our theory is a nonequilibrium nature of the ac Josephson effect in layered superconductors [18,20–22]. It means that superconducting layers are in the nonstationary nonequilibrium state due to the injection of quasiparticles and Cooper pairs, and a nonzero invariant potential

$$\Phi_i(t) = \phi_i - (\hbar/2e)(\partial\theta_i/\partial t)$$

is generated inside them, where ϕ_i is the electrostatic potential and θ_i is the phase of superconducting condensate, $\Phi = 0$ in the equilibrium state. (Here and below $e = |e|$.) The important point to notice is that in the nonequilibrium regime an ordinary Josephson relation $(d\varphi/dt) = (2e/\hbar)V$ between the Josephson phase difference $\varphi_{ij} = \theta_i - \theta_j$ and voltage $V_{ij} = \phi_i - \phi_j$ is violated [18,20–25]. Instead, we have (from the definition of Φ)

$$\frac{d\varphi_{ij}}{dt} = \frac{2e}{\hbar} V_{ij} + \frac{2e}{\hbar}(\Phi_j - \Phi_i). \tag{1}$$

Thus $\Phi_i(t)$ are the new important dynamical variables of the theory. The layer thickness d_0 is smaller than the characteristic length of the disequilibrium relaxation l_E and the Debye length r_D, so that superconducting layers are in a homogeneous nonequilibrium state. Note that the shift of the chemical potential of the superconducting condensate from its equilibrium value is $\delta\mu_s = e\Phi$ and is determined from the expression for charge density inside

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a superconducting layer [23–26]

$$\rho_i = -2e^2 N(0)(\Phi_i - \Psi_i) = -\frac{1}{4\pi r_D^2}(\Phi_i - \Psi_i), \tag{2}$$

where Ψ_i is determined by the electron-hole charge imbalance

$$e\Psi_i = -\int_\Delta^\infty (n_\epsilon^i - n_{-\epsilon}^i)\, d\epsilon, \tag{3}$$

where we use the averaged-over-momentum-direction quasiparticle distribution function n_ϵ^i introduced by Eliashberg [26], which describes quasielectron (at $\epsilon > 0$) and quasihole (at $\epsilon < 0$) energy distributions, $|\epsilon|$ is the quasiparticle energy. In equilibrium $n_\epsilon^i = n_{-\epsilon}^i = n_\epsilon^{(0)} = 1/2[1 - th(|\epsilon|/2T)]$.

From Eq. (2) one can see that the quasineutrality breakdown effect ($\rho \neq 0$) as well as the charge imbalance effect ($\Psi \neq 0$) lead to generation of a nonzero Φ. Both these mechanisms can take place in layered superconductors. Quasineutrality breakdown has been considered from this point of view by Koyama and Tachiki [20]. The importance of charge imbalance generation has been stressed by Artemenko and Kobelkov [21] and Ryndyk [22]. In this work we consider dynamical equations of a general type and apply them to the problem of the one-dimensional intrinsic Josephson effect.

To obtain the full set of equations we use, following Refs. [16,18–21], the continuity equation $d_0 S(d\rho_i/dt) = J_{i-1\,i} - J_{i\,i+1}$, and the expression describing voltage (electric field) boundary condition on a superconducting layer $V_{i\,i+1} - V_{i-1\,i} = (4\pi d_0 d/\epsilon_0)\rho_i$, here d is the interlayer distance, S is the area of the intrinsic junction, and ϵ_0 is the dielectric constant. From these equations it follows that

$$J_{ij} + \frac{\epsilon_0 S}{4\pi d}\frac{dV_{ij}}{dt} = J(t), \tag{4}$$

$J(t)$ is the external current.

The dynamics of quasiparticles can be taken into account with the help of the kinetic equation. Following the method presented in Ref. [22], where the quasineutral dynamics of a multilayer structure was considered ($\rho_i = 0$ and $\Phi_i = \Psi_i$), we propose the equation for the charge imbalance in linear and low frequency limit

$$\tau_q \frac{d\Psi_i}{dt} + \Psi_i + \eta(2\Psi_i - \Psi_{1-i} - \Psi_{i+1}) = \eta\frac{\hbar}{2e}\left(\frac{d\varphi_{i-1\,i}}{dt} - \frac{d\varphi_{i\,i+1}}{dt}\right), \tag{5}$$

where $\eta = 2\nu\tau_q f(T)$ is the parameter of disequilibrium, $\nu = [4e^2 N(0) R_N S d_0]^{-1}$ is the "tunnel frequency," R_N is the normal resistivity of the tunnel junction, $V = Sd_0$ is the volume of the superconducting layer, $N(0) = mp_F/2\pi^2\hbar^3$, and τ_q is the well-known charge imbalance relaxation time, which can be estimated as $\tau_q \sim \tau_\epsilon$ at low temperatures and $\tau_q \sim (4T/\pi\Delta)\tau_\epsilon$ at $T \sim T_c$, $\tau_\epsilon^{-1} = 14 g\hbar^{-1}\theta_D^{-2}T^3\zeta(3)$ is the inelastic electron-phonon scattering frequency. $f(T)$ is the temperature factor describing quasiparticle freezing at low temperatures, $f \to 1$ at $T \to T_c$. In ordinary isotropic BCS superconductors $f(t) = 1 - \tanh(\Delta/2T)$, and the number of quasiparticles is exponentially small at low temperatures. However, in layered superconductors, especially in the case of d pairing, it may be zeros in the energy gap in k space and the quasiparticle effects may be more significant. This possibility is strongly supported by recent experiments [9–13], so it is better to use for estimations temperature dependent resistivity $R = R_N/f(T)$, which can be determined as $R = V_c/J_c$; J_c and V_c are the critical current and the voltage at this current belonging to the resistive branch. Finally, if we express $N(0)$ through Debye radius r_D and assume $r_D \approx d_0 = 3$ Å, we obtain useful formula

$$\nu f(T) = 1.7 \times 10^8 \frac{(J_c/S)\,[\mathrm{A/cm^2}]}{V_c[\mathrm{mV}]}. \tag{6}$$

In the same approximation we derive the expression for the nonequilibrium interlayer current

$$J_{ij} = J_c\left[1 - \frac{(e\Psi_i)^2}{\Delta^2}\right]\left[1 - \frac{(e\Psi_j)^2}{\Delta^2}\right]\sin(\varphi_{ij}) + \frac{\hbar}{2eR}\frac{d\varphi_{ij}}{dt} + \frac{\Psi_i - \Psi_j}{R}. \tag{7}$$

Here we take into account variation of the energy gap Δ_i depending on potential Ψ_i [27] and, consequently, variation of critical currents $J_{c\,ij} \propto \Delta_i\Delta_j$.

Finally, we obtain the set of equations in the dimensionless form

$$\beta\frac{d^2\varphi_{ij}}{d\tau^2} + \frac{d\varphi_{ij}}{d\tau} + (1 - \kappa\psi_i^2)(1 - \kappa\psi_j^2)\sin(\varphi_{ij}) + \psi_i - \psi_j + \beta\left(\frac{d\mu_i}{d\tau} - \frac{d\mu_j}{d\tau}\right) = j(t), \tag{8}$$

$$\alpha\frac{d\psi_i}{d\tau} + \psi_i + \eta(2\psi_i - \psi_{i-1} - \psi_{i+1}) = \eta\left(\frac{d\varphi_{i-1\,i}}{d\tau} - \frac{d\varphi_{i\,i+1}}{d\tau}\right), \tag{9}$$

$$\mu_i + \zeta(2\mu_i - \mu_{i-1} - \mu_{i+1}) = \psi_i + \zeta\left(\frac{d\varphi_{i-1\,i}}{d\tau} - \frac{d\varphi_{i\,i+1}}{d\tau}\right), \tag{10}$$

$$\sum_i \frac{d\varphi_{i-1\,i}}{d\tau} = v(t), \tag{11}$$

where $j(t)$ is the external current in J_c units, $v(t)$ is the external voltage in V_c units, $\mu(t) = \Phi(t)/V_c$, $\psi(t) = \Psi(t)/V_c$, and $\alpha = \tau_q \omega_c$, $\beta = \omega_c^2/\omega_p^2$, $\zeta = (\epsilon_0 r_D^2)/(d_0 d)$, $\omega_c = 2eRJ_c/\hbar$, $\omega_p^2 = (8\pi e d J_c)/(\hbar \epsilon_0 S)$, $\kappa = (eV_c/\Delta)^2$, $\tau = \omega_c t$. These equations may be considered as a good phenomenological approximation at all temperatures. In the limit $\eta \rightarrow 0$ the quasiparticle contribution can be neglected (if the voltage is less than 2Δ and pair braking is forbidden), so that $\psi = 0$ and we have the equations for φ and μ similar to that obtained in Ref. [20]. If $\zeta \rightarrow 0$ then $\mu = \psi$, and we obtain the equations of Ref. [22], which are valid for thick enough layers or large interlayer distance ($r_D \ll d, d_0$).

Now we go to discuss various regimes of intrinsic Josephson effect qualitatively and also present the results of numerical calculations of the current-voltage characteristics in the case of ten junctions with equilibrium edge layers in the fixed-current limit (Figs. 1–4). The parameters of intrinsic junctions are taken from [1–13]. The weak dispersion of critical currents (less than 1 percent) has been introduced additionally.

The shape of the current-voltage characteristic is determined by the parameter β (at $\beta \gg 1$ there is hysteresis) and by the competition of two quasiparticle effects: "current effect" and gap suppression effect. If one of intrinsic junctions switches into resistive state, then nonequilibrium quasiparticle distribution is induced in the neighbor junctions. Inhomogeneity of charge imbalance distribution leads to the quasiparticle current through the neighbor junctions, and because of that supercurrent through these junctions decreases and larger external current is needed to switch these junctions into resistive state ("current effect"). In the case of $\beta \gg 1$ this leads to the enhanced multibranching. The effect of gap suppression is opposite to the current effect because the critical current is suppressed (κ terms in the equations). In the case of $\beta \gg 1$ this leads to single hysteresis instead of multibranching. At low η, multibranching is preferable because gap suppression is proportional to ψ^2. At large η gap suppression "wins the game" and collective switching takes place. At low β hysteresis is absent and the I-V curve is single valued.

There are four typical regimes corresponding to four experimentally observed situations.

(i) *Multiple branches* (Fig. 1) similar to those observed experimentally appear in the strong anisotropic case $J_c/S \sim 10^3$ A/cm^2, $V_c \sim 10$ mV, thus $\nu f(T) \sim 10^{10}$, $\omega_c \sim 10^{13}$, $\kappa \sim 0.1$. Taken $\epsilon_0 \sim 10$ we get $\omega_p \sim 10^{12}$, $\beta \sim 100$, $\zeta \sim 2$. For $\tau_q \sim 10^{-11}$ sec we obtain $\eta \sim 0.1$, $\alpha \sim 100$. The observed dispersion of critical currents may be significantly larger than the initial one (1 percent in our calculations) because of the current effect.

(ii) In the less anisotropic case $J_c/S \sim 10^4$ A/cm^2, thus $\nu f(T) \sim 10^{11}$, $\omega_p \sim 10^{13}$, $\beta \sim 1$–10, and $\eta \sim 1$–10. In this case the structure of multibranches is fine and a single large hysteresis appears (Fig. 2) due to the gap suppression effect.

(iii) In the case of parasitic shunts $V_c \sim 1$ mV is less than in an ordinary case and hysteresis is absent, $\omega_c \sim 10^{12}$, $\beta < 1$. The example at $\alpha = 1$, $\beta = 0.1$, $\eta = 0.5$, $\zeta = 0.2$ is shown in Fig. 3. This regime is phased locked and $\dot{\varphi}_{3\,4}(t)$ and $\dot{\varphi}_{5\,6}(t)$ dynamics at $j = 2$ is shown in the inset.

(iv) Finally, at small β and larger coupling ($\eta > 1$), which can take place in shunted junctions, we consider the case $\alpha = 0.1$, $\beta = 0.1$, $\eta = 2$, $\zeta = 2$ (Fig. 4). The current-voltage characteristic looks like the previous one, but the dynamical regime is rather different. The time dependencies of $\dot{\varphi}_{4\,5}(t)$ and $\dot{\varphi}_{5\,6}(t)$ are shown in the inset. Unlike the coherent state, intrinsic Josephson junctions are phase unlocked. This result can explain the broadband microwave emission observed experimentally. The same result (broadband emission) may be explained by thermal

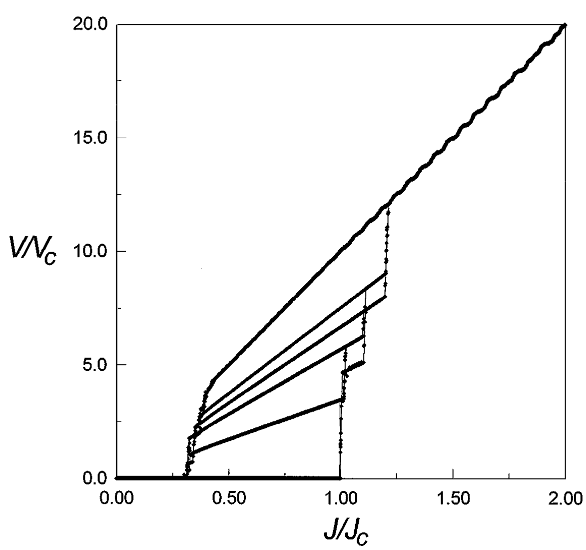

FIG. 1. The current-voltage characteristic of most anisotropic structure with $J_c/S \sim 10^3$ A/cm^2 ($\alpha = 100$, $\beta = 100$, $\eta = 0.1$, $\zeta = 2$). Multiple branches are clearly observed.

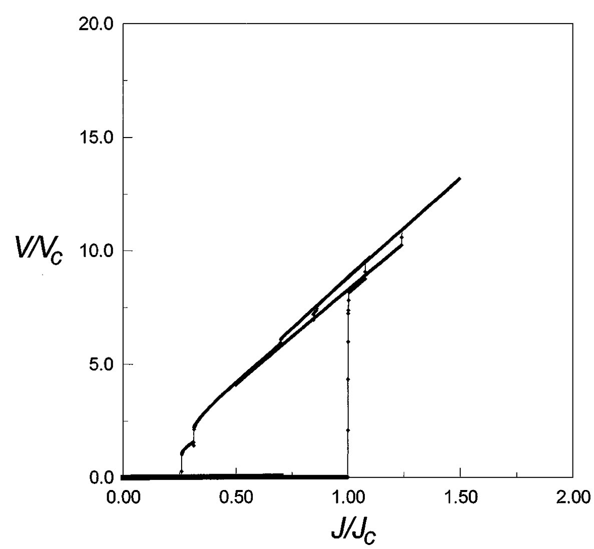

FIG. 2. The current-voltage characteristic of less anisotropic structure with $J_c/S \sim 10^4$ A/cm^2 ($\alpha = 100$, $\beta = 10$, $\eta = 10$, $\zeta = 2$). Collective switching takes place at $J = J_c$.

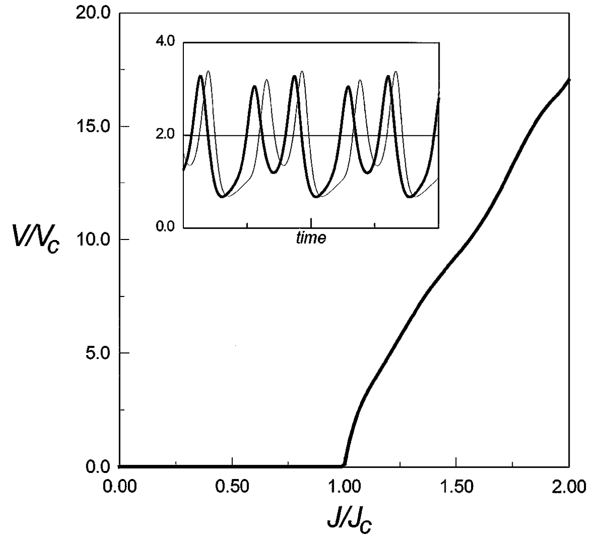

FIG. 3. The nonhysteretic current-voltage characteristic of the structure with parasitic shunts ($\alpha = 1$, $\beta = 0.1$, $\eta = 0.5$, $\zeta = 0.2$). Phase-locked regime. $\dot{\varphi}_{3\,4}(t)$ and $\dot{\varphi}_{5\,6}(t)$ dynamics is shown in the inset.

fluctuations. These two mechanisms can be distinguished experimentally, but this problem is beyond the scope of this paper.

The results presented in this paper can explain a number of experimental features. We show that quasiparticle dynamics determines the shape of the current-voltage characteristics, explains phase locking at small coupling and phase unlocking at larger coupling. In all probability the theory of the nonequilibrium Josephson effect [based on relation (1)] corresponds to the physical nature of the intrinsic Josephson effect in high-T_c superconductors, including the problems of plasma waves and vortex motion.

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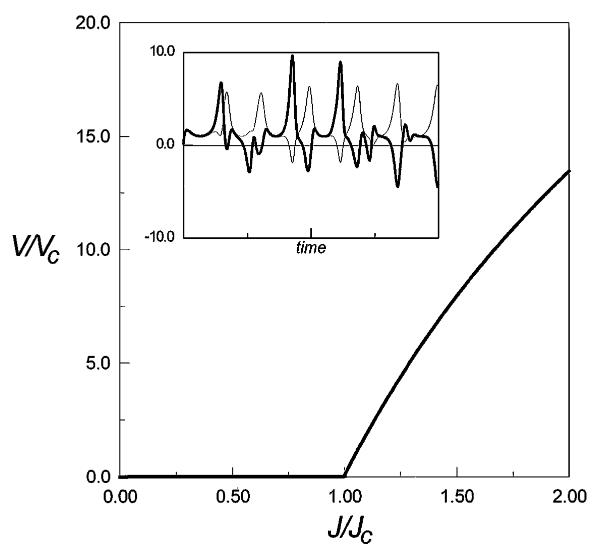

FIG. 4. The nonhysteretic current-voltage characteristic with large η ($\alpha = 0.1$, $\beta = 0.1$, $\eta = 2$, $\zeta = 2$). Phase-unlocked regime. $\dot{\varphi}_{4\,5}(t)$ and $\dot{\varphi}_{5\,6}(t)$ dynamics is shown in the inset.

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