New Liquid Phase and Metal-Insulator Transition in Si MOSFETs

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We argue that there is a new liquid phase in the two-dimensional electron system in Si MOSFETs at low enough electron densities. The recently observed metal-insulator transition results as a crossover from the percolation transition of the liquid phase through the disorder landscape in the system below the liquid-gas critical temperature. The consequences of our theory are discussed for variety of physical properties relevant to the recent experiments. [S0031-9007(98)05758-5]

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The scaling theory of localization [1] of noninteracting electrons tells us that disorder is a relevant perturbation in two dimensions: The system is an insulator at low enough temperatures. The recent discovery of a possible metal-insulator transition in Si MOSFETs by Kravchenko *et al.* [2] apparently defies this long-held belief. This should not be surprising because the dominant Coulomb interaction in Si MOSFETs may invalidate the noninteracting scaling theory. In a typical Si MOSFET sample where the transition is observed, the ratio between the average Coulomb interaction and the Fermi energy is about 20. These intriguing experiments [2–6] generate renewed interests in the properties of low density two-dimensional electron systems [7], especially in the combined effects of interaction and disorder in such systems [8,9]. One of the striking features of the system is the *I-V* nonlinearity at average electric fields much weaker than the expected value determined by the effective temperature of the electrons [10]. This raises questions of whether the system can be described by a theory for homogeneous systems. (See the discussion on electric field dependence in the text for more detail.) As we argue in this Letter, the mobile positive background in Si MOSFETs allows macroscopic inhomogeneity to occur more easily at low enough average electron densities. The metal-insulator transition, the small field nonlinear *I-V*, as well as a host of other phenomena result from the combined effect of the inhomogeneity and the disorder in the system.

In a remote doped or modulation doped GaAs/AlGaAs sample, the electron system is well described by the classic jellium model of electron gas. The roughly uniform positive background is fixed by the bulk crystal and it does not participate in the dynamics of the electron system. On the other hand, in an inverted Si MOSFET, the positive background charges at the metal-oxide interface are mobile. This allows for more possibilities for the physical properties of the electron system in Si MOSFETs. However, we want to emphasize that the mobile positive background is not absolutely essential for the macroscopic inhomogeneity to occur. A slowly varying disorder potential combined with electron-electron correlation, for example, may also favor macroscopic inhomogeneity in the system. Obviously detail microscopic calculations are needed to address this issue. This is deferred to a future publication.

We now consider what happens at low enough electron densities in a Si MOSFET. For the convenience of our argument, we treat the mobile positive background charges as holes. We consider the adiabatic evolution of the system as the oxide thickness increases. At zero thickness, the phase diagram of such an electron-hole system, shown in Fig. 1, is well established [11,12]. The high density phase is a metallic electron-hole liquid. The low density phase is an exciton gas. There is a two-phase coexistence region below the critical temperature T_c. As the oxide thickness increases beyond the Bohr radius a_B, the excitons in the gas phase dissolve into separated electrons and holes. In a disordered sample, these electrons occupy the lowest lying localized states in the system. The internal degrees of freedom, such as spin and valley indices, do not play important roles in determining the energy of the gas phase. This implies that there is a large amount of spin and valley entropy

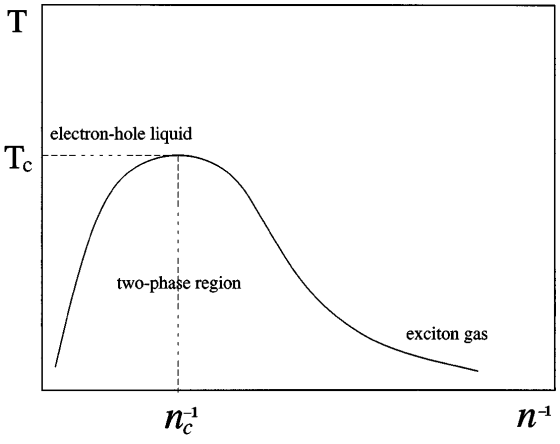

FIG. 1. Phase diagram of an electron-hole system.

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at energies close to the ground state of the localized gas phase and that the energy cost to polarize these internal degrees of freedom is small compared to the cohesive energy of the liquid phase, which is given by the Coulomb interaction. Since there is no excitonic correlation in the electron-hole liquid phase, the length scale set by the Bohr radius does not have a special meaning to the liquid. As long as the oxide thickness is comparable to the interelectron distance, the liquid phase remains intact except that its cohesive and surface energies are reduced. Therefore the new liquid phase in low density Si MOSFETs is adiabatically connected to the electron component of the electron-hole liquid which is an entanglement of two normal Fermi liquids of electrons and holes [11]. It is a singlet of the spin and valley degrees of freedom. In a disordered system, the liquid phase further lowers its energy by occupying the valleys of the disorder landscape. It percolates through the sample in a fashion determined by the competition among its cohesive energy, its surface energy, and the disorder potential. This gives rise to a metal-insulator transition at the percolation threshold in the classical limit at zero temperature. Apparently, quantum tunneling between separated liquid regions or a finite temperature destroys this transition. This is illustrated in Fig. 2. Along the n axis, i.e., at zero temperature and when the coupling constant determining the phase breaking mechanism is infinity, there is a second order phase transition at the percolation threshold n_c. Away from the n axis, the transition is destroyed by a finite temperature or quantum tunneling. However, in the region of temperature and coupling constant close to the critical point, the physics of the system is influenced by the very existence of the critical point, similar to the situation in a quantum phase transition [13,14]. Therefore it is the crossover behavior near the critical point that is observed in the experiments.

We now discuss the related experiments in Si MOSFETs and show how they are consistent with the consequences of our proposed theoretical framework.

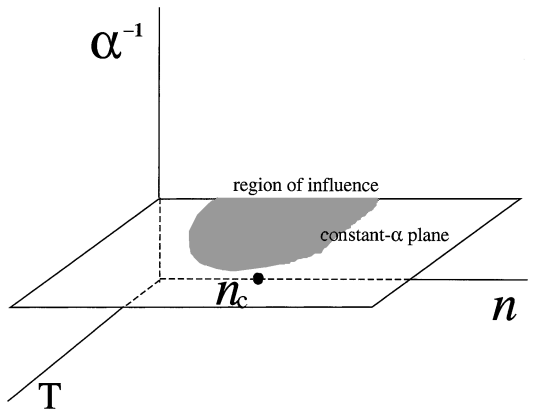

FIG. 2. Effects of a finite temperature and quantum tunneling on the percolation transition.

Temperature dependence of the resistivity.—One of the most striking features of the experiments [2–6] is that the temperature dependence of the resistivity changes from insulator-like to metal-like as the electron density increases. This is readily understood in our theory. At low enough densities and when $T < T_c$, the system is in the two-phase regime. When $n < n_c$, the conduction in the system is dominated by the phonon-assisted hopping through the localized gas phase. This gives rise to the famous Coulomb-gap behavior [15]

$$\rho \sim e^{\sqrt{E_0/T}} \tag{1}$$

at low temperatures. On the other hand, when $n > n_c$, the sample is able to conduct electric current. Using a simple two-fluid model for the conduction, we have

$$\sigma = f_l \sigma_l + f_g \sigma_g \quad \text{and} \quad f_l + f_g = 1, \tag{2}$$

where f_l and f_g are, respectively, the fractions of the electrons in the liquid and the gas phases. σ, σ_l, and σ_g are, respectively, the total conductivity, the conductivity of the liquid, and the conductivity of the localized gas. At T much lower than the cohesive energy Δ_c of the liquid, we estimate f_g to be

$$f_g \simeq A e^{-(\Delta_c/T)}. \tag{3}$$

Combining the above equations, we obtain the low T behavior of the resistivity on the metallic side

$$\rho = \tilde{\rho}_0 + \tilde{\rho}_1 e^{-(\Delta_c/T)}, \tag{4}$$

where $\tilde{\rho}_0 = 1/\sigma_l$ and $\tilde{\rho}_1 = A\tilde{\rho}_0(1 - \sigma_g/\sigma_l)$. This temperature dependence of ρ on the metallic side was suggested by Pudalov [16] based on a completely different interpretation of the experiments. It was observed by Hanein *et al.* [5] in recent experiments on GaAs/AlGaAs hole samples with a conducting backgate. For $T > \Delta_c$, the liquid phase ceases to exist. The resistivity of the system depends weakly on temperature.

Scaling and duality.—In a realistic sample, the coupling constant of the phase breaking mechanism is fixed by the property of the sample. We need to consider what happens in a constant-α plane in Fig. 2. Because of the diverging length scale ξ and vanishing energy scale $\varepsilon \sim e^2/\xi$ close to the critical point n_c, using the usual arguments [14], we write the finite temperature resistivity of the system in the scaling form

$$\rho(T, n) = \tilde{\rho}(\delta/T^{1/\nu}), \tag{5}$$

where $\delta = (n - n_c)/n_c$ and ν is the correlation length exponent. The value of ν predicted by the percolation theory is $\nu = 4/3$. This is within the range of the measured values of $\sim$1.2–1.6.

The duality observed in the experiments [2,3] can be readily understood by the following argument. Because there is no critical point on the constant-α plane at a finite

temperature, the function $\tilde{\rho}(x)$ is an analytic function of its variable. Taylor expanding it at $x = 0$, we have

$$\rho(T, \delta) = \tilde{\rho}^{(0)} + \tilde{\rho}^{(1)} \frac{\delta}{T^{1/\nu}} + \frac{1}{2} \tilde{\rho}^{(2)} \left(\frac{\delta}{T^{1/\nu}} \right)^2 + \ldots . \tag{6}$$

Immediately, we have

$$\rho(T, n_c + \delta)\rho(T, n_c - \delta) = (\tilde{\rho}^{(0)})^2 + \mathrm{O}(\delta^2) . \tag{7}$$

Obviously, this operationally defined duality relation is a generic feature of the critical region. It should be interesting to test this by looking at the temperature dependence of the coefficient of the δ^2 term.

Electric-field dependence of the resistivity.—There are two important features associated with the electric-field experiments [10]. First, the nonlinear *I-V* occurs at very small currents (or electric fields). Second, the nonlinear resistivity exhibits scaling as a function of electric field on both the metallic and insulating sides. There are two known mechanisms for nonlinearity in a homogeneous system [14]. The first requires that the electric field energy scale be the dominant energy scale in the system. In the critical region, this energy should be larger than the temperature. However, in a typical nonlinear *I-V* experiment in Si MOSFETs, the nonlinearity occurs at electric fields as low as 0.25 mV/cm [10]. Using a phase breaking length of 10 000 Å, which is an upper bound of the localization length, we estimate the electric field energy scale to be about 0.3 mK, much smaller than the temperature 220 mK. No nonlinearity should result from this mechanism. The second mechanism is Joule heating. Joule heating raises the effective temperature T_e of the electrons according to $T_e \sim E^{1/2}$ [17]. The experiments in Ref. [10] indicate that T_e is about 2 K at an electric field of 50 mV/cm. Using an electric field of 0.25 mV/cm, we estimate T_e to be 150 mK, which is lower than the lattice temperature 220 mK. Therefore Joule heating is not significant at such an electric field and no nonlinearity should result. The above arguments lead us to believe that the nonlinear *I-V* is caused by the intrinsic inhomogeneity in the system, which greatly enhances the electric field by a factor given by the ratio of the sample size to the typical gap between two liquid regions.

The scaling behavior which appeared at large electric fields can be readily understood following the arguments in Ref. [14]. Close to the critical point, the vanishing energy scale $\epsilon \sim e^2/\xi$ is cut off by the electric field through $\epsilon \sim eE\xi$. Thus, the zero temperature resistivity can be written in the scaling form

$$\rho(E, \delta) = \tilde{\rho}(\delta/E^{1/\nu(z+1)}) , \tag{8}$$

with $z = 1$. Using $\nu = 4/3$, we find $\nu(z + 1) = 8/3 \simeq 2.66$, which is very close to the experimental value of 2.70 [10].

Effects of an in-plane magnetic field. — When the Zeeman energy of the magnetic field is larger than the Fermi energy of the gas phase, it fully polarizes the electrons in the gas phase. The portion of electrons in the gas phase at low temperatures can be estimated to be

$$f_g \sim e^{(g\mu_B S_z H_\parallel - \Delta_c/T)}, \tag{9}$$

when $g\mu_B S_z H_\parallel < \Delta_c$. Using the two-fluid model above, we find, in agreement with experiments [18], the in-plane magnetic field dependence of the resistivity on the metallic side to be

$$\rho \sim \rho_0 + \rho_1 e^{g\mu_B S_z H_\parallel / T} . \tag{10}$$

When the Zeeman energy goes beyond Δ_c, the liquid phase ceases to exist because the critical temperature T_c is zero. The system is a fully polarized gas and it is not affected by further increases of $H_\parallel$. Figure 3 shows this generalized situation of Eq. (10) for a given temperature T. The resistivity increases exponentially with $H_\parallel$ from its zero-field value $\rho_0(T)$ to the saturated value $\rho_m(T)$ at the critical field $H_{\parallel,c}$ given by $g\mu_B S_z H_{\parallel,c} = \Delta_c$, i.e.,

$$\ln \rho(T, H_\parallel) = \begin{cases} \ln \rho_0(T) + \frac{H_\parallel}{H_{\parallel,c}} \ln \frac{\rho_m(T)}{\rho_0(T)}, & \text{if } H_\parallel < H_{\parallel,c} , \\ \ln \rho_m(T), & \text{if } H_\parallel > H_{\parallel,c} . \end{cases} \tag{11}$$

When $T \ll g\mu_B S_z H_{\parallel,c}$, $\rho(T, H_\parallel)$ should be exponential in $H_\parallel/T$ for intermediate values of $H_\parallel$. Using this we have

$$\ln \rho_m(T) \sim \ln \tilde{\rho}_m + \frac{g\mu_B S_z H_{\parallel,c}}{T} . \tag{12}$$

The preliminary experimental data [18] are consistent with this prediction. It should be interesting to test it in more detail.

Next, we look at how the low temperature behaviors of the resistivity on the metallic side evolve for different in-plane magnetic fields. Using Eqs. (11) and (12), we have

$$\ln \rho(T, H_\parallel) = \left(1 - \frac{H_\parallel}{H_{\parallel,c}} \right) \ln \rho_0(T) + \frac{g\mu_B S_z H_\parallel}{T} + \frac{H_\parallel \ln \tilde{\rho}_m}{H_{\parallel,c}} . \tag{13}$$

This equation predicts different behaviors of $\rho(T, H_\parallel)$, shown in the inset of Fig. 3, for $H_\parallel < H_{\parallel,0}$ and $H_\parallel > H_{\parallel,0}$, where $H_{\parallel,0} = \tilde{\rho}_1 H_{\parallel,c}/(\tilde{\rho}_0 + 2\tilde{\rho}_1)$. Using the experimental data of $\tilde{\rho}_1/\tilde{\rho}_0 \simeq 10$ and $H_{\parallel,c} \simeq 20$ kOe in the experiments of Simonian *et al.* [18], we predict $H_{\parallel,0} \simeq 10$ kOe. This is consistent with the data shown in Fig. 4 in Ref. [18]. For $H_\parallel \ll H_{\parallel,0}$, the temperature T_m at which $\rho(T, H_\parallel)$ reaches a minimum is given by

$$T_m \sim \frac{\Delta_c}{\ln(\frac{\Delta_c}{g\mu_B S_z H_\parallel})} . \tag{14}$$

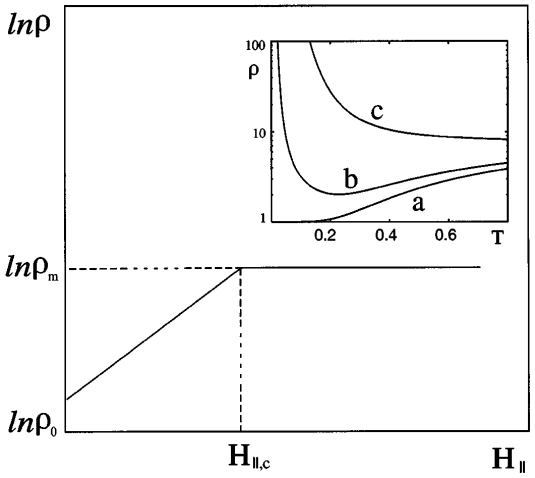

FIG. 3. In-plane magnetic field dependence of the resistivity. The inset shows the temperature dependence of ρ for various in-plane magnetic fields: (a) $H_{\parallel} \ll \tilde{H}_{\parallel,0}$, (b) $H_{\parallel} < \tilde{H}_{\parallel,0}$, (c) $H_{\parallel} > \tilde{H}_{\parallel,0}$. The temperature is measured in units of $g\mu_B S_z H_{\parallel,c}$.

This is consistent with the preliminary experimental data [18]. It should be interesting to test this prediction quantitatively.

Further experiments.—(1) One of the most interesting experiments is to probe directly the inhomogeneity in the electron density. This can be achieved through scanning techniques using a single-electron-transistor as the probe for the electric field in the system. According to our theory, we expect the system to phase separate into regions of different densities. The regions of the sample occupied by the metallic liquid phase should be easily observable in one of the implementations of such scanning techniques [19]. (2) In our theory, the spin and valley indices play similar roles in determining the physics of the system. We expect a strain field, which splits the valley degeneracy of the two-dimensional electron system in Si MOSFETs, to have effects similar to those of an in-plane magnetic field. This should distinguish our theory from those in which the spin degrees of freedom play a nontrivial role in producing the observed experiments. (3) Light scattering experiments in such systems can also provide useful information. Because of the occurrence of macroscopic inhomogeneity in the system below the critical temperature T_c, we should see an increase in the wave-vector-breaking components of the light scattering as the temperature gets below T_c. In the vicinity of T_c, observation of critical opalescence in light scattering should be strong evidence for the existence of the new liquid phase.

In summary, we have argued that there is a new liquid phase in the two-dimensional electron system in Si MOSFETs at low enough average electron densities. The percolation transition of this liquid through the disorder landscape in the classical limit in a disordered system causes the metal-insulator crossover behaviors observed in recent experiments. We have made quantitative predictions and proposed new experiments to further investigate the physical properties of the system.

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