

Anomalous Interaction of Spatial Solitons in Photorefractive Media

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We investigate the interaction of mutually incoherent spatial solitons in photorefractive media with anisotropic nonlocal nonlinear response. We show that the photorefractive nonlinearity leads to an anomalous interaction between solitons. Theoretical and experimental results reveal that an incoherent soliton pair may experience both attractive and repulsive forces, depending on their mutual separation. [S0031-9007(98)05808-6]

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Because of their stability and robustness [1], optical solitons have attracted considerable attention in recent years. While temporal solitons are applicable in data transmission systems [2], their spatial counterparts are ideally suited for all-optical switching and beam manipulation [3–5]. Spatial soliton circuitry is based on the ability to implement logic operations by allowing solitons to interact in a nonlinear medium, as well as the possibility of soliton induced waveguides being used to guide and switch additional optical beams [6].

The nature of the forces between mutually coherent interacting solitons has been discussed in the literature for both temporal [7,8] as well as spatial solitons [9,10]. When solitons are in phase the effective light intensity in the area between the beams increases. This, in turn, results in a local increase of the refractive index which effectively attracts both beams. For out-of-phase solitons the light intensity drops in the interaction region and so does the refractive index. This results in the beams moving away from each other, which indicates a repulsive force. Phase-sensitive interaction of spatial solitons has been demonstrated experimentally in liquids [11], glass waveguides [12], atomic vapors [13], and photorefractive crystals [14,15].

Utilization of coherent interaction effects for spatial switching is, however, troublesome. Phase control of a large number of beams in a switching fabric may be difficult to achieve. Furthermore, efficient interaction requires that the relative phase between solitons is maintained under propagation. This condition is actually difficult to satisfy experimentally. This is because soliton phase varies during propagation at a rate determined by the so-called propagation constant. Any difference in propagation constants, or distance traveled by two solitons, results in a variation of their relative phase and, subsequently, periodic changes of the mutual force.

To overcome these limitations, one may consider the use of mutually incoherent solitons, whose interaction is independent of their relative phase. Incoherent solitons provide, however, a restricted range of spatial interactions

since in typical isotropic self-focusing media they always attract each other. This is because for incoherently interacting beams the total light intensity always increases in the region where they overlap. Recent works have studied incoherent soliton interactions in photorefractive media under conditions where the physics is quite similar to that pertaining to saturable Kerr-type media [16].

In this Letter we show that the restriction to attractive interactions is not fundamental, and that it is possible to achieve both attractive *and* repulsive forces between mutually incoherent solitons. This opens the possibility of accessing a significantly wider range of soliton logic operations with incoherent beams. This anomalous situation occurs in photorefractive media, where the particular anisotropic and nonlocal structure of the nonlinearity results in both attraction and repulsion of parallel beams depending on their relative spatial separation.

The existence of anomalous repulsive forces between mutually incoherent solitons may be understood by considering the structure of the nonlinear increment to the refractive index. In the present situation the optical field consists of two beams $B(\vec{r}, t) = [B_1(\vec{r}, t) + B_2(\vec{r}, t) \times \exp(-i\Omega t)] \exp[i(kz - \omega t)]$, separated by a frequency shift Ω such that $\Omega\tau \gg 1$, where τ is the characteristic response time of the nonlinearity. When the characteristic spatial scales are larger than the photorefractive Debye length, and the diffusion field may be neglected, steady state propagation along the z axis of a photorefractive crystal with an externally applied electric field along the x axis is described by an evident generalization of the model of Zozulya and Anderson [17]

$$\left[\frac{\partial}{\partial z} - \frac{i}{2} \nabla^2 \right] B_{1,2}(\vec{r}) = i \frac{\partial \varphi}{\partial x} B_{1,2}(\vec{r}), \quad (1a)$$

$$\begin{aligned} \nabla^2 \varphi + \nabla \varphi \cdot \nabla \ln(1 + |B_1|^2 + |B_2|^2) \\ = \frac{\partial}{\partial x} \ln(1 + |B_1|^2 + |B_2|^2), \end{aligned} \quad (1b)$$

where $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$ and φ is the dimensionless electrostatic potential induced by the light with the boundary conditions $\nabla\varphi(\vec{r} \rightarrow \infty) \rightarrow 0$ [18]. Numerical solutions of Eq. (1b) for $B_{1,2} = \sqrt{2.3} \times \exp(-[(x \pm d/2)^2 + y^2]/4)$ for several values of the separation d along the direction of the applied field (x axis) are shown in Fig. 1, where the nonlinear refractive index $\delta n \sim \partial\varphi/\partial x$ has been visualized. For small separation [Fig. 1(a)] the structure of the nonlinear refractive index is similar to that due to a single beam [17,19]. The peak centered at $x = y = 0$ leads to self-focusing and attraction of the beams. The essentially anisotropic nature of the nonlinearity is seen in the tails of the refractive index distribution. Along the y axis, where the refractive index decays monotonically to zero, the structure is analogous to that of a Kerr-type nonlinearity ($\delta n \sim I$). Therefore, one may expect that beams separated along the y axis will attract each other, as in Kerr-type media.

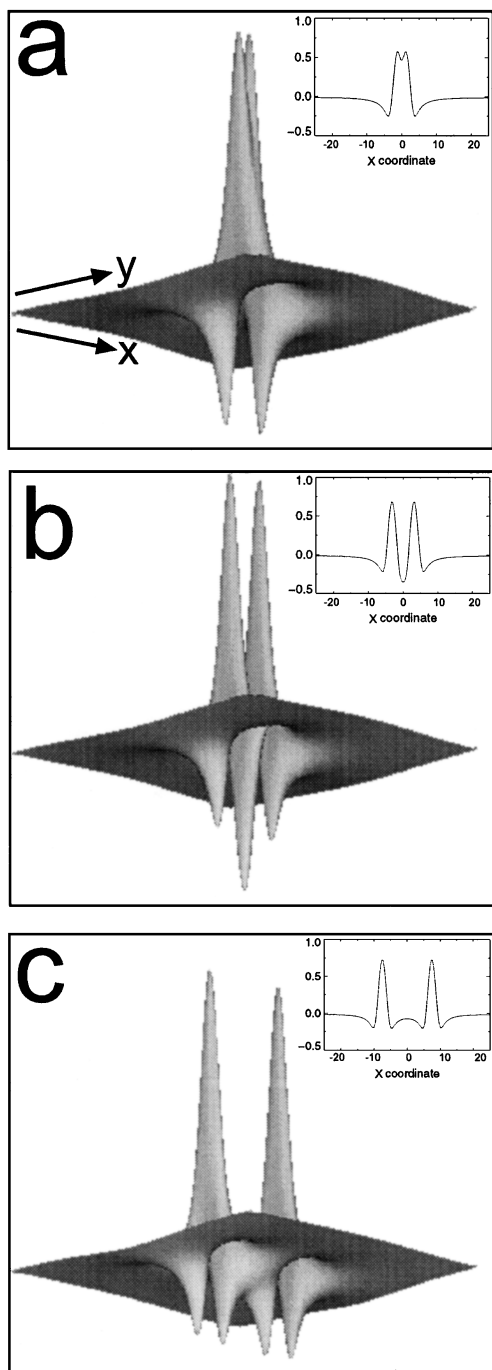


FIG. 1. Nonlinear refractive index for beam separation (along the x axis) d equal to 3 (a), 7 (b), and 15 (c). The computational window was a square with widths $l_x = l_y = 50$. The inset plots show the variation of the refractive index in units of 10^{-4} along the x coordinate for $y = 0$.

Along the x axis, however, the central peak is surrounded by regions of both positive and negative δn . Sufficiently far from the center of the beam the refractive index actually increases with decreasing light intensity, which corresponds to a self-defocusing behavior. When the separation is increased to $d = 7$ [Fig. 1(b)] the negative index regions induced by each beam overlap, resulting in an even larger dip in the index between the beams. It is exactly this circumstance that results in the observed repulsion of mutually incoherent beams. For larger values of the separation [Fig. 1(c)] the regions of negative refractive index are less deep, and the beams repel each other weakly. Analogous calculations show that for beams separated along the y axis the refractive index always increases in the region between them leading to their attraction.

The propagation dynamics were investigated by numerical integration of Eqs. (1) for initial separations corresponding to those shown in Fig. 1. The closely spaced input beams [Fig. 2(a), top row] attract each other strongly, and eventually coalesce into a single beam after propagating over a distance $l_z = 38$ [Fig. 2(a), bottom row]. This behavior is generic for solitons in saturable nonlinear media colliding at a very small angle [20–22]. Essentially different behavior can be seen in Fig. 2(b), where the initial separation is $d_x = 5.3$. The separation between the solitons increases with the propagation distance indicating mutual repulsion. Note also that each beam attains an elliptical shape with the diameter ratio ~ 1.5 , characteristic for photorefractive solitons [18]. For the same initial separation, but oriented along the y axis [Fig. 2(c)], the beams attract each other, and coalesce partially. Additional calculations over longer propagation distances show full fusion of the beams. Finally, for initial separations along both x and y , more complex dynamics, including the initial stages of a counterclockwise spiraling motion about the center of the two beams [23], is observed, as shown in Fig. 2(d).

The interaction of a pair of mutually incoherent spatial solitons was investigated experimentally using a photorefractive strontium barium niobate crystal measuring 6 mm along each side, and doped with Ce (0.002% by weight). The experimental arrangement was analogous to that used recently in a study of soliton collisions [15]. Two circular beams ($2 \mu\text{W}$ each) derived from an argon ion laser ($\lambda = 514.5 \text{ nm}$) were focused with Gaussian diameters of $15 \mu\text{m}$ on the entrance face of the crystal, and were polarized along x to make use of the r_{33} electro-optic coefficient, which had a measured value of

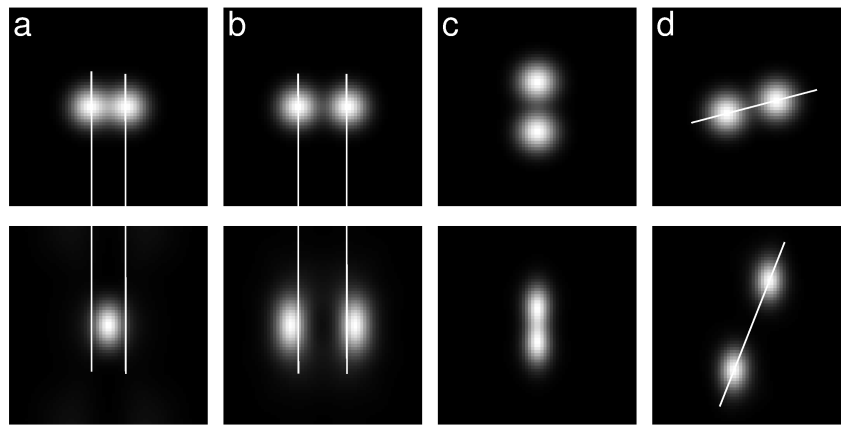


FIG. 2. Numerical calculations of propagation over a distance $l_z = 38$ for initial separations of $d_x = 3.8$ (a), $d_x = 5.3$ (b), and $d_y = 5.3$ (c). In (d), $d_x = 5$ and $d_y = 1.5$. Other computational parameters: $l_x = l_y = 20$, $l_z = 38$, peak intensity = 2, and Gaussian input radius = 2.5. The top (bottom) rows show input (output) beams.

180 pM/V. A voltage of 2 kV was applied along the x axis (= crystalline c axis). One of the beams was phase modulated by reflection from a mirror mounted on a piezoelectric transducer driven by an ac voltage at several kHz, such that $\Omega\tau \gg 1$. A white light source was used to control the value of the saturation intensity such that $|B_i|^2 \sim 2$ [18]. In order to ensure that the trajectories of the solitons did not intersect, they were launched with a slight divergence angle between them. The input and output light intensity distributions were recorded with a CCD camera.

Experimental results are presented in Fig. 3, where we show input and output light intensity distributions for various initial separation of the beams. The first row shows images of both beams at the input face of the crystal. The second row shows the output intensity distribution for noninteracting solitons. These images were obtained by allowing each beam to propagate separately in the crystal and superimposing the resulting images. Finally, the third row contains images of *interacting* solitons.

The first column in Fig. 3 depicts results for closely spaced solitons. In that case, for separation $\approx 13 \mu\text{m}$, the interaction is strongly attractive and the beams fuse emerging from the crystal as a single elliptically shaped solitary beam. This behavior is essentially the same as that found in a saturable Kerr-type nonlinearity. The situation changes dramatically when the initial separation is increased to $27 \mu\text{m}$ (second column in Fig. 3). This time both input beams evolve into separate solitary waves while their separation increases, indicating mutual repulsion. Evidence of the repulsive nature of the interaction was also seen clearly by blocking one of the input beams. This resulted in the other beam moving towards the now blocked beam. We wish to stress that the interaction of incoherent solitons separated only along y always resulted in their attraction.

A direct check between the experimental observations and numerics, given the experimental parameters, and taking account of the small initial angle between the beams,

was made. Calculations showed fusion of the output beams for the small initial separation in Fig. 3 (left column), and an output separation of $40 \mu\text{m}$ for the data in the right-hand column, which agrees well with the measured value of $43 \mu\text{m}$.

As shown in Fig. 2(d) more complex behavior featuring mutual rotation occurs when the input beams are separated along both x and y . This was observed experimentally as shown in Fig. 4 [24]. The first image [Fig. 4(a)] shows the intensity distribution of the input beams. They are separated along an axis tilted at $\approx 13^\circ$ with respect to the x axis. Frames 4(b) and 4(c) contain output intensity distributions for noninteracting (independent propagation) and interacting solitons, respectively. As is clear from the figure, mutual interaction results not only in increased separation between solitons but also in a counterclockwise spiral motion about the center of the beams, in exactly the same manner as found numerically in Fig. 2(d). It should be emphasized that the spiraling motion is due to the anisotropy of the potential created by the two beams,

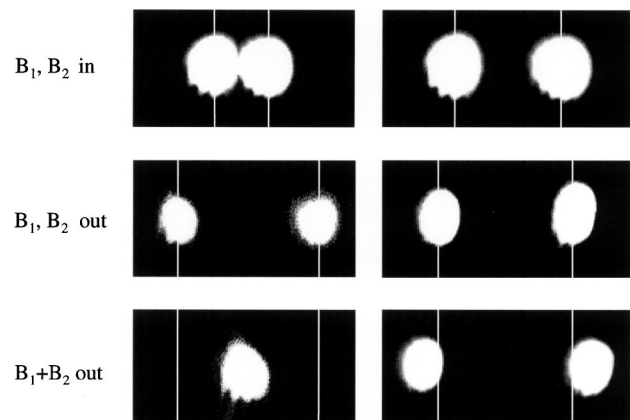


FIG. 3. Experimental observation of separation-dependent interaction of incoherent solitons. The x axis is horizontal. The pictures have been corrected to remove the displacement along x due to self-bending of the solitons [17].

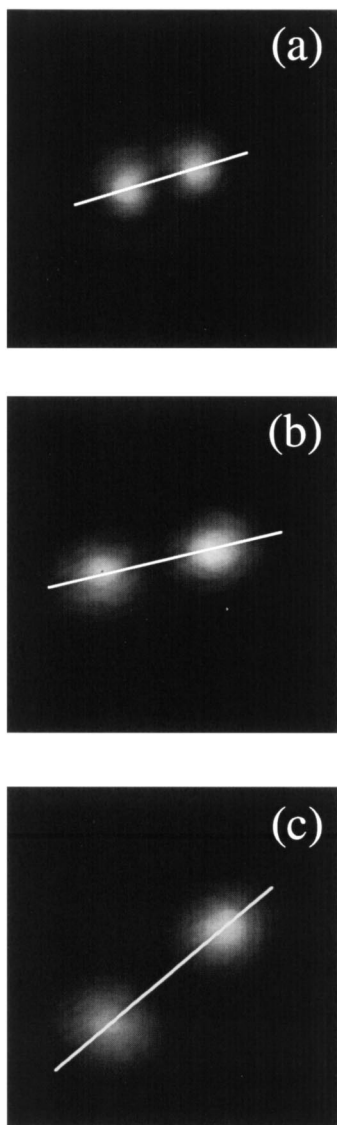


FIG. 4. Experimental observation of mutual repulsion and rotation. Again, the x axis is horizontal. [See text for in-depth explanation of (a)–(c).]

and occurs even though they are launched without any tangential velocity.

In conclusion, we have shown that incoherent spatial solitons in photorefractive media exhibit anomalous interaction properties not seen in isotropic nonlinear media. In particular, while closely overlapping, incoherent solitons always attract each other; larger separations may result in repulsive forces, depending on the separation and its orientation. This effect is a result of the nonlocal character of the anisotropic self-focusing in photorefractive media which leads to an effective self-defocusing in parts of the outer regions of the optical beam. Close agreement between experiments and the predictions of a three-dimensional model were obtained.

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