Measurement of the Rotational Frequency Shift Imparted to a Rotating Light Beam Possessing Orbital Angular Momentum

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We observe the frequency shift, $l\Omega$, imparted to a mm-wave beam with an orbital angular momentum of $l\hbar$ per photon, when the beam is rotated at angular frequency Ω. We show that this shift, and those found in a number of experiments on the rotation of circularly polarized beams, are special cases of the rotational frequency shift recently predicted by Bialynicki-Birula and Bialynicka-Birula. The measurement also explicitly confirms a theoretical prediction by Nienhuis. [S0031-9007(98)05836-0]

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The nonrelativistic translational Doppler effect is a well known phenomenon. The relative velocity between a source and a detector leads to a frequency shift proportional to the product of the velocity and the unshifted frequency.

The angular Doppler shift associated with circularly polarized light and a rotating wave plate was described nearly 20 years ago by Garetz and Arnold [1]. Recently Bialynicki-Birula and Bialynicka-Birula [2] predicted a rotational frequency shift due to the uniform rotation of an atomic system. They stress that this shift should not be confused with the Doppler shift observed for rotating objects due to the rotation having a linear velocity with respect to the observer. Unlike the linear Doppler shift which is maximal in the plane of rotation, their effect is maximal in the direction of the angular velocity vector where the linear Doppler shift is zero.

Bialynicki-Birula and Bialynicka-Birula perform a detailed specific quantum-mechanical calculation for the shift of the atomic resonances when an atom is rotated. The effect occurs for atomic systems which lack rotational symmetry, but which still have stationary states in a rotating frame. The frequency shift is shown to be equal to the scalar product of the angular frequency Ω and the angular momentum of the emitted photon in units of $\hbar$. They emphasize that this rotational frequency shift is in addition to any shifts between the energy levels which may arise from the centrifugal and Coriolis forces associated with the rotation. Consequently, for the rotational frequency shift to be observed, care must be taken in the design of an appropriate experiment. They suggest that the rotational frequency shift might be observed in a Mössbauer-like regime, where the angular recoil is negligible.

In this paper we report the observation of a frequency shift for a light beam possessing orbital angular momentum. This orbital angular momentum is distinct from that associated with intrinsic spin and arises from the $e^{il\phi}$ phase structure within the beam, Fig. 1. This leads to an orbital angular momentum of $l\hbar$ per photon [3]. Nienhuis [4] predicted a frequency shift for a light beam possessing orbital angular momentum when transmitted through a rotating mode converter consisting of a system of cylindrical lenses. We show that this is the shift we measure and that it is a special case of the rotational frequency shift. We also suggest that the shifts observed in the work of Garetz [1,5] and others [6,7] associated with intrinsic spin are further examples.

In our work the potential frequency shifts associated with the centrifugal and Coriolis forces are eliminated by keeping the light source stationary. The rotation is applied to the light beam only, by means of a rotating Dove prism [8]. To rotate the beam without any associated translation or angular deviation, the axis of the Dove prism must be aligned to within a fraction of the wavelength of the light. At optical frequencies this requirement is extremely challenging. Therefore, we have performed the experiment at mm-wave frequencies where the longer wavelength eases the mechanical tolerance needed both for the rotation of the prism and for overall system alignment. In the mm-wave region of the spectrum, components such as lenses and prisms can all be fabricated from polyethylene, which has a refractive index at this frequency of 1.52 [9]. Consequently, it is possible to design and construct an *optical* experiment where the wavelength of the *light* is several mm.

The light source used in this work is an Indium Phosphide Gunn Diode, configured as an oscillator at 94 GHz [10]. An impedance-matched feed horn launches

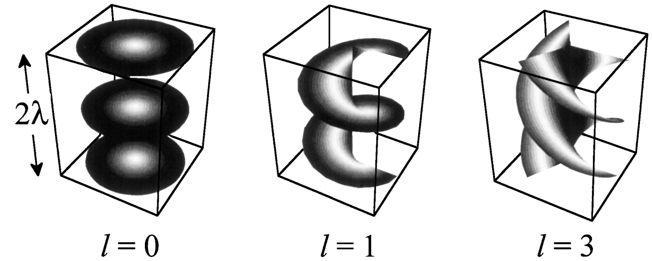

FIG. 1. Helical wave fronts resulting from an azimuthal phase structure of the form $e^{il\phi}$.

a monochromatic Gaussian beam into free space. Orbital angular momentum is introduced into the beam by passing it through a spiral phase plate [11], the optical thickness of which varies linearly with azimuthal angle ϕ. The transmitted beam has the required $e^{il\phi}$ phase structure and associated orbital angular momentum of $l\hbar$ per photon, which is uniform at all points in the beam [3,12].

We measure the rotationally induced frequency shift by dividing the beam in two and recording the interference between that transmitted through the rotating Dove prism and that transmitted through a nonrotating Dove prism. The introduction of the second Dove prism ensures that the helical wave front of each beam has the same sense. Consequently, interference between the beams is uniform across the whole aperture. This allows the two beams to be focused onto a single detector, and the interference recorded as one Dove prism is rotated. Although the phase structure of the beam is rotated by the Dove prism, its polarization state remains essentially unchanged. A simple Jones matrix analysis shows that if the prism is rotated between parallel polarizers the transmitted intensity does not vary by more than 15%. The system is configured as a Mach-Zehnder interferometer with polarizing beam splitters, Fig. 2. This ensures that the light remains linearly polarized and that intrinsic spin plays no role in our observation.

Figure 3 shows the interference between two beams with the rotation of the Dove prism. In each case the observed interference implies that the frequency shift is given by $l\Omega$, where Ω is the angular rotation frequency of the beam. The slight amplitude modulation in the observed difference frequency, particularly evident for $l = 0$, arises from the small change in the relative intensities of the two beams as the Dove prism is rotated.

For a light beam with an $e^{il\phi}$ phase structure with l intertwined helical wave fronts, the physical origin of the frequency shift is readily understood. For a beam propagating in free space the number of wave fronts that pass through a plane in unit time is equal to the optical frequency. If the beam has l intertwined helical wave fronts, then each rotation of the beam will change the number of wave fronts passing through this plane by l. Hence, the frequency shift is simply equal to $l\Omega$.

Garetz and Arnold [1] predicted that a rotating half-wave plate inserted into a beam of circularly polarized light results in a frequency shift which they called the angular Doppler effect. This has subsequently been confirmed experimentally by a number of groups [6,7]. The wave plate reverses the circular polarization of the beam. Although sometimes analyzed in terms of an energy exchange [7], the rotation of the wave plate may also be considered as inducing a rotation of the polarization of the beam. We should note that the fast axis of the wave plate breaks the rotational symmetry of the system.

Nienhuis [4] predicted that a rotating mode converter, which changes the sense of the orbital angular momentum of the beam, would shift the frequency of the light. His

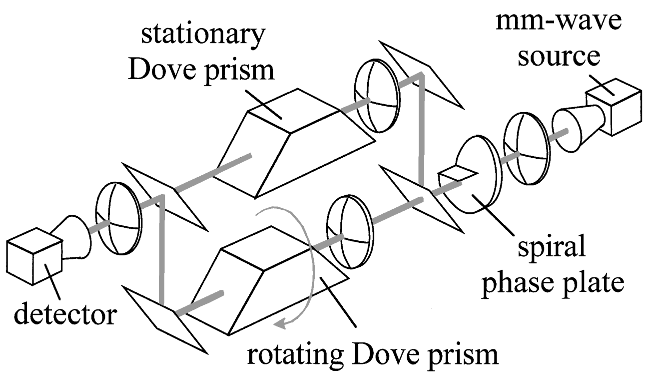

FIG. 2. Interferometer with a rotating Dove prism for the observation of the rotational frequency shift.

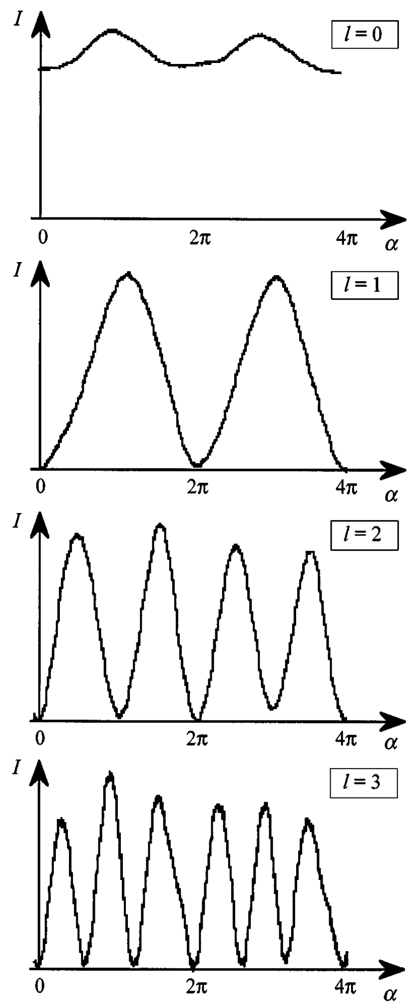

FIG. 3. Interference between two beams with an azimuthal phase structure of the form $e^{il\phi}$ after one of them has been transmitted through the rotating Dove prism; α denotes the beam rotation angle; the curves have been plotted over one rotation of the Dove prism, which corresponds to two rotations of the beam.

lens configuration may be shown to be an image rotator exactly equivalent to our Dove prism. Our experiment is thus a direct observation of his prediction, with the experimental merit that the Dove prism is much easier to align than the lens system actually proposed.

When rotated, the asymmetry of the cylindrical lenses in the experiment proposed by Nienhuis, the inversion axis of the Dove prism, and the fast axis of the wave plate analyzed in the work of Garetz and Arnold are all responsible for the rotation of the transmitted beam. The rotation of an atomic system with an asymmetric potential, as discussed by Bialynicki-Birula and Bialynicka-Birula, produces a rotation of the emitted beam. Consequently, these observations are all examples of the rotational frequency shift.

Garetz clearly had a deep understanding and insight into experiments of this kind involving intrinsic spin. He showed [5] that circularly polarized light phenomena as diverse as fluorescence doublets, interactions with rotating half-wave plates and rotation-induced optical activity were all examples of the same effect. More significantly he demonstrated classically that spectra due to rotating nuclei arise from just such a shift, as does rotational Raman scattering.

It is, perhaps, also possible to make a connection between the rotational frequency shift and various other atomic effects where rotation or effective rotation play a role. For example, Millman [13] observed a shift in resonance frequency in a molecular beam experiment due to a rotating magnetic field, while frequency shifts also occurred in the NMR rotating-sample nuclear quadrupole resonance experiment of Tycko [14]. Certainly, in each case there is a term in the Hamiltonian of the form $S_z\Omega$, but their experiments, unlike ours, do not give a shift consisting unambiguously of the product of an integer angular momentum and the angular frequency due to rotation. These experiments also depend entirely upon intrinsic spin and not orbital angular momentum. The azimuthal Doppler shift [15] experienced by an atom or ion moving in a field possessing orbital angular momentum has also been shown theoretically [16] to be a specific example of the rotational frequency shift.

In conclusion, we report the first observation of the rotational frequency shift for light possessing orbital angular momentum. The rotation of the beam was accomplished using a rotating Dove prism. The observation extends the work of Garetz [5] to include orbital angular momentum and constitutes the experimental verification of both the work of Nienhuis [4] and of Bialynicki-Birula and Bialynicka-Birula [2].

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