A Baryon Model in Covariant Constraint Dynamics

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An important ingredient of parton or string cascade models for ultrarelativistic heavy-ion reactions is a parton description of the baryon. We have developed a new model using Dirac's approach of Poincaré-covariant many-body dynamics with constraints. The baryon is described as a dynamical set of three valence quarks and a fourth particle, the "junction," which carries the momentum fraction of the sea quarks as well as all of the glue. The model's parameters are the quark current masses, and one interaction strength, determined by the proton radius. We obtain a remarkably good fit to the valence quark structure functions of the baryon. [S0031-9007(98)05696-8]

Among the methods employed in describing ultrarelativistic heavy ion reactions theoretically, various cascade models [1,2], (cf. also [3]), play an important role. These models follow the space-time development of the reaction dynamics in phase space; they are therefore essentially classical models. They simulate the nuclear reaction by a series of elementary propagations, fusions, and decays of partons or strings. When successful, cascade models not only reproduce the experimental data as asymptotic states of the reaction but give some information on the dynamical, nonequilibrium development of the intermediate states, including a phase transition to a quark-gluon plasma—if such a phase transition should indeed occur.

The basic objects in [1] are strings; (confined) baryons or mesons are described by such strings sweeping $(1 + 1)$-dimensional periodic hypersurfaces in space-time ("yo-yo motions"). A baryon is thus modeled as a system of a quark and a diquark. Parton distributions are not included in this ansatz.

In this Letter we will present a new Poincaré-covariant dynamical model for a baryon, leading to yo-yo-like motion of its constituents. Three valence quarks and a "junction" (representing sea quarks and all gluonic degrees of freedom) interact as classical point particles via a quasipotential. This model, which is free of adjustable parameters, approximately reproduces the valence quark structure functions. The Q^2 dependency of the structure functions will be discussed at the end of this Letter.

Constraint Dynamics.—In constructing a Poincaré-covariant system of N interacting point particles, one has to face the consequences of the no-interaction theorem [4]. This theorem asserts that the only canonical Hamiltonian theory that is fully covariant under Poincaré transformations is that of a system of noninteracting particles. Dirac's method of introducing dynamical constraints [5] circumvents this problem.

In this paper we use a set of $(2N)$ second-class constraints, written in the scheme of [6] (cf. also [7]). The phase space is expanded to $8N$ dimensions:

$$\Gamma := \{q_{(i)}^{\mu}, p_{(i)}^{\mu} | i = 1 \ldots N, \mu = 0 \ldots 3\}.$$

An evolution parameter s—without any direct physical meaning—is introduced for the parametrization of the phase space trajectories. A set of $(2N - 1)$ Poincaré-covariant constraints $\{\Phi_r | r = 1 \ldots (2N - 1)\}$ and one connection Φ_{2N} between the evolution parameter s and the phase space variables are introduced, which reduce the degrees of freedom of the system to a physical $6N$-dimensional hypersurface Γ'. We use the standard notation of writing $A \approx B$ to denote that an equation holds only on Γ'; we also employ a summation convention over equal indices wherever applicable.

The equations of motion are generated by a Poincaré-invariant "Hamiltonian" $H := \lambda^j \Phi_j$ via the Poisson brackets,

$$\frac{dq_{(i)}}{ds} \approx \{q_{(i)}, H\} \approx \{q_{(i)}, \Phi_j\}\lambda^j, \tag{1a}$$

$$\frac{dp_{(i)}}{ds} \approx \{p_{(i)}, H\} \approx \{p_{(i)}, \Phi_j\}\lambda^j. \tag{1b}$$

The Lagrange parameters λ^j are determined by the requirement that H preserve the constraints: $\frac{d\Phi_i}{ds} \approx \frac{\partial \Phi_i}{\partial s} + \{\Phi_i, \Phi_j\}\lambda^j \approx 0$. Denoting the matrix of the Poisson brackets $\{\Phi_i, \Phi_j\}$ by $\mathcal{P}_{i,j}$, the λ^j are given by $\lambda^j \approx -(\mathcal{P}^{-1})^{j,k} \frac{\partial \Phi_k}{\partial s}$; since of all the constraints only Φ_{2N} depends explicitly on s, we obtain

$$\lambda^j \approx -(\mathcal{P}^{-1})^{j,2N} \frac{\partial \Phi_{2N}}{\partial s}. \tag{2}$$

(It is straightforward to see that $\lambda^{2N} \approx 0$; thus H is independent of s.) Using Eqs. (1) and (2), one obtains the equations of motion for physically interpretable variables on the hypersurface Γ'.

With constraint dynamics it is possible to construct a dynamical system, consisting of two massive particles $m_{(1,2)}$, which reproduces the essential features of the yo-yo solution of the Lund model (cf. also [8]). Specifically, the effective masses of the two particles increase due to a quadratic quasipotential between them:

$$(m_{(1,2)\text{eff}})^2 := (p_{(1,2)})^2 = (m_{(1,2)})^2 - V$$
$$= (m_{(1,2)})^2 - \kappa^2(q_{(1)} - q_{(2)})^2.$$

[Note that, in this Poincaré-covariant formulation, the quasipotential has a dimension of (mass)^2, and thus the *energy* (in the rest frame of each particle) increases linearly with distance, as is the case in the Lund model.]

The dynamical system is defined by the constraints

$$\Phi_{1,2} := (p_{(1,2)})^2 - (m_{(1,2)})^2 + \kappa^2(q_{(1)} - q_{(2)})^2, \quad (3a)$$

$$\Phi_3 := (q_{(1)} - q_{(2)})(p_{(1)} + p_{(2)}), \quad (3b)$$

$$\Phi_4 := \frac{p_{(1)} + p_{(2)}}{|p_{(1)} + p_{(2)}|} q_{(1)} - s. \quad (3c)$$

The physical meaning of these constraints is the following: $\Phi_{1,2}$ guarantee that the particles are on mass shell wherever the quasipotential vanishes. Their effective masses increase with increasing distance. Φ_3 ensures that in the center-of-momentum system (CMS) the particle distance is spacelike, and Φ_4 connects the evolution parameter s with the particle times in the CMS.

The solutions of the equations of motion for this system are trigonometric functions (Fig. 1). In the CMS and with vanishing particle masses, the period of this yo-yo motion is $T := \pi M/(2\kappa)$, where $M^2 := (p_{(1)} + p_{(2)})^2$. Setting $\kappa = \pi/4\kappa_{\text{Lund}}$, one obtains the same period as in the Lund model. As a consequence, the maximum time for one fragmentation [1] would be the same in both models.

This fully Poincaré-covariant model can thus be thought of as a naive simulation of a meson of total mass M, composed of two quarks with current masses $m_{(1,2)}$. As in the Lund model, the quarks in the meson are confined, and this increases their effective masses. In contrast, the yo-yo solution of the Lund model is only boost invariant in one direction. Current quark masses are not included in the Lund model.

This simple model does not, of course, purport to give a fully adequate description of a meson—no account is given, e.g., for gluonic degrees of freedom. It is presented here merely as an example of the principal method employed.

Model of the baryon.—In the spirit of the above, a naive model for a baryon would be a system of three valence quarks, each carrying one-third of the baryon momentum. This, however, is too gross a simplification, since from deep inelastic scattering we know that about one-half of the momentum is carried by sea quarks and gluonic degrees of freedom. In the model to be described now, we will introduce a fourth (classical) particle, the junction [9,10], which binds the valence quarks, and thus models these degrees of freedom (cf. Fig. 2).

In keeping with the discussion in the previous section, we will again use a quadratic quasipotential depending on the distances between the junction and the valence quarks (again, constraints will ensure that these distances are spacelike in the CMS). This quasipotential should affect the effective mass $(p_{(J)})^2$ of the junction, because pushing away a valence quark will increase the number of sea quarks and gluons, which are modeled by the junction. For
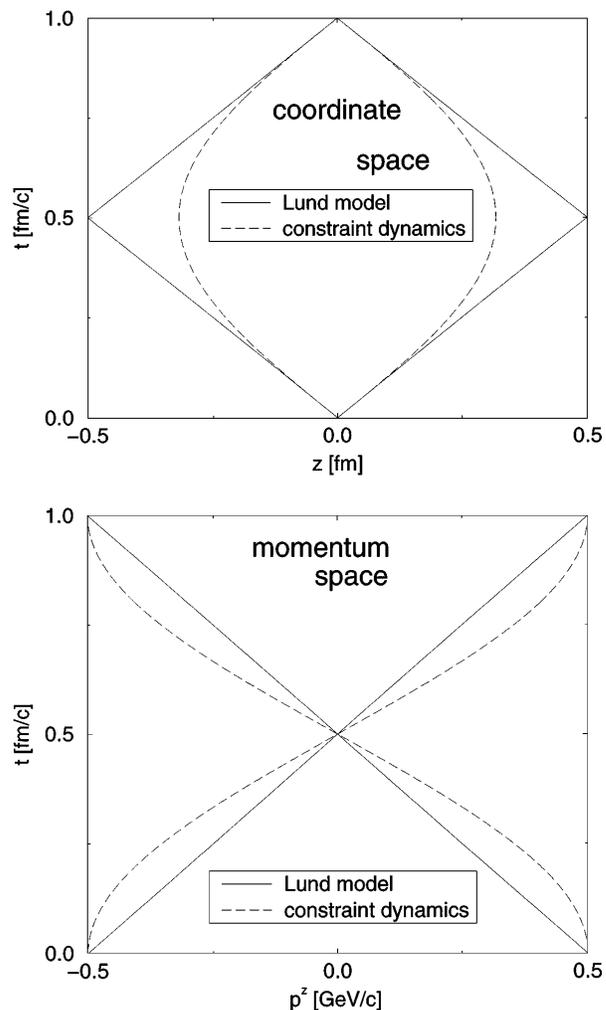

FIG. 1. Solution of the equations of motion in the CMS of the constraint dynamics model [Eqs. (3)] compared to the Lund solution, in *coordinate* and in *momentum* space ($m_{(1,2)} = 0$).

simplicity, we will assume the valence quarks to always remain on mass shell.

The above physical requirements are embodied in the following constraints:

$$\Phi_{1,2,3} := (p_{(1,2,3)})^2 - (m_{(1,2,3)})^2, \quad (4a)$$

$$\Phi_J := (p_{(J)})^2 + \kappa^2 \sum_{i=1}^{3} (q_{(i)} - q_{(J)})^2, \quad (4b)$$

$$\Phi_{5,6,7} := (q_{(1,2,3)} - q_{(J)})P, \quad (4c)$$

$$\Phi_8 := \frac{P}{|P|} q_{(J)} - s, \quad (4d)$$

where $P := p_{(J)} + \sum_{i=1}^{3} p_{(i)}$ is the total four-momentum of the system.

The equations of motion for this system have no simple solution in closed form; but they are, of course, easily solved numerically. For the simulation of a proton, we use $M_p = |P| = 0.938$ GeV (proton mass), $\kappa = 0.5$ GeV/fm (leading to a proton radius of ≈ 0.8 fm), and
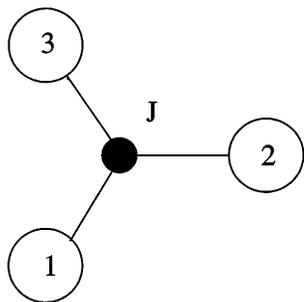
FIG. 2. Model for the baryon: A junction (J) is binding the valence quarks (1,2,3) by a confinement potential. The junction carries the average momentum of sea quarks and gluonic degrees of freedom.
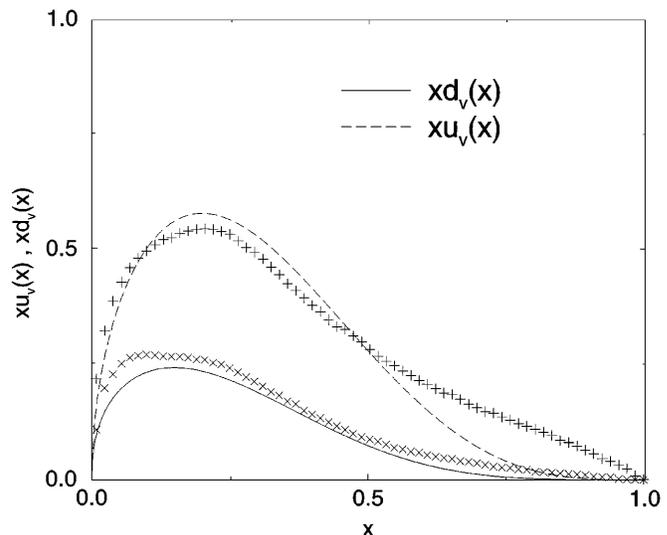

FIG. 3. Sampled distributions of the valence quark longitudinal momentum fraction $xu_v(x)$ (up quark) and $xd_v(x)$ (down quark). The solid and dashed lines are the results obtained with the parametrization [12].

estimated values for the current masses of the quarks [11]: $m_{(1)} = m_{(2)} = m_{(u)} = 5$ MeV, $m_{(3)} = m_{(d)} = 10$ MeV.

Q^2-*independent structure functions.*—For fixed values of the variable $x := Q^2/2M\nu$, the substructure of nucleons is rather independent of the four-momentum transfer (Q^2). (M denotes the nucleon mass and ν the transferred energy.) In the parton model this scaling behavior is explained in terms of the presence of pointlike charged constituents, called "partons." Using the parton hypothesis, x can be interpreted as the fractional momentum of a parton in the infinite momentum frame. The distribution functions of this variable can be obtained from the data of deep inelastic e-p collision experiments. For comparison with our results, we shall use the parametrization of the valence quark distribution functions $u_v(x), d_v(x)$ given in [12].

To obtain valence quark structure functions in our model described by Eqs. (4), we first need the variable x in the context of our model. We define the longitudinal momentum fraction of particle i in terms of the ratio of the light-cone variables: $x_{(i)} := \frac{p_{(i)}^+}{P^+} := \frac{p_{(i)}^0 + p_{(i)}^z}{P^0 + P^z}$. Note that this definition is flavor dependent because we are using flavor-dependent quark masses (cf. end of the previous section).

We have then sampled distributions of these momentum fractions by randomly choosing different initial conditions for the dynamical system of Eqs. (4); during the evolution of the system with one set of initial conditions, values of $x_{(i)}$ are sampled and averaged over. The results of these calculations are given in Fig. 3 and compared with the parametrization of [12].

The rather good overall agreement of our result with the data seems rather remarkable, given the fact that ours is a purely classical particle model, and no free parameters were fitted in obtaining these results.

The tails of these distributions are definitely overestimated by our model. However, one should note that for $x \approx 0.8$ the number of events in the sample is vastly less than for $x \approx 0.1$ (by about a factor of 10^{-6}), so any error in the tail of the distribution hardly contributes to the momentum distribution of the proton.

A fragmentation mechanism using the baryon model described here has been presented elsewhere [13].

Q^2-*dependent structure functions.*—Experiments at HERA with high four-momentum transfer show that the total momentum carried by the valence quarks decreases with increasing Q^2, and that the number of gluons and sea quarks increases [14]. In terms of our model, this would mean that the effective mass $(p_{(J)})^2$ of the junction should increase with Q^2, as it is the junction which models these degrees of freedom. However, since by construction the momentum transfer Q^2 does not enter the dynamics of our model baryon at all, the only way to include this effect is by introducing an additional *phenomenological* mass term in the equations of motion for the junction. We thus replace the constraint Φ_J in Eqs. (4) by

$$\Phi_J := (p_{(J)})^2 - [m_{(J)}(Q^2)]^2 + \kappa^2 \sum_{i=1}^{3} (q_{(i)} - q_{(J)})^2,$$

(5)

with a phenomenological Q^2-dependent term $m_{(J)}(Q^2)$. Instead of simply fitting $m_{(J)}(Q^2)$ to the data, we proceed as follows: The measured parton distribution functions for valence as well as for sea quarks and gluons, $f_k(x, Q^2)$, are parametrized in [14]. From these we calculate *integrated* momentum fractions $\bar{x}_k(Q^2) := \int_0^1 x f_k(x, Q^2) \, dx$. $\bar{x}_{\text{glue}}$ and $\bar{x}_{\text{sea}}$ are a measure of the amount of the corresponding "mean field" carried by the J particle in our model and which we want to represent by $m_{(J)}$. However, since a certain part of this mean field is generated dynamically by the system in the form of the effective mass squared of the junction $(p_{(J)})^2$, we have to subtract this part to avoid double counting. We are thus led to the ansatz,

$$m_{(J)} := \left[\bar{x}_{\text{glue}} + \bar{x}_{\text{sea}} - \frac{\alpha}{1-\alpha} (\bar{x}_{d_v} + \bar{x}_{u_v}) \right] M_p,$$
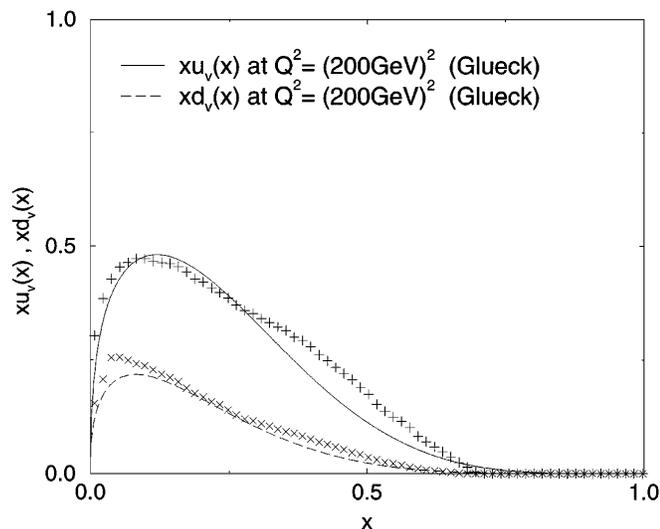
FIG. 4. Sampled distributions of the valence quark longitudinal momentum fraction $xu_v(x)$ (up quark) and $xd_v(x)$ (down quark), at $Q^2 = (200 \text{ GeV})^2$. The solid and dashed lines are the results obtained with the parametrization [14].

FIG. 5. Sampled integrated momentum fractions $\bar{x}_u, \bar{x}_d$ of the valence up quarks ($\square$) and down quarks ($\diamondsuit$), as a function of Q^2. The solid and dashed lines are the results obtained with the parametrization [14].

where the proton mass M_p is needed for dimensional reasons. The factor α, finally, is determined by the requirement that $m_{(J)}(Q^2)$ vanish at the point where we connect with the Q^2-independent calculation: $Q^2 = 2 \text{ GeV}^2$. Thus we take $m_{(J)}(Q^2)$ to be given by

$$m_{(J)}(Q^2) := \left[\bar{x}_{\text{glue}}(Q^2) + \bar{x}_{\text{sea}}(Q^2) \right.$$
$$\left. - \frac{0.55}{0.45} [\bar{x}_{d_v}(Q^2) + \bar{x}_{u_v}(Q^2)] \right] M_p . \quad (6)$$

With this Q^2-dependent term added, we again solve the equations of motion of the system and sample the distributions of the valence quark structure functions in the manner described above. The results are presented in Figs. 4 and 5, in comparison with [14].

In conclusion, we have presented a truly Poincaré-covariant model for a baryon [15] in terms of a completely classical particle description which, in spite of its simplicity, seems to describe some of the features of the baryon surprisingly well. We believe this model (possibly with further refinements) to be useful in constructing models for ultrarelativistic heavy-ion reactions of the parton cascade type.

In such a cascade model, scattering of baryons (or, rather, their constituents, including the junction) would be implemented by a suitable generalization to many particles of the constraint approach, and hadronization would be introduced via phenomenological fragmentation functions
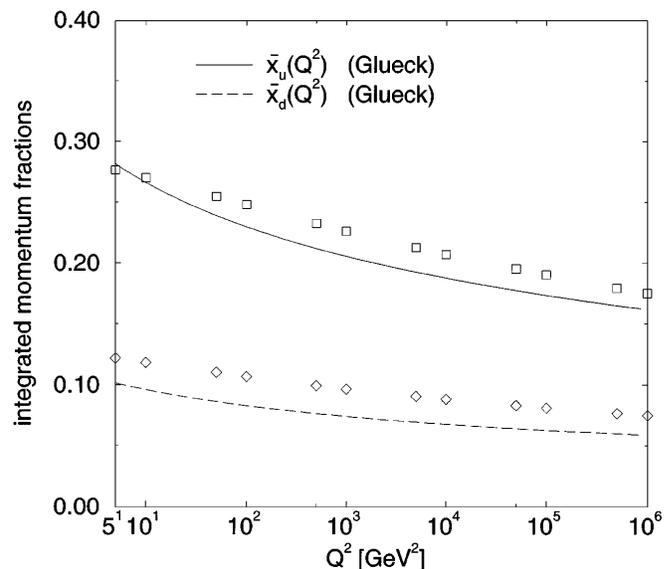
in much the same way as in [8]. We are currently in the process of developing such a code.

[1] K. Werner, Phys. Rep. **232**, 87 (1993).
[2] K. Geiger, Phys. Rep. **258**, 237 (1995).
[3] D. J. Dean, A. S. Umar, J. S. Wu, and M. R. Strayer, Phys. Rev. C **45**, 400 (1992).
[4] D. G. Currie, T. F. Jordan, and E. C. G. Sudarshan, Rev. Mod. Phys. **35**, 350 (1963).
[5] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949); "Lectures on Quantum Mechanics," Yeshiva Lectures, New York, 1964.
[6] J. Samuel, Phys. Rev. D **26**, 3475 (1982).
[7] H. Sorge, H. Stöcker, and W. Greiner, Ann. Phys. (N.Y.) **192**, 266 (1989).
[8] D. Behrens, G. Peter, and C. C. Noack, Phys. Rev. C **49**, 3253 (1994).
[9] G. C. Rossi and G. Veneziano, Phys. Rep. **63**, 153 (1980).
[10] D. Kharzeev, Phys. Lett. B **378**, 238 (1996).
[11] Particle Data Group, R. M. Barnett *et al.,* Phys. Rev. D **54**, 1 (1996).
[12] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984); A. D. Martin, R. G. Roberts, and W. J. Stirling, Phys. Rev. D **37**, 1161 (1988).
[13] H. W. Fricke, Verh. Dtsch. Phys. Ges. **1**, HK 11.7 (1997).
[14] M. Glück, E. Reya, and A. Vogt, Z. Phys. C **67**, 433 (1995).
[15] A meson would be simulated accordingly with the number of valence quarks in Eqs. (4) and (5) set to 2.