Nonsupersymmetric SU(5) Resuscitated by New Quarks and Leptons

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(Received 11 December 1997)

The issue of gauge unification in the (nonsupersymmetric) standard model is reinvestigated. It is found that, with just an additional fourth generation of quarks and leptons, SU(3) ⊗ SU(2) ⊗ U(1) gauge couplings converge to a common point $\sim 3.5 \times 10^{15}$ GeV ($\tau_p \sim 3.3 \times 10^{34\pm 2}$ yr). This result is due to the non-negligible, but still perturbative, contributions of the top and fourth generation Yukawa couplings to the gauge two-loop β functions, in contrast with the three generation case where such a contribution is too small to play an important role in unification. [S0031-9007(98)05731-7]

PACS numbers: 12.10.Kt, 11.10.Hi, 12.10.Dm, 12.60.–i

It is a standard lore that present measurements of the gauge couplings at the Z mass appear to indicate that all three couplings actually do not converge at the same point if they are to evolve according to the minimal standard model (SM) with three generations [1]. This is somewhat problematic for the simple idea of grand unification [2,3], and, in particular, the minimal SU(5) model [3]. It is also a standard lore that, with low-energy supersymmetry (generically referred to as MSSM from here on) broken at around 1 TeV or less, such a unification is possible and occurs at an energy scale $\sim 10^{16}$ GeV corresponding to a proton lifetime of $\sim 10^{36}$ yr (roughly 4 orders of magnitude above the current limit). As such, the idea of grand unification with a desert (beyond 1 TeV) fits snugly with low-energy supersymmetry. There is, however, a catch. The lightest scalar in MSSM cannot be heavier than ~ 150 GeV [4]. What would happen to grand unification if no scalar is found below, say, $2m_Z$, thus ruling out low-energy supersymmetry (or at least the simplest version of it)?

It is not entirely clear that one has exhausted all possibilities concerning the SM. One may ask, for example, what role the mass of the Higgs boson, m_H, has, especially when it is larger than ~ 174 GeV. (For a lighter Higgs boson in the presence of a heavy fermion, several constraints, especially from vacuum stability, have been discussed [5].) It is known from previous studies that when $m_H \geq 174$ GeV, the Higgs quartic coupling develops Landau poles below the Planck scale $\sim 10^{19}$ GeV [6]. How do these Landau poles influence the evolution of the gauge couplings?

One may also ask whether or not the addition of a fourth generation might change the evolution of the gauge couplings in such a way as to unify them again. It is well known that the addition of an extra family does not change the result at one loop. However, the two-loop β functions for the gauge couplings contain contributions from Yukawa couplings. With just three generations, the dominant Yukawa contribution comes from the top quark. However, it can be seen that the top Yukawa coupling actually *decreases* with energy. As a result, it practically does not help the convergence of the three gauge couplings. With four generations and with the fourth generation being sufficiently heavy (an issue explored below), it turns out that all Yukawa couplings *grow* with energy, and this can significantly affect the evolution of the gauge couplings. In fact, when the fourth generation quarks and leptons are sufficiently heavy, all Yukawa couplings (including that of the top quark) develop Landau poles *below* the Planck scale. In order to make sensible statements based strictly on the validity of perturbation theory, we shall restrict ourselves to the range of mass where these Landau poles lie above a few times 10^{15} GeV.

There are two reasons for doing so. The first one is the fact that, in order to satisfy the current lower bound on the proton lifetime, the unification scale (if there is one) has to be larger than 10^{15} GeV. The second reason is the fact that, if the three gauge couplings were to converge at the same point—with that point being $\sim 10^{15}$ GeV—due to the effects of the Yukawa couplings which show up at two loops, we would like it to happen when these Yukawa couplings are *still in the perturbative domain*. (That does not mean, however, that, if the fourth generation is sufficiently massive so that the Landau poles are *below* 10^{15} GeV, one could not have unification.)

For the minimal SU(5) [3], the well-known prediction for the proton lifetime is roughly 2 orders of magnitude lower than the current experimental lower bound of 5.5×10^{32} yr [7]. Is it possible that, if the proton does decay, its lifetime might be within reach of, say, SuperKamiokande which presumably could extend its search up to 10^{34} yr? We would like to point out in this Letter that this might be possible.

We shall use two-loop renormalization group (RG) equations throughout this paper. For case I, they are well known, and the explicit expressions can be found in the literature [8]. For the second case with four generations, we shall write down explicitly the two-loop RG equations below [9]. To set the notations straight, our definition of the quartic coupling in terms of the Higgs mass is the conventional $m_H^2 = 2\lambda v^2$. The RG equations given below reflect our convention on the quartic coupling.

We begin with the minimal SM with three generations. As mentioned earlier, we are particularly interested in the Higgs mass range, $m_H \geq 174$ GeV. In particular, if we restrict ourselves to the values of m_H where the associated Landau poles would lie above 10^{15} GeV, we are then looking at the range $174 \leq m_H \leq 180$ GeV. Figure 1 shows the evolution of the three gauge couplings g_1, g_2, g_3 for $m_H = 174$ GeV. (The results are practically the same for $m_H = 180$ GeV.) One clearly sees that they *do not* converge to the same point, a result similar to the already well-known one. Here, we actually include the indirect effect of the Higgs mass, namely, its effect on the top Yukawa coupling which feeds into the RG equations for the gauge couplings. As one might have suspected, we have found no effect: the three gauge couplings still do not meet. For heavier Higgs, i.e., $m_H > 180$ GeV, the Landau poles will appear below 10^{15} GeV. We cannot say for sure, at least within the context of the perturbation theory, what influence such a "heavy" Higgs might have on the evolution of the gauge couplings.

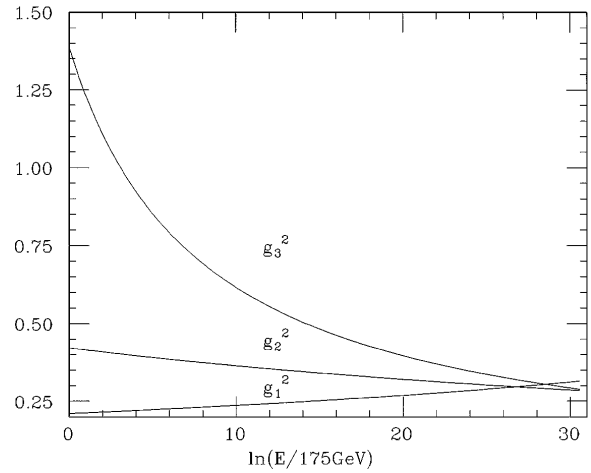

FIG. 1. The evolution of the SM gauge couplings squared versus $\ln(E/175\ \text{GeV})$ for the three generation case. g_3, g_2, and g_1 are the couplings of SU(3), SU(2), and U(1), respectively. We have used $g_3^2(m_t) = 1.392$, $g_2^2(m_t) = 0.421$, and $g_3^2(m_t) = 0.2114$. The Higgs mass is $m_H = 174$ GeV.

We now turn to the second scenario with four generations and one Higgs doublet. These four generations fit snugly into $\bar{5} + 10$ representations of SU(5), except for the right-handed neutrinos which we should need if we were to give a mass to the neutrinos. For example, this could be incorporated into a 16-dimensional representation of SO(10) which splits into $\bar{5} + 10 + 1$ under SU(5). We could then have a pattern of symmetry breaking like SO(10) $\longrightarrow$ SU(5), for example. All four neutrinos can acquire a mass via the seesaw mechanism, for example. Furthermore there is no reason why the fourth one cannot be much heavier than the other three, namely, its mass could be at least half the Z mass. These details are, however, beyond the scope of this paper.

The appropriate two-loop RG equations are given by

$$16\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 + 4\lambda(3g_t^2 + 6g_q^2 + 2g_l^2 - 2.25g_2^2 - 0.45g_1^2) - 2(3g_t^4 + 6g_q^4 + 2g_l^4) + (16\pi^2)^{-1} \times \{30g_t^6 + 48g_q^6 + 16g_l^6 - [3g_t^4 + 6g_q^4 + 2g_l^4 - 80g_3^2(g_t^2 + 2g_q^2)]\lambda - 6\lambda^2(24g_t^2 + 48g_q^2 + 16g_l^2) - 312\lambda^3 - 32g_3^2(g_t^4 + 2g_q^4)\}, \tag{1a}$$

$$16\pi^2 \frac{dg_t^2}{dt} = g_t^2\{9g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2(8\pi^2)^{-1} \times [1.5g_t^4 - 2.25g_t^2(6g_q^2 + 3g_t^2 + 2g_l^2) - 12g_q^4 - (27/4)g_t^4 - 3g_l^4 + 6\lambda^2 + g_t^2(-12\lambda + 36g_3^2) - (892/9)g_3^4]\}, \tag{1b}$$

$$16\pi^2 \frac{dg_q^2}{dt} = g_q^2\{6g_t^2 + 12g_q^2 + 4g_l^2 - 16g_3^2 - 4.5g_2^2 - 1.7g_1^2(8\pi^2)^{-1} \times [3g_q^4 - g_q^2(6g_q^2 + 3g_t^2 + 2g_l^2) - 12g_q^4 - (27/4)g_t^4 - 3g_l^4 + 6\lambda^2 + g_q^2(-16\lambda + 40g_3^2) - (892/9)g_3^4]\}, \tag{1c}$$

$$16\pi^2 \frac{dg_l^2}{dt} = g_l^2\{6g_t^2 + 12g_q^2 + 4g_l^2 - 4.5(g_2^2 + g_1^2)(8\pi^2)^{-1} \times [3g_q^4 - g_q^2(6g_q^2 + 3g_t^2 + 2g_l^2) - 12g_q^4 - (27/4)g_t^4 - 3g_l^4 + 6\lambda^2 - 16\lambda g_l^2]\}, \tag{1d}$$

$$16\pi^2 \frac{dg_1^2}{dt} = g_1^4\{(163/15) + (16\pi^2)^{-1}[(787/75)g_1^2 + 6.6g_2^2(352/15)g_3^2 - 3.4g_t^2 - 4.4g_q^2 - 3.6g_l^2]\}, \tag{1e}$$

$$16\pi^2 \frac{dg_2^2}{dt} = g_2^4\{-(11/3) + (16\pi^2)^{-1}[2.2g_1^2 + (133/3)g_2^2 32g_3^2 - 3g_t^2 - 3g_q^2 - 2g_l^2]\}, \tag{1f}$$

$$16\pi^2 \frac{dg_3^2}{dt} = g_3^4\{-(34/3) + (16\pi^2)^{-1}[(44/15)g_1^2 + 12g_2^2 - (4/3)g_3^2 - 4g_t^2 - 8g_q^2]\}. \tag{1g}$$

In the above equations, we have assumed for the fourth family, for simplicity, a Dirac neutrino mass and, in order to satisfy the constraints of electroweak precision measurements, that both quarks and leptons are degenerate $SU(2)_L$ doublets. The respective Yukawa couplings are denoted by g_q and g_l. Also, in the evolution of λ and the Yukawa couplings, we have neglected, in the two-loop terms, contributions involving τ and bottom Yukawa couplings as well as the electroweak gauge couplings, g_1 and g_2. For the range of Higgs and heavy quark (including the top quark) masses considered in this paper, these two-loop contributions are not important to the evolution of λ and the Yukawa couplings. Also, as long as the mixing between the fourth generation and the other three is small, the results will be unaffected by such a mixing. (We shall briefly comment on the phenomenological implications of this fourth generation below.)

In what follows, we shall assume that whatever mechanism (a right-handed neutrino in this case) that is responsible for giving a mass to at least the fourth neutrino will not affect the evolution of the three SM gauge couplings. Also there are reasons to believe that this fourth generation might be rather special, distinct from the other three and having very little mixing with them. The physics scenario behind the fourth neutrino mass might be quite unconventional.

What masses for the fourth generation are we allowed to use in our analysis? For the quarks, the mass can even be lower than the top quark mass. As of now, there is no strict limit on the mass of the fourth generation quarks if the fourth family is *nonsequential*, i.e., having very little mixing with the other three. As discussed in [10], the current accessible but unexplored decay length for a long-lived heavy quark to be detected is between 100 μm and 1 m. As long as a member of the fourth generation quark doublet (e.g., the down-type quark) decays in that range, its mass can even be lower than the top mass. The phenomenology of a near degenerate long-lived doublet of quarks and its detection is discussed in full length in Ref. [10]. As for the fourth generation leptons, we shall assume that the mass is greater than m_Z.

As we have stated above, we shall restrict ourselves to the mass range of the fourth generation that will have Landau poles only above 10^{15} GeV. We shall require that, if there is convergence of the three gauge couplings, it should occur when the Higgs quartic coupling and the Yukawa couplings are still perturbative in the sense that one can neglect contributions coming from three-loop (and higher) terms to the β functions.

Figure 2 shows the evolution of g_1^2, g_2^2, and g_3^2 for one particular set of masses, namely, $m_Q = 151$ GeV, $m_L = 95.3$ GeV, and $m_t = 175$ GeV, where m_Q, m_L, and m_t denote the fourth quark, lepton, and top masses, respectively. Vacuum stability ($\lambda > 0$) and the requirement that $\lambda/4\pi \sim 1$ above 10^{15} GeV, for the fermion masses listed above, give a prediction for the mass for the Higgs boson to be $m_H = 188$ GeV which is *larger* than the fourth generation quark mass. Two remarks are in order here.

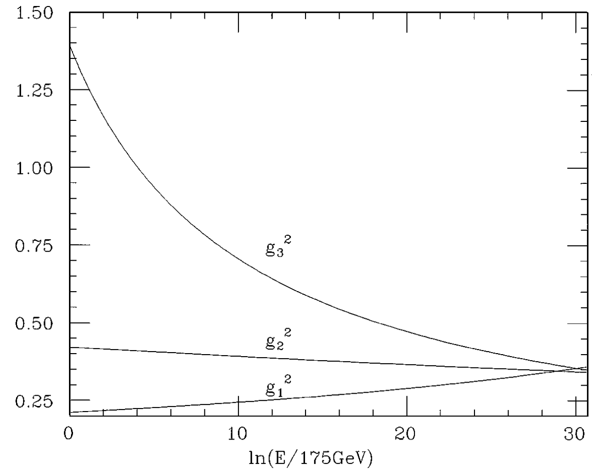

FIG. 2. The evolution of the SM gauge couplings squared versus $\ln(E/175\ \mathrm{GeV})$ for the four generation case. g_3, g_2, and g_1 are the couplings of SU(3), SU(2), and U(1), respectively. We have used $g_3^2(m_t) = 1.392$, $g_2^2(m_t) = 0.421$, and $g_3^2(m_t) = 0.2114$. Also we use $m_Q = 151$ GeV, $m_L = 95.3$ GeV, and $m_t = 175$ GeV, where m_Q, m_L, and m_t denote the fourth quark, lepton, and top masses, respectively. The heavy threshold effects are not taken into account here. They are discussed in the text and are shown to improve the unification point.

First, Fig. 2 shows the evolution of the gauge couplings *without* taking into account the effects of the heavy particle threshold near the unification point. For example, that threshold could come from the 24 and 5 Higgs scalars of SU(5). In fact, as one can see from Fig. 2, the three gauge couplings come *close* (to 4% or less) to each other but do not actually meet at the same point if one does not include heavy threshold effects. By itself, within errors, it is already a good indication of possible unification. We would like, nevertheless, to discuss the issues of heavy threshold for completeness. One may ask the following question: if we choose a scale, M_G, where the uncorrected couplings $\alpha_i \equiv g_i^2/4\pi$ $(i = 1, 2, 3)$ are within, say, 4% of each other, can one bring them together after the inclusion of heavy threshold effects? As an example, let us take the following point (last point in Fig. 2): $\ln(E/175\ \mathrm{GeV}) = 30.62$ which corresponds to $M_G = 3.48 \times 10^{15}$ GeV. At this point, one has $\alpha_3(M_G) = 0.0278$, $\alpha_2(M_G) = 0.0273$, and $\alpha_1(M_G) = 0.0285$. If one defines $\Delta\alpha/\alpha \equiv (\alpha_{\mathrm{larger}} - \alpha_{\mathrm{smaller}})/\alpha_{\mathrm{larger}}$, one can immediately see that $\Delta\alpha/\alpha \approx 2\%$–$4\%$.

Let us, e.g., assume the minimal SU(5) with the following heavy particles: $(X, Y) = (\bar{3}, 2, 5/6)$ + c.c. with mass M_V, real scalars $(8, 1, 0) + (1, 3, 1) + (1, 1, 0)$ (belonging to the 24-dimensional Higgs field) with mass M_{24}, and the complex scalars $(3, 1, -1/3)$ (belonging to the 5-dimensional Higgs field) with mass M_5. [The quantum numbers are with respect to $SU(3) \otimes SU(2) \otimes U(1)$.] The heavy threshold corrections are then [1] $\Delta_1 = \frac{35}{4\pi}\ln(\frac{M_G}{M_V}) - \frac{1}{30\pi}\ln(\frac{M_G}{M_5}) + \Delta_1^{NRO}$, $\Delta_2 = -\frac{1}{6\pi} + \frac{21}{4\pi}\ln(\frac{M_G}{M_V}) - \frac{1}{6\pi}\ln(\frac{M_G}{M_{24}}) + \Delta_2^{NRO}$, $\Delta_3 = -\frac{1}{4\pi} + \frac{7}{2\pi}\ln(\frac{M_G}{M_V}) - \frac{1}{12\pi}\ln(\frac{M_G}{M_5}) - \frac{1}{4\pi}\ln(\frac{M_G}{M_{24}}) + \Delta_3^{NRO}$, where $\Delta_i^{NRO} = -\eta k_i(\frac{2}{25\pi\alpha_G^3})^{1/2}\frac{M_G}{M_{\rm Planck}}$, with $k_i = 1/2, 3/2, -1$ for $i = 1, 2, 3$, is the correction coming from possible dimension 5 operators present between M_G and $M_{\rm Planck}$. The magnitude of the coefficient η is constrained to be less than or equal to 10. The corrected couplings can be written as $\frac{1}{\tilde{\alpha}_i(M_G)} = \frac{1}{\alpha_i(M_G)} + \Delta_i$. There are, of course, many choices for the different mass scales. As an example, we shall choose $M_5 = M_G$, $M_{24} = M_G$, $M_V = 0.5M_G$, and $\eta = 10$. With this choice and taking as α_G the average of $\alpha_3(M_G) = 0.0278$, $\alpha_2(M_G) = 0.0273$, and $\alpha_1(M_G) = 0.0285$, we obtain $\tilde{\alpha}_1(M_G) = 0.027\,05$, $\tilde{\alpha}_2(M_G) = 0.026\,62$, and $\tilde{\alpha}_3(M_G) = 0.027\,35$. From the above values, one can say that the couplings are practically the same with all three $\sim$0.027 or $1/\tilde{\alpha}_G \sim 37$. This little exercise shows that heavy threshold effects can indeed bring about better unification.

The second remark we wish to make is the question of the validity of perturbation theory. The usual requirement encountered in the literature is that $g_t^2/4\pi$, $g_q^2/4\pi$, $g_l^2/4\pi$, and $\lambda/4\pi$ have to be less than unity. In particular, lattice calculations suggest that perturbation theory breaks down when the Higgs mass is $\sim$750 GeV. With $m_H^2 = 2\lambda v^2$, this translates into $\lambda/4\pi \sim 0.37$. This criterion is more restrictive than just simply setting $\lambda/4\pi \sim 1$. What are our corresponding values at the unification point? At the point that we refer to as the unification point, namely, $M_G = 3.48 \times 10^{15}$ GeV, we found the following values for the Higgs quartic and Yukawa couplings: $g_t^2/4\pi = 0.4$, $g_q^2/4\pi = 0.16$, $g_l^2/4\pi = 0.48$, and $\lambda/4\pi = 0.19$. This is not a strong coupling regime and perturbation theory is presumably reliable. A side remark is in order here. For the three generation case, the value of the top Yukawa coupling at a similar scale is $g_t^2/4\pi \sim 0.016$, a factor of 20 smaller than in the four generation case. This explains why the Yukawa couplings are important enough in this case, but not in the three generation case, to modify the evolution of the gauge couplings.

An exhaustive study of different mass combinations for the fourth generation is beyond the scope of this paper.

The proton partial mean lifetime as represented by $\tau_{p\to e^+\pi^0}$ is predicted to be $\tau_{p\to e^+\pi^0}(\mathrm{yr}) \approx 10^{31}(M_G/4.6 \times 10^{14})^4$. In our case, we obtain the following prediction: $\tau_{p\to e^+\pi^0}(\mathrm{yr}) \approx 3.28 \times 10^{34\pm2}$. Part of the uncertainty in the lifetime is due to the uncertainty in heavy threshold effects. The central value is larger than the current lower limit of 5.5×10^{32} yr and is within reach of the next generation of SuperKamiokande proton decay search.

This scenario made the following predictions: (1) the proton decays at an accessible rate $\sim 3.3 \times 10^{34\pm2}$ yr; (2) there is a fourth generation of long-lived quarks and leptons with a quark mass $\sim$151 GeV; and (3) the Higgs mass is predicted to be $\sim$188 GeV $> 2m_Z$, a value which is well suited for the "gold plated" signal $H \to l^+l^-l^+l^-$ at the LHC. All of these features can be tested in a not-too-distant future. For example, the fourth generation can be *nonsequential* and can have exceptionally long lifetimes. This could provide a distinct signature [10].

I thank Paul Frampton and Gino Isidori for helpful discussions and comments on the manuscript. I also thank the theory groups at the University of Rome "La Sapienza" and at the Ecole Polytechnique, Palaiseau, for the warm hospitality where part of this work was carried out. This work is supported in part by the U.S. Department of Energy under Grant No. DE-A505-89ER40518.

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