Creation of Dark Solitons and Vortices in Bose-Einstein Condensates

R. Dum,[1,2,3] J. I. Cirac,[1,2] M. Lewenstein,[4] and P. Zoller[1]
[1]*Institute für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*
[2]*Departamento de F'isica Aplicada, Universidad de Castilla–La Mancha, E-13071 Ciudad Real, Spain*
[3]*Ecole Normale Supérieure, Laboratoire Kastler Brossel, F-75231 Paris Cedex 05, France*
[4]*DSM/DRECAM/SPAM, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette, France*
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We propose and analyze a scheme to create dark solitons and vortices in Bose-Einstein condensates. This is achieved starting from a condensate in the internal state $|a\rangle$ and transferring the atoms to the internal state $|b\rangle$ via a Raman transition induced by laser light. By scanning adiabatically the Raman detuning, dark solitons and vortices are created. [S0031-9007(98)05713-5]

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Recently Bose-Einstein condensation has been demonstrated for dilute atomic gases in magnetic traps [1]. This state of matter resembles other states found in the fields of superfluidity, superconductivity, and nonlinear optics. It is, thus, natural to expect to observe phenomena similar to the ones in those fields, such as solitons and vortices. In fact, the Gross-Pitaevskii equation (GPE) [2] describing the wave function of the macroscopically occupied state allows stationary solutions that represent dark solitons (see below) and vortices (see also [3]). We propose to "engineer" these solutions in a *controlled way:* once the condensate has formed, we use a coherent Raman process to couple the internal state $|a\rangle$ where the condensate is formed with another internal state $|b\rangle$ (Fig. 1). The laser parameters are chosen such that the state after the transfer is a stationary state of the GPE corresponding to solitons or vortices. Given the nonlinear character of the problem due to atom-atom interactions, straightforward generalizations of single atom methods of quantum state engineering that mainly use resonant Raman pulses are not possible [4]. In fact, atom-atom interactions modify the spatial wave function as well as the energy of the atoms during the transfer process, making the problem highly nontrivial. Our method relies on an *adiabatic transfer* process that takes fully into account these interactions, and is very robust against uncertainties in the experimental parameters.

Let us start by showing that one can have stationary states of a Bose condensate that represent solitons or vortices. In the Hartree-Fock approximation a stationary state Φ of a condensate of N bosons confined in a potential $V(\vec{r})$ is described by the time-independent GPE

$$\left[-\frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r}) + Ng|\Phi(\vec{r},t)|^2\right]\Phi(\vec{r},t) = E\Phi(\vec{r},t). \tag{1}$$

The nonlinear term with $g > 0$ describes the mean field due to the atomic interaction.

In the following we will concentrate on the Thomas-Fermi limit [5] valid for current experiments. In this limit, the lowest stationary solution of Eq. (1) varies slowly along the condensate, and, therefore, one can neglect the kinetic energy compared to the mean interaction energy. Here, we generalize this approach to find other stationary solutions that contain solitons and vortices centered near the origin, say at positions $\vec{r}$ with $|\vec{r}| \lesssim r_0$ (where r_0 has to be determined consistently at the end). We, therefore, look for solutions of the GPE that vary slowly along the condensate except at the center; this suggests the following ansatz:

$$\Phi^{sv}(\vec{r}) = \phi(\vec{r})\Phi_E(\vec{r}), \tag{2}$$

where $\Phi_E(\vec{r})$ is an envelope function that varies slowly along the condensate, whereas $\phi(\vec{r})$ can vary strongly around the center of the potential and fulfills $|\phi(\vec{r})| \to 1$ for $|\vec{r}| \gtrsim r_0$. We substitute (2) in the GPE and distinguish two regions in space: (i) for $|\vec{r}| \gtrsim r_0$, we can take $|\phi(\vec{r})| \simeq 1$ and neglect the spatial derivatives of $\Phi_E(\vec{r})$ obtaining

$$\Phi_E(\vec{r}) = \{[E - V(\vec{r})]/(Ng)\}^{1/2}, \tag{3}$$

for $\vec{r}$ such that $V(\vec{r}) < E$ and zero otherwise; (ii) for $|\vec{r}| \lesssim r_0$, we can neglect $V(\vec{r})$ compared to E both in $\Phi_E(\vec{r})$ [see Eq. (3)] and in the GPE (1) obtaining

$$\left[-\frac{\hbar^2\vec{\nabla}^2}{2m} + E|\phi(\vec{r})|^2\right]\phi(\vec{r}) = E\phi(\vec{r}). \tag{4}$$

The energy E is determined by imposing $\int d^3r|\Phi_E(\vec{r})|^2 = 1$, which reflects particle conservation.

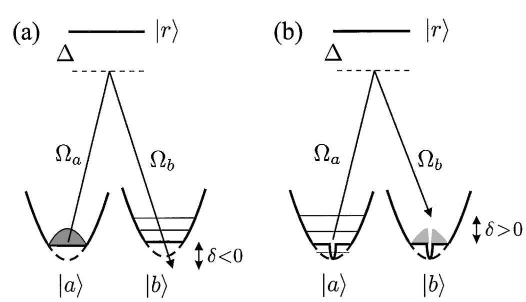

FIG. 1. Schematic representation of the process: (a) initial state; (b) final state.

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Equation (4) is the familiar GPE for the homogeneous case which is known to give rise to dark solitons [6] and vortices. The ground state solution of the GPE corresponds to $\phi(\vec{r}) = 1$ so that it is identical to $\Phi_E(\vec{r})$ in Eq. (3) with $E = \mu$, the chemical potential.

In our model we take bosons with two internal levels $|a\rangle$ and $|b\rangle$ as in Rb [7] (see also [8]). After condensation in level $|a\rangle$, we drive the transition $|a\rangle \to |b\rangle$ with a Raman laser configuration (Fig. 1). The spinor $\vec{\Phi} = (\Phi_a, \Phi_b)$ obeys a two-component time-dependent GPE

$$i\hbar \frac{d}{dt}\vec{\Phi}(\vec{r},t) = \mathcal{H}_\delta \vec{\Phi}(\vec{r},t) = \begin{pmatrix} H & \frac{1}{2}\lambda(\vec{r}) \\ \frac{1}{2}\lambda(\vec{r})^* & H - \delta \end{pmatrix} \vec{\Phi}(\vec{r},t), \quad (5)$$

where the scalar (nonlinear) Hamiltonian

$$H = -\frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r}) + Ng[|\Phi_a(\vec{r},t)|^2 + |\Phi_b(\vec{r},t)|^2] \quad (6)$$

describes the evolution in the potential $V(\vec{r})$ (for which we choose an anisotropic harmonic potential with frequencies $\omega_{x,y,z}$) in the presence of interactions [9]. In (5) $\lambda(\vec{r}) = \Omega_a(\vec{r})\Omega_b(\vec{r})/(4\Delta)$, where Δ is the detuning from the intermediate level $|r\rangle$, and $\Omega_{a,b}$ are the Rabi frequencies coupling $|a\rangle$ and $|b\rangle$ to $|r\rangle$, respectively (see Fig. 1). The Raman two-photon detuning is denoted by δ. The conservation of the number of particles gives $\int d^3\vec{r}[|\Phi_a(\vec{r})|^2 + |\Phi_b(\vec{r})|^2] = 1$.

As the initial state we take $(\Phi_\mu, 0)$ [see Eq. (3)] that is the lowest lying stationary state of the GPE formed in the internal level $|a\rangle$. We will design $\lambda(\vec{r})$ and $\delta(t)$ such that the atoms are transferred to $|b\rangle$ with a wave function that corresponds to dark solitons or vortices. In the absence of interactions ($g = 0$) the problem reduces to the one of a single trapped particle. In that case, one can simply use a resonant laser pulse of a well defined area to carry out the population transfer [4]. In presence of interactions, this method will not work: as soon as particles are transferred to a different state, the shape of the wave function as well as the interaction energy change [Figs. 1(a) and 1(b)]. Therefore, an initially resonant Raman laser pulse soon becomes off resonant, and the transfer process will stop unless one tailors the Raman pulse (detuning and area) which is extremely difficult due to the narrowness of the Raman resonance. We circumvent this problem by using *adiabatic passage:* we start from a *negative* Raman detuning so that the atoms do not feel the laser [Fig. 1(a)] and change it adiabatically to sufficiently large *positive* values [Fig. 1(b)]. As soon as the laser frequency approaches the Raman resonance, the atoms will start flowing to the state $|b\rangle$. The fact that the interaction energy changes will shift the Raman resonance. This will not affect the process, provided the final value of δ is large enough so that at the end the atoms do not feel the off-resonant laser anymore. The reason is that adiabatic transfer depends only on the initial and final values of δ.

In order to describe this process, we wish to determine the relevant stationary states of the Hamiltonian $\mathcal{H}_\delta$ in Eq. (5) for each particular value of the Raman detuning δ, i.e.,

$$\mathcal{H}_\delta \vec{\Phi}_\delta = E_\delta \vec{\Phi}_\delta . \quad (7)$$

In analogy to the noninteracting case, we will call the states $\vec{\Phi}_\delta$ *dressed states* [10]. According to the adiabatic theorem, if the initial state fulfills $\vec{\Phi}(t=0) = \vec{\Phi}_{\delta_0}$ and the change of the detuning is sufficiently slow from δ_0 to δ_f, we will have $\vec{\Phi}(t_f) = \vec{\Phi}_{\delta_f}$. We have to choose δ_0 such that $\vec{\Phi}_{\delta_0} \simeq (\Phi_\mu, 0)$, and design the laser so that $\vec{\Phi}_{\delta_f} = (0, \Phi^{sv})$ where Φ^{sv} is the desired solution of (2).

In order to construct the relevant solutions of (7) we have developed the following method. First, we look for pairs of functions forming a spinor $\vec{\Phi}^{\pi_a} = (\Phi_a^{\pi_a}, \Phi_b^{\pi_a})$ that fulfills the coupled equations

$$\begin{aligned} H\Phi_a^{\pi_a}(\vec{r}) &= \epsilon_a^{\pi_a}\Phi_a^{\pi_a}(\vec{r}), \\ H\Phi_b^{\pi_a}(\vec{r}) &= \epsilon_b^{\pi_a}\Phi_b^{\pi_a}(\vec{r}), \end{aligned} \quad (8)$$

for a given value of the population $\pi_a = \int d^3\vec{r}|\Phi_a^{\pi_a}(\vec{r})|^2$ in level a (note $\pi_b + \pi_a = 1$, where π_b denotes the population in level b). There are many pairs of solutions of Eq. (8). We choose those that fulfill $\Phi_a^{\pi_a \simeq 1} = \Phi_\mu$ and $\Phi_b^{\pi_a \simeq 0} = \Phi^{sv}$; that is, for $\pi_a = 1$ and $\pi_a = 0$ they represent the desired initial and final states. These *bare states* can be found using either the generalized Thomas-Fermi approach outlined in the context of Eq. (2), or simply numerically. This gives for each value of π_a the energy separation $\Delta_{a,b}^{\pi_a} \equiv \epsilon_b^{\pi_a} - \epsilon_a^{\pi_a}$ and, thus, the resonance condition $\delta \to \Delta_{a,b}^{\pi_a}$. For an appropriately designed $\lambda(\vec{r})$ we can restrict the solution of Eq. (7) to the solutions of Eq. (8); that is, we substitute $\vec{\Phi}_\delta \to \vec{\Phi}^{\pi_a(\delta)}$ in (7). Multiplying the two resulting equations by $\Phi_a^{\pi_a}(\vec{r})^*$ and $\Phi_b^{\pi_a}(\vec{r})^*$, respectively, using (8) and integrating we obtain

$$\begin{pmatrix} \epsilon_a^{\pi_a} & \lambda^{\pi_a} \\ \lambda^{\pi_a *} & \epsilon_b^{\pi_a} - \delta \end{pmatrix} \begin{pmatrix} \sqrt{\pi_a} \\ \sqrt{\pi_b} \end{pmatrix} = E_\delta \begin{pmatrix} \sqrt{\pi_a} \\ \sqrt{\pi_b} \end{pmatrix}, \quad (9)$$

where we have defined

$$\lambda^{\pi_a} \equiv \frac{1}{2\sqrt{\pi_a \pi_b}} \int d^3\vec{r}\, \lambda(\vec{r})\, \Phi_a^{\pi_a}(\vec{r})^*\, \Phi_b^{\pi_a}(\vec{r}). \quad (10)$$

From Eq. (9) we can determine the values of π_a corresponding to a given δ as well as E_δ, the dressed state energy. The restriction to this particular solution is justified given that the coupling strength between the levels of $\vec{\Phi}^{\pi_a}$ which is given by λ^{π_a}/δ is much larger than the coupling strength of $\Phi_a^{\pi_a}$ to other levels. Typically $\Phi_a^{\pi_a}$ in level

a corresponds to the lowest lying state (zero nodes) and $\Phi_b^{\pi_a}$ in level b to one of the excited states, with say one or two nodes (remember we finally want a soliton or vortex in level b); therefore, $\lambda(\vec{r})$ has to be slowly varying along the condensate as otherwise coupling to solutions with many nodes will be dominant (see later). The condition for adiabaticity (and, therefore, for the time scale for the change of δ) is given by the avoided crossing, which is λ^{π_a} for $\pi_a \simeq 1/2$.

We illustrate this procedure for the 1D case, that is, the limit $\omega_{x,y} \gg \omega_z$, so that the dynamics along the x and y direction is frozen. Our goal is to create a dark soliton with a zero at the trap center starting from $\Phi_\mu(z)$. This requires that the laser interaction changes the parity of the wave function when the atoms are transferred from a to b: we choose the simplest laser configuration, so that $\lambda(z) = \lambda_0 \sin(kz)$, i.e., a standing wave. In order to achieve an efficient coupling between the ground state and the soliton and to avoid coupling to other states we take $k \le 1/z_0$, where z_0 is the size of Φ_μ [11]; note that the effective λ defined in (10) will be very small if $kz_0 \gg 1$. In this case the avoided crossing of the dressed energy levels will be of the order λ_0, which sets the time scale for the adiabaticity. On the other hand, λ_0 has to be smaller than the energy separations $|\Delta_{ab}^{\pi_a}|$ so that the Stark shifts do not mix these wave functions with others of higher energies ϵ. The initial value of the Raman detuning must be $\delta_0 \ll \Delta_{a,b}^{\pi_a=1}$, whereas the final value must fulfill $\delta_f \gg \Delta_{a,b}^{\pi_a=0}$. In Fig. 2 we have plotted numerical results of the solutions of the time-dependent GPE (5). Figure 2(a) shows the spatial distribution $P_{a,b}(z) = |\Phi_{a,b}(z)|^2$ corresponding to states $|a\rangle$ (solid line) and $|b\rangle$ (dashed line). As the transfer progresses, we see that the wave function of the atoms in $|a\rangle$ narrows and one of the atoms in $|b\rangle$ develops a hole in the center; that is, a dark soliton is formed. This manifests itself also in the effective (trap plus mean field) potential; it is initially flat, and later it develops a narrow dip [see Fig. 1(b)]. Those atoms still in $|a\rangle$ become trapped in a bound state of this dip, which becomes deeper as we move more atoms in the excited state. In Fig. 2(b) we have plotted the fraction $\pi_{a,b}$ of atoms in a and b. As this figure shows, the transfer efficiency is essentially 100%. We have performed a full 3D integration of the time dependent GPE (5) in order to make sure that the 1D results are still valid in the presence of the transverse degrees of freedom [12].

For the analytical understanding of these results, we proceed as explained above. First, we calculate the bare states: in the limit $\pi_a \to 1$ one can estimate the value of $\Delta_{ab}^{\pi_a}$ using a square well of length equal to the size of the Thomas-Fermi solution Φ_μ; that is, we take as Φ_b the first excited state in the effective potential $[V(\vec{r}) + Ng|\Phi_\mu(\vec{r})|^2]$; when $\pi_a \lesssim 1/2$ we can calculate $\Delta_{ab}^{\pi_a}$ using a generalized Thomas-Fermi ansatz for the bare states. To this end we write as in Eq. (2)

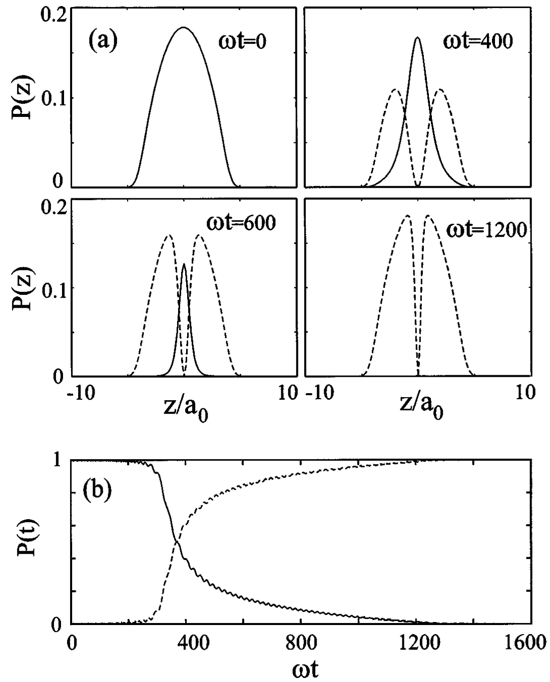

FIG. 2. Generation of a dark soliton. The detuning is varied linearly from $\delta = -1.5\omega$ to $\delta = 6.5\omega$ in a time $\omega t = 1500$. Other parameters: $\lambda_0 = 0.15\omega$, $k_L = 0.5/a_0$, and $Ng = 50\hbar\omega a_0$, where $a_0^2 = \hbar/(m\omega)$. (a) Snapshots of the position distributions of the wave functions corresponding to atoms in level $|a\rangle$ (solid line) and $|b\rangle$ (dashed line) for different times; (b) populations of these levels as a function of time.

$\vec{\Phi}(z) = [\phi_a(z), \phi_b(z)]\Phi_E(z)$, with $\Phi_E(z)$ defined in (3). Near the trap center Eq. (8) becomes

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + E[|\phi_a(z)|^2 + |\phi_b(z)|^2] - \epsilon_{a,b}\right]\phi_{a,b}(z) = 0\,, \tag{11}$$

with the boundary conditions $|\phi_a(z)| \to 0$ and $|\phi_b(z)| \to 1$ as $|z| \to \infty$. We find the solution

$$\vec{\Phi}(z) = \begin{pmatrix} A\,\mathrm{sech}(\sqrt{\frac{mE}{\hbar^2}}\alpha z) \\ \tanh(\sqrt{\frac{mE}{\hbar^2}}\alpha z) \end{pmatrix} \Phi_E(z)\,, \tag{12}$$

with $1 - A^2 = \alpha^2$, $\epsilon_b = E$, and $\epsilon_a = E(1 - \alpha^2/2)$ corresponding to the energy of a state localized in the dip of the effective potential. E is determined via the normalization condition. The generalized dressed states from Eq. (9) finally give δ and E_δ for a given value of π_a. The *analytical* solutions agree perfectly with our numerical results.

A two-soliton solution is obtained by starting from the ground state in a, and coupling with a laser configuration that preserves the parity, $\lambda(z) = \lambda_0 \cos(kz)$. In order not to couple to the ground state in b, the initial detuning has to be $\delta_0 > |\lambda_0|$. We then increase the detuning adiabatically to a sufficiently large value. In Fig. 3 we

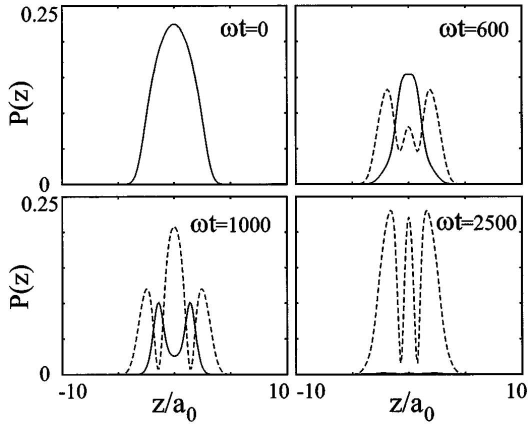

FIG. 3. Generation of two dark solitons. The detuning is varied linearly from $\delta = 0.25\omega$ to $\delta = 5\omega$ in a time $\omega t = 2500$. Other parameters: $\lambda_0 = 0.15\omega$, $k_L = 0.5/a_0$, and $Ng = 20\hbar\omega a_0$.

show plots of numerical solutions. At the end of the process all the particles are in the state $|b\rangle$ with a wave function that includes two dark solitons.

A 2D situation arises in the limit $\omega_z \gg \omega_\perp = \omega_x = \omega_y$. In this case we are interested in creating vortex solutions of the form $\Phi(\rho, \varphi) = f(\rho)e^{i\varphi}$, where ρ and φ are cylindrical coordinates, and $f(\rho)$ is a function with a zero at $\rho = 0$. In order to provide the required angular momentum to the atoms that are transferred, we choose a laser configuration such that $\lambda(x,y) = \lambda_0[\sin(k_L x) + i\sin(k_L y)] \simeq \lambda_0 k_L \rho e^{i\varphi}$ for $k_L\rho \lesssim 1$ [11]. The density distribution $|\Phi(x,y)|^2$ after an adiabatic switch of the detuning is plotted in Fig. 4. The inset shows that all the population is transferred to the vortex state.

One way of observing the shape of the density $n(\vec{r})$ is by opening of the trap at $t = t_f$. As in Ref. [13] we have the relation $n(\vec{r}, t > t_f) = n[\vec{r}/\gamma(t), t_f]/\gamma^{2d}(t)$; the scaling factors obey $\ddot{\gamma} = \omega^2/\gamma^{d+1}$ (d is the dimension) and $\gamma(t_f) = 1$, $\dot{\gamma}(t_f) = 0$. The asymptotic behavior $\gamma(t) \to \sqrt{2}\omega_z t$ for the 1D dark soliton and $\gamma(t) \to \omega_\perp t$ for the vortex solution can be measured.

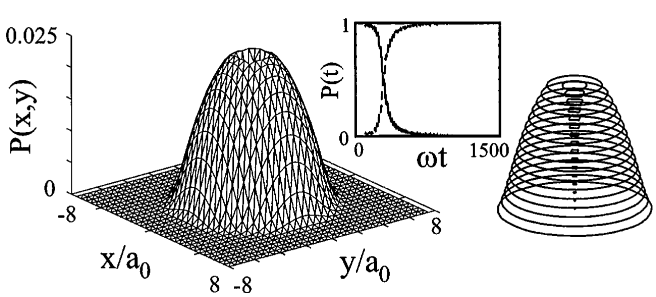

FIG. 4. Generation of a vortex: position distribution of the final state. The detuning is varied linearly from $\delta = -0.6\omega_\perp$ to $\delta = 5\omega_\perp$ in a time $\omega_\perp t = 2000$. The parameters are $Ng = 500\hbar\omega_\perp a_\perp^2$, $\lambda = 0.15\hbar\omega_\perp$, and $k_L = 0.5a_\perp$. The inset shows the evolution of the populations $P(t)$ in levels $|a\rangle$ and $|b\rangle$ (solid and dashed lines, respectively).

An issue to address is the stability of vortices and dark solitons. In [3] it is shown that states localized near the center of the trap will be preferentially occupied by collisions thereby destabilizing the vortex. However, we expect the destabilization time to be much longer than that required for the creation of vortices and solitons. We have numerically verified that adding a small perturbation to the initial condensate function does not change the conclusions of the present work.

We have demonstrated that one can engineer the macroscopic wave function of a Bose-Einstein condensed sample by coupling the internal atomic levels with a laser. The method is based on adiabatic transfer of population along generalized dressed states that include the nonlinear atom-atom interactions.

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Note added.—After submission of the present Letter an article has appeared [14] that proposes to create vortices using resonant Raman pulses.

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[9] For simplicity, we have assumed equal interaction strengths ($a - a$, $b - b$, and $a - b$) characterized by g and identical trapping frequencies for both levels. Our conclusions hold for the general case. We have also neglected the Stark shift due to the presence of the laser, since it is small compared to the potential $V(\vec{r})$.

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[11] For typical experimental conditions a typical wavelength is 500 nm and a typical extension of the condensate is $l_0 \approx 5\ \mu$m. To achieve $k_L l_0 \simeq 1$ one can use Raman beams with wave vectors $\vec{k}_{1,2}$ such that $|\vec{k}_1 - \vec{k}_2| < k_L$ [see, for example, T. Müller-Seydlitz *et al.,* Phys. Rev. Lett. **78**, 1038 (1997)].

[12] We take $\omega_z/\omega_\perp = 0.1$, and we relate the interactions by $g_{1D} = \frac{1}{\pi}\frac{g_{3D}}{a_\perp^2}$ where $a_\perp = (\hbar/m\omega_\perp)^{1/2}$.

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