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Searching for Evolutions of Pure States into Mixed States in the Two-State System $K^0\bar{K}^0$

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Oscillations between two quantum states may be a sensitive probe for a loss of quantum coherence due to possible violations of quantum mechanics, e.g., caused by gravitation. We show, for the strangeness oscillations in the neutral kaon system, that there exist four experiments which uniquely determine all nine non-quantum-mechanical parameters that occur in a density matrix description using the Pauli matrix basis. [S0031-9007(98)05768-8]

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Gravitation might cause violations of quantum mechanics by influencing the coherence of the wave function, and thereby creating transitions from pure states to mixed states [1]. A decrease in coherence should manifest itself in sensitive interference experiments. A system with oscillations between two quasistable states can be described by a 2×2 density matrix, whose time development is determined by a linear system of differential equations with 16 real parameters. Quantum mechanics requires that 9 of them vanish [2].

This work shows, using the $K^0\bar{K}^0$ system as an example, that all of these 16 parameters are measurable in experiments which allow the identification of energy states as well as states of another, not exactly conserved, quantity (flavor), and which, in addition, have the ability to act differently upon states of opposite flavor quantum number. We also find a general prescription on the presentation of experimental results which allows one to express the full information on all 16 parameters.

This subject has been treated with restriction to three parameters violating quantum mechanics [2–6].

The time evolution of neutral K mesons is given by the four elements H^{ik} of the matrix H, with respect to the basis $(K^0\bar{K}^0)$ of the form $\dot{\Psi} = -iH\Psi$. Ψ has two components, the amplitudes Ψ_K and $\Psi_{\bar{K}}$, and describes a pure state. Mixed states are described by the density matrix ρ, whose time evolution is given by the *Liouville* equation

$$\dot{\rho} = -i(H\rho - \rho H^+).$$

For pure states $\rho = \Psi\Psi^+$, and, thus, $\det(\rho) = 0$. We often represent 2×2 matrices as linear combinations of the Pauli matrices σ^μ, $\mu = 0, \ldots, 3$ (with $\sigma^0 =$ unit matrix): $H = H^\mu\sigma^\mu$, (H^μ complex), $\rho = \rho^\mu\sigma^\mu$, $\dot{\rho} = \dot{\rho}^\mu\sigma^\mu$, ($\rho^\mu, \dot{\rho}^\mu$ real). A summation over multiple indices on the right-hand side of the equations that do not appear on the left-hand side, is understood. The *Liouville* equation implies the linear relation

$$\dot{\rho}^\mu = T^{\mu\nu}\rho^\nu, \tag{1}$$

with the (real) 4×4 matrix [2]

$$T = (T^{\mu\nu}) = 2\,\mathrm{Im}\,H^0 \mathbf{1}_{4\times4} + 2\begin{pmatrix} 0 & \mathrm{Im}\,H^1 & \mathrm{Im}\,H^2 & \mathrm{Im}\,H^3 \\ \mathrm{Im}\,H^1 & 0 & -\mathrm{Re}\,H^3 & \mathrm{Re}\,H^2 \\ \mathrm{Im}\,H^2 & \mathrm{Re}\,H^3 & 0 & -\mathrm{Re}\,H^1 \\ \mathrm{Im}\,H^3 & -\mathrm{Re}\,H^2 & \mathrm{Re}\,H^1 & 0 \end{pmatrix}.$$

Here, the matrix T has seven parameters, and is composed of a multiple of the unit matrix $\mathbf{1}_{4\times4}$ and of a matrix from the Lie algebra of the Lorentz transformations. This is typical for quantum mechanical time evolution where the quadratic form $\rho^\mu\rho_\mu = \det(\rho) = 0$ is left unchanged, implying pure states stay pure. The remaining nine parameters may serve to express manifestations of violations of quantum mechanics. A deviation from the symmetry $T^{0\nu} = T^{\nu 0}$, the asymmetry $T^{mn} = -T^{nm}$ ($m, n = 1, 2, 3$), or the equality of the diagonal elements would indicate such violations.

Equation (1) has the solution $\rho^\mu(t) = (e^{Tt})^{\mu\nu}\rho^\nu(0)$. The time evolution of an observable $\langle B\rangle$ (represented by the matrix B) is then calculated as a trace. Let B^μ be the Pauli components of B, $B^\mu = \mathrm{Tr}(B\sigma^\mu)/2$, then

$$\langle B\rangle = \mathrm{Tr}(B\rho) = 2B^\mu\rho^\mu(t) = 2B^\mu(e^{Tt})^{\mu\nu}\rho^\nu(0).$$

$\rho^\nu(0)$ describes the preparation of the kaons, $(e^{Tt})^{\mu\nu}$ describes the evolution from 0 to t, and B^μ describes the measurement. According to the four independent quadruplets (B^μ) and $(\rho^\nu(0))$, there are 16 different types of experiments.

We will now show that there is a choice of four experiments which uniquely determine all the values of the 16 parameters of a general matrix T. Let $T = T_Q + X$, where T_Q is known and obeys quantum mechanics with $H^2 = H^3 = 0$, and where $X = (X^{\alpha\beta})$ is small, $|X^{\alpha\beta}| \ll |H^1|$, and unknown. Then, $\langle B\rangle$ has as an approximation

$$\begin{aligned} B(t) &= 2B^\mu[(e^{T_Q t})^{\mu\nu} + X^{\alpha\beta}D^{\mu\nu}_{\alpha\beta}]\rho^\nu(0) \\ &= B_0(t) + X^{\alpha\beta}B_{\alpha\beta}(t), \end{aligned}$$

where the derivative $D^{\mu\nu}_{\alpha\beta}$ is calculated at $X^{\alpha\beta} = 0$ as follows:

$$\begin{aligned} D^{\mu\nu}_{\alpha\beta} &\equiv \frac{\partial}{\partial X^{\alpha\beta}}(e^{(T_Q+X)t})^{\mu\nu} \\ &= \left(e^{T_Q t}\int_0^t dt' e^{-T_Q t'}\frac{\partial X}{\partial X^{\alpha\beta}}e^{T_Q t'}\right)^{\mu\nu}. \end{aligned}$$

Diagonalizing $T_Q = VdW$, where $W = V^{-1}$, and where $d = (d^{\tau\sigma}) = (\delta^{\tau\sigma}\Lambda^\tau)$, we obtain

$$\begin{aligned} B_0(t) &= 2B^\mu V^{\mu\tau}W^{\tau\nu}e^{\Lambda^\tau t}\rho^\nu(0), \\ B_{\alpha\beta}(t) &= 2B^\mu V^{\mu\tau}W^{\tau\alpha}V^{\beta\varepsilon}W^{\varepsilon\nu} \\ &\quad\times[(e^{\Lambda^\varepsilon t} - e^{\Lambda^\tau t})/(\Lambda^\varepsilon - \Lambda^\tau)]\rho^\nu(0), \\ \text{and}\quad B(t) &= B_0(t) + X^{\alpha\beta}C^\kappa_{\alpha\beta}N^\kappa(t), \end{aligned}$$

where the sum over κ runs from 1 to 8. We note that

$$N^{\varepsilon+1}(t) = e^{\Lambda^\varepsilon t} \quad\text{and}\quad N^{\varepsilon+5}(t) = te^{\Lambda^\varepsilon t},$$
$$\varepsilon = 0, \ldots, 3.$$

For $\tau \neq \varepsilon$ we obtain

$$\begin{aligned} C^{\varepsilon+1}_{\alpha\beta} = 2B^\mu\rho^\nu(0)\,(&V^{\mu\tau}W^{\tau\alpha}V^{\beta\varepsilon}W^{\varepsilon\nu} \\ &+ V^{\mu\varepsilon}W^{\varepsilon\alpha}V^{\beta\tau}W^{\tau\nu})/(\Lambda^\varepsilon - \Lambda^\tau), \end{aligned}$$

otherwise ($\tau = \varepsilon$) we obtain

$$C^{\varepsilon+5}_{\alpha\beta} = 2B^\mu\rho^\nu(0)V^{\mu\varepsilon}W^{\varepsilon\alpha}V^{\beta\varepsilon}W^{\varepsilon\nu}.$$

For V (and $W = V^{-1}$), we use the limit $H^2 \to 0$ with $H^3 = 0$:

$$V = (V^{\mu\tau}) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & i & -i \\ 0 & 0 & -1 & -1 \end{pmatrix}.$$

The eigenvalues Λ^ε of T_Q are expressed by those of H as

$$\Lambda^0 = 2\,\mathrm{Im}\,\lambda_S, \qquad \Lambda^1 = 2\,\mathrm{Im}\,\lambda_L,$$
$$\Lambda^2 = i(\lambda_L^* - \lambda_S), \quad\text{and}\quad \Lambda^3 = i(\lambda_S^* - \lambda_L) = \Lambda^{2*},$$

where $\lambda_S = m_S - i\gamma_S/2$, and $\lambda_L = m_L - i\gamma_L/2$, are the eigenvalues of H.

A measurement of $B(t)$, for which $B_0(t)$ is an approximation, is thus represented as a superposition of the 8 functions $N^\kappa(t)$: $B_{\mathrm{exp}}(t) = B_0(t) + b^\kappa N^\kappa(t)$, and leads to the 8 equations $X^{\alpha\beta}C^\kappa_{\alpha\beta} = b^\kappa$, $\kappa = 1, \ldots, 8$, for the 16 unknowns $X^{\alpha\beta}$. The n measurements $B^1(t), \ldots, B^n(t)$ lead to $8n$ equations

$$M^{kl}x^l = b^k, \qquad k = 1, \ldots, 8n, \tag{2}$$

where $x^l = X^{\alpha\beta}$, $l = l(\alpha, \beta) = 1, \ldots, 16$, are the 16 unknowns consecutively labeled, and $M = (M^{kl})$ is the corresponding $8n \times 16$ matrix of the $C^\kappa_{\alpha\beta}$ values. The least squares solution of Eq. (2) for $8n > 16$, yields the unique result $x^l = (N^{-1})^{lm}(M^{km}b^{k*} + M^{km*}b^k)$, when the determinant of the 16×16 matrix $N = M^+M + (M^+M)^*$ is finite.

Modern experiments [7,8] are able to produce neutral kaons with definite and individually known strangeness in great numbers as K^0 with $(\rho^\nu(0)) \equiv (\rho_K) = (\frac{1}{2}\ 0\ 0\ \frac{1}{2})$ or as $\bar{K}^0$ with $(\rho_{\bar{K}}) = (\frac{1}{2}\ 0\ 0\ -\frac{1}{2})$. A complete set of four measurements is now obtained as follows: The semileptonic decays identify K^0 and $\bar{K}^0$, i.e., $(B^{1\mu}) = (\frac{1}{2}\ 0\ 0\ \frac{1}{2})$ and $(B^{2\mu}) = (\frac{1}{2}\ 0\ 0\ -\frac{1}{2})$, and the decay into 2π identifies K_1, i.e., $(B^{3\mu}) = (\frac{1}{2}\ \frac{1}{2}\ 0\ 0)$. Additional regeneration experiments enable one to obtain the data corresponding to a measurement of the type $(B^{4\mu}) = (\frac{1}{2}\ 0\ -\frac{1}{2}\ 0)$, the important property being $B^{42} \neq 0$. See appendix below.

The explicit calculation of the determinant $|N|$ for the set of four measurements $B^i(t)$, with $(\rho^\nu(0)) = (\rho_K)$ and $(B^\mu) = (B^{i\mu})$, $i = 1, \ldots, 4$, yields the general result

$$|N| = \frac{K}{(|\Delta\lambda|^4\Delta\gamma\Delta m)^4},$$

(with $K = 3^3 \times 7/2^{26}s^8 = 2.8 \times 10^{-6}s^8$, $\Delta\lambda = \lambda_S - \lambda_L$). This represents a finite value if $\Delta\gamma = \gamma_S - \gamma_L \neq 0$ and $\Delta m = m_L - m_S \neq 0$, or for arbitrary values of the Λ^ε, as long as they are pairwise different. Numerical difficulties in calculating N^{-1} should not arise, as the condition number of N in a typical numerical example is as low as 40.

These results show the possibility of the determination of all the 16 elements of the matrix X, and thus of T, starting from the measured amplitudes b^k of the four experiments mentioned. [A set of experiments where none is of the type $(B^{4\mu})$ is insufficient, as the corresponding determinant $|N|$ vanishes.]

From a purely phenomenological viewpoint, we can imagine nine experimentally distinguishable manifestations of quantum mechanics violations. The corresponding empirical information is contained in the values of the b^k.

A numerical study shows that the statistical significance of the resulting x^l values is greatly enhanced by the use of data from more than four experiments.

Present experimental bounds [5] for the three violating parameters, as proposed in Ref. [2], are in the range of model expectations [2,6] of $O(10^{-19}$ GeV). Similar bounds for all nine violating parameters using K^0 and $\bar{K}^0$ as initial states, would require only about 5 times the number of events observed in Ref. [5].

Experiments at DAΦNE [8] with correlated neutral kaon pairs will open a new field where, e.g., unconventional aspects of conservation of energy and angular momentum (as pointed out in Ref. [4]) or irreducible two-particle parameters [6] arise. A treatment of the two-state systems using the complete set of the 16 parameters might provide more basic insight into the phenomenology of loss of coherence.

Appendix on regeneration.—The observation of K_1 mesons behind a suitable regenerator constitutes an experiment described by (B^μ) having the element $B^2 \neq 0$. Let ρ^ν_{inc} be the Pauli components of neutral kaons in front of an infinitely thin sheet of matter with an areal density of nuclei n. The components ρ^μ_r of the kaons having traversed the sheet are then given by $\rho^\mu_r = L^{\mu\nu}\rho^\nu_{\text{inc}}$ with

$$(L^{\mu\nu}) = \exp\left(-n\frac{(\bar{\sigma} + \sigma)}{2}\right)\begin{pmatrix} C & 0 & 0 & S \\ 0 & c & s & 0 \\ 0 & -s & c & 0 \\ S & 0 & 0 & C \end{pmatrix},$$

where $C = \cosh(n\Delta/2)$, $S = \sinh(n\Delta/2)$, $c = \cos(\Omega n)$, $s = \sin(\Omega n)$ with $\Delta = \bar{\sigma} - \sigma$, and $\Omega = 2\pi\hbar\,\mathrm{Re}[f(0) - \bar{f}(0)]/p$. Here, $f(0)$ is the forward scattering amplitude, σ is the total cross section for kaons, and p is the kaon momentum. The bar refers to antikaons. Combining this matrix with $(B^\mu) = (\frac{1}{2}\ \frac{1}{2}\ 0\ 0)$ gives $B^\nu_r = B^\mu L^{\mu\nu}$ which contains an element $B^2_r \neq 0$ if $s \neq 0$.

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