Enhancement of Coherent Emission by Energy-Phase Correlation in a Bunched Electron Beam

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(Received 16 October 1997)

A radio-frequency (rf) modulated electron beam passing through a magnetic undulator emits coherent radiation at harmonics of the rf with a phase which depends on the electron drift velocity. Treating the bunches as ensembles of electrons, each with its energy and phase, the radiated field is calculated as a sum over the particle distribution in the phase space. At long wavelengths a proper correlation between the energy and phase distributions of the electrons in the bunch can be exploited to lock the radiated field in phase, resulting in a significant enhancement of the coherent emission. [S0031-9007(98)05672-5]

PACS numbers: 41.60.Cr

In recent years the coherent emission from a radio-frequency (rf) modulated electron beam at wavelengths comparable to the electron bunch length has been the object of renewed interest because of its practical relevance in the design of compact free electron generators of coherent radiation [1–4]. Issues like the dependence of the emitted radiation on the bunch shape [5], the observation of emission at discrete frequencies which are harmonics of the fundamental rf, and the limits on the ultimate linewidth achievable on the individual harmonics [6] have been addressed both theoretically and experimentally. Most physical models and calculations assume monoenergetic electron bunches with a given longitudinal charge density distribution, or a distribution in electron energy independent of the longitudinal position in the bunch. In reality the longitudinal phase-space distribution of the electron bunches generated in rf accelerators shows, in most cases [3], a strong correlation between energy and phase, which has to be taken into account when calculating the field radiated in a magnetic undulator or in another suitable coupling structure.

In Ref. [1] the physical model utilized to calculate the coherent emission is based upon the following assumptions: (i) A monoenergetic beam of normalized energy γ is assumed. (ii) Electron bunches, regularly spaced at the rf period $T_{\rm rf}$, move along z with average drift velocity $\beta_z = \sqrt{1 - (1 + K^2)/\gamma^2}$ through a purely sinusoidal undulator magnetic field $\vec{B} = B_0 \cos(k_u z)\hat{y}$, where $k_u = 2\pi/\lambda_u$, λ_u is the undulator period and $K = eB_0\lambda_u/(2\sqrt{2}\pi m_0 c^2)$ is the so-called undulator parameter governing the amplitude of the electron oscillations in the plane perpendicular to $\vec{B}$. (iii) The transverse motion of the electron bunches can be described by a transverse current density $\vec{J}_T$, which is localized inside the undulator of length L in a rectangular waveguide of cross section $\Sigma = ab$, and has a temporal profile which can be expanded in a series of harmonics of the fundamental $\omega_l = 2\pi l/T_{\rm rf}$, where l is a positive integer.

Each harmonic current source plays the role of an external driving term in the inhomogeneous Maxwell's wave equation leading to an amplitude of the normalized radiated field coefficient in a given $\mathrm{TE}_{0,n}$ waveguide mode [1]:

$$A_{l,0,n} = -\frac{Z_0}{\beta_g} I_p \frac{C_l}{2} \frac{K}{\beta\gamma} \frac{L}{\sqrt{ab}} F \frac{\sin(\theta_l/2)}{(\theta_l/2)} i e^{i\theta_l/2} \tag{1}$$

where

$$\theta_l = \left(\frac{\omega_l}{c\beta_z} - k_u - k_{0,n}\right) L \tag{2}$$

is the usual definition of the phase shift parameter describing the free electron laser (FEL) resonance condition in a waveguide [7], $Z_0 = 377\ \Omega$ is the free space impedance, I_p is the peak current in the bunch, β_g is the normalized group velocity of the waveguide mode at the frequency ω_l with wave vector $k_{0,n}$, F is a form factor describing the overlapping between the e-beam transverse distribution and the waveguide mode, and C_l is the coefficient of the Fourier expansion of the normalized bunch profile at the lth harmonic as defined in [1]. Radiation is emitted at harmonics of the fundamental rf in a frequency band defined by the line-shape function $\sin(\theta_l/2)/(\theta_l/2)$. The phase factor $e^{i\theta_l/2}$ in (1) is of fundamental nature since, like the line-shape function, it does not depend on the specific physical model but directly derives from the integration of the $\vec{J} \cdot \vec{E}$ product over the interaction volume $V = \Sigma \cdot L$, which represents the rate of energy exchange between the beam current and the radiation field. It has the important physical meaning of showing that the external driving current and the radiated field are no longer in phase when the electron drift velocity does not match the resonance condition $\theta = 0$. If each electron bunch is treated as a collection of N_e particles of charge q, each with its energy γ_j and position or phase ψ_j along z, the total radiated field can be simply calculated as a sum over the particle distribution:

$$A_{l,0,n} = -\frac{Z_0}{\beta_g} \frac{q}{T_{\rm rf}} \frac{KL}{\sqrt{ab}} F \times \sum_{j=1}^{N_e} \frac{1}{\beta_j\gamma_j} \frac{\sin(\theta_j/2)}{(\theta_j/2)} i e^{i[(\theta_j/2) + l\psi_j]}, \tag{3}$$

where the Fourier components $I_p C_l$ of regularly spaced charges represented by a train of delta functions are independent of the harmonic number l and have been calculated as $2q/T_{\rm rf}$, $\theta_j = [(\omega_l/c\beta_{zj}) - k_u - k_{0,n}]L$ describes the dependence of the phase shift from resonance on the electron drift velocity, and $\psi_j = \omega_{\rm rf} t_j$ is the phase at the fundamental frequency $\omega_{\rm rf}$ of the jth charge injected at the time t_j with respect to a reference charge injected at $t = 0$ with velocity β_{z0}. The radiated power at the lth rf harmonic is then calculated as

$$P_l = \frac{\beta_{gl}}{2Z_0} |A_{l,0,n}|^2 , \tag{4}$$

and the total power of the coherent spontaneous emission (CSE) is obtained by summing (4) over the number of harmonics in the FEL band. Although this model is essentially one dimensional, the effect of transverse beam emittance can be taken into account, in a first approximation, by expressing the longitudinal drift velocity as $\beta_{zj} = \sqrt{1 - (1 + K^2 + \gamma_j^2\theta_j^2)/\gamma_j^2}$, with $\theta_j^2 = x_j'^2 + y_j'^2$, where θ_j is the average angle between the electron trajectory and the undulator axis, which can be written in terms of the angular deviations $x_j' = dx/dz$ and $y_j' = dy/dz$. Similarly the form factor F can be calculated as the integral of the mode profile over the particle distribution in the transverse phase space.

It is evident from (3) that constructive interference is achieved when the electrons are distributed in the phase space (ψ, γ) as close as possible to the "phase-matching" curve given by

$$\left(\frac{\theta_l(\gamma)}{2}\right) + l\psi = \Phi_l , \tag{5}$$

where the dependence on l and γ is explicitly indicated for clarity, and Φ_l is a constant phase at the harmonic l. Substituting (2) into (5) and remembering that $\psi = 0$ for $\beta_z = \beta_{z0}$, the phase-matching relation can be reformulated as

$$\psi = -\pi \frac{L}{\lambda_{\rm rf}} \left(\frac{1}{\beta_z(\gamma)} - \frac{1}{\beta_{z0}} \right), \qquad \text{where } \lambda_{\rm rf} = cT_{\rm rf} . \tag{6}$$

This relation is of general validity since it is independent of the harmonic number l and explicitly shows the dependence on the drift velocity. An angular spread σ' of the electrons around the undulator axis will imply a spread in the drift velocity and, hence, a spread in phase around the phase-matching curve. Constructive interference will not be affected by the beam emittance as long as the phase spread is much less than half a period of the wave, which from (6) leads to the condition $L\sigma'^2/\beta_{z0}^3 \ll \lambda/2$, in agreement with the usual requirement that the beam emittance be smaller than the FEL operating wavelength. It is also clear that the energy-phase correlation method is most effective at low energy. The phase interval per unit relative energy variation is indeed proportional to the inverse square of the energy:

$$\Delta\psi/(\Delta\gamma/\gamma) = \pi \frac{L}{\lambda_{\rm rf}} \frac{1 + K^2}{\beta_z^3 \gamma^2} . \tag{7}$$

The useful range of γ over which Eqs. (5) and (6) can be satisfied to generate coherent radiation with constructive interference is given by the interval of energy corresponding to the main lobe in the line-shape function of the FEL resonance:

$$\gamma_1 < \gamma < \gamma_2 \quad \text{such that } \pi > \frac{\theta_l(\gamma)}{2} > -\pi . \tag{8}$$

From (5) one observes that such an interval of γ results in a phase interval $2\pi/l$ corresponding to one wavelength at the operating frequency ω_l.

The longitudinal phase-space distribution plays a crucial role in the enhancement or suppression of CSE. We have calculated the effect of phase matching for the experimental parameters of the ENEA millimeter-wave FEL facility in Frascati [8]. Electron energy, waveguide dimensions, and undulator parameters in this device are chosen to operate at the so-called "zero slippage" condition which provides a broad-band emission in the frequency range between 60 and 150 GHz. In Fig. 1(a) we show an ensemble of 1250 macroparticles, each with charge $q = 0.1$ pC, distributed according to (6) in the phase-space region bounded by a phase shift of $\pm\lambda/4$ around the upper frequency of 150 GHz, within which the radiated field keeps the same sign to generate constructive interference. The corresponding bunch profile, given by the histogram as a function of phase of the above distribution, is shown in Fig. 1(c). The dramatic effect of

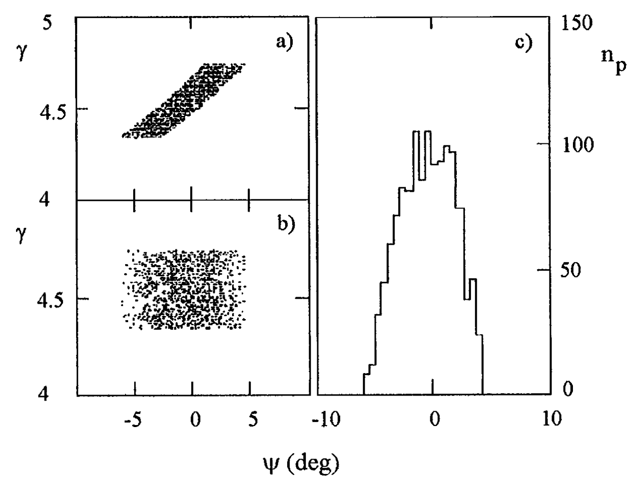

FIG. 1. (a) Distribution of 1250 particles in the phase space (ψ, γ) showing a correlation between energy and phase according to (6) with $\beta_{z0} = 0.95$. (b) Phase-space distribution of particles having the same longitudinal profile as in case (a), but a random distribution of energy in the interval $[\gamma_1, \gamma_2]$ defined by (8). (c) Histogram of the number of particles n_p as a function of phase (longitudinal bunch profile) of both distributions (a) and (b).

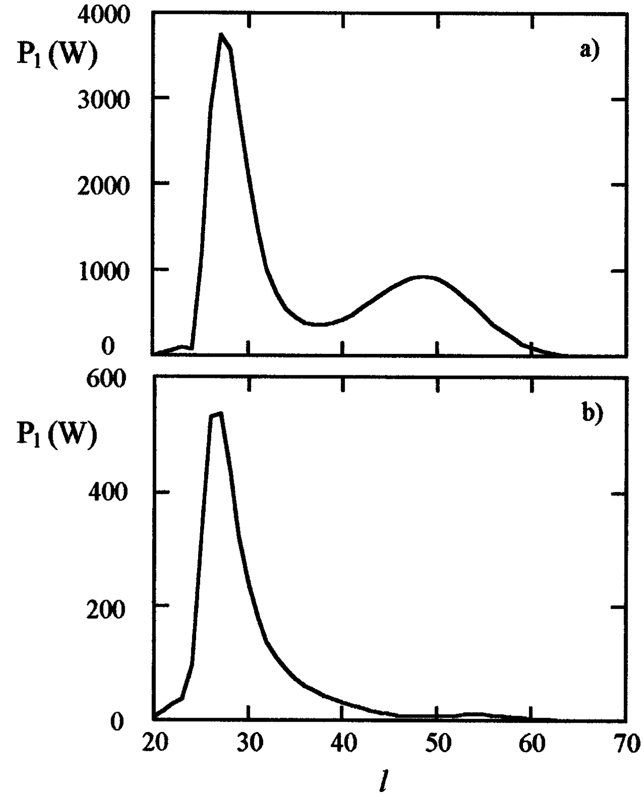

FIG. 2. CSE power as a function of the harmonic number l of the fundamental frequency $\nu_{\mathrm{rf}} = 3$ GHz for the correlated (a) and uncorrelated (b) energy-phase distributions of Fig. 1, respectively. Operating parameters: $\lambda_u = 2.5$ cm, $N = 16$, $K = 1$, $\mathrm{TE}_{0,1}$ mode in rectangular waveguide $\Sigma = 1.067 \times 0.432$ cm^2.

energy-phase correlation becomes evident by comparing the total radiated power by this distribution of charges with that of a bunch with the same longitudinal profile as in the correlated case, but a random distribution of energy in the interval $[\gamma_1, \gamma_2]$ [see Fig. 1(b)]. The spectral distribution of power for the correlated and uncorrelated cases is shown in Figs. 2(a) and 2(b), respectively. Coherent spontaneous emission occurs in both cases due to the short bunch duration. However, in spite of the identical histograms as a function of phase of the two distributions, the correlated case shows an enhancement of the high frequency components in the spectrum, with a total radiated power of 36 kW compared to the 3.6 kW of the uncorrelated case. The error on the total radiated power due to the small number of particles used (1250) compared to the actual number of electrons in the bunch (about 10^9) has been estimated to be 1% and 5% for the correlated and uncorrelated cases, respectively.

A strong sensitivity of the CSE power to the rf settings in the accelerator components was reported in several experiments [2,9,10]. Off peak acceleration and bunch compression were exploited in [2] to enhance CSE of about 2 orders of magnitude. Bunch compression and the presence of a structure in the bunch profile do indeed affect

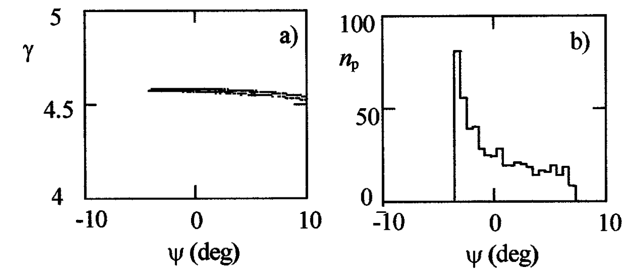

FIG. 3. PARMELA simulation of the phase-space distribution (a) and longitudinal bunch profile (b) at the linac output.

the amplitude of the Fourier coefficients in (1). Further investigations on the two-dimensional distribution of the bunched electrons in the longitudinal phase space would certainly help to clarify whether interference effects related to the phase-matching condition also occurred in the reported experiments.

The manipulation of the particle distribution in the phase space can provide an extremely powerful tool for the realization of high efficiency generators of coherent radiation in the mm-wave and far-infrared regions and at even shorter wavelengths. To test the feasibility of a device suitable for a systematic investigation of energy-phase correlation effects, we have run a computer simulation based on PARMELA code [11] to describe an accelerating structure composed of a $3 + \frac{1}{2}$ cell beta-graded self-focusing rf linac operating at 3 GHz. The linac is injected by a 13 kV, 1 A electron gun and is followed by a "phase-matching section" (PMS) placed 2.5 cm downstream of the linac output. The PMS is made of two coupled cavities, each 5 cm long, operating in the π mode. The distribution of the bunched electrons in the phase space at the output of the PMS can be changed by varying the phase and amplitude of the rf field driving the PMS with respect to the linac. The calculation utilizes 3000 particles leaving the gun with a charge per particle of 0.1 pC and accounts for space charge effects. About 1250 particles reach the linac output with a maximum kinetic energy of 1.82 MeV, a total average current of 0.4 A, and an rms emittance of 5π mm mrad. To obtain the positive slope of the phase-matching curve in the phase space shown in Fig. 1(a), the drift space and the phase shift of the PMS are set so as to have the reference

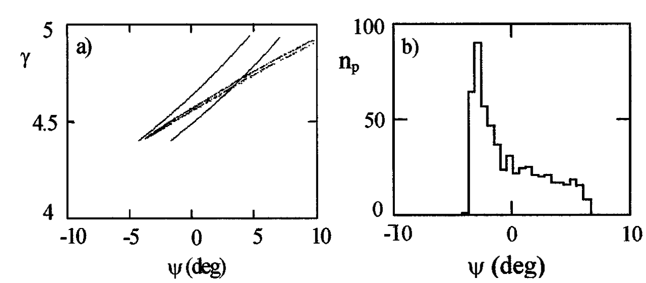

FIG. 4. PARMELA simulation of the phase-space distribution (a) and longitudinal bunch profile (b) at the PMS output.

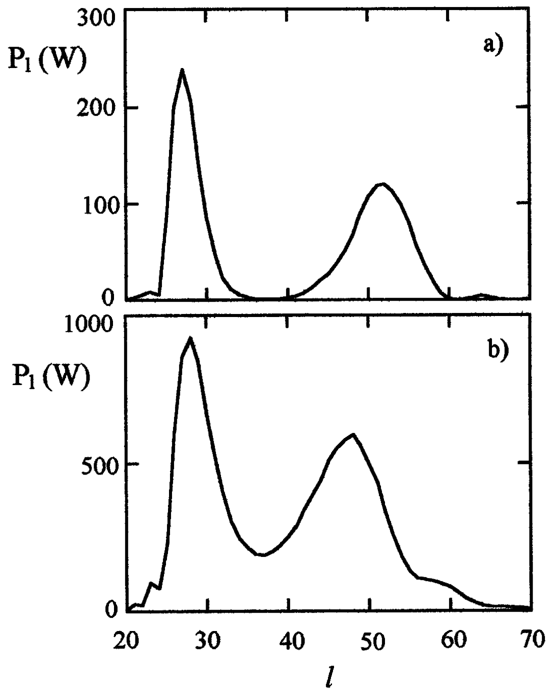

FIG. 5. CSE power as a function of the harmonic number l at the linac output (a) and at the PMS output (b), respectively, with operating parameters as in Fig. 2.

electron passing through the center of the first cavity in the PMS with a phase close to zero. In this way higher energy electrons arriving first at the PMS are decelerated, while lower energy electrons are accelerated and emerge first from the PMS. The main difference of this technique, when compared to bunch compression methods [12], is that bunch manipulation in the PMS results in a strong correlation between energy and phase of the electrons, which are distributed in a narrow strip over a relatively wide phase interval with a positive energy vs phase slope. The expected phase-space distribution and bunch profile of the particles at the linac output are shown in Figs. 3(a) and 3(b), respectively, while the expected phase-space distribution and bunch profile after manipulation at the PMS output are shown in Figs. 4(a) and 4(b), respectively. The corresponding power spectra are also plotted in Figs. 5(a) and 5(b) showing again a strong increase in the total CSE power from about 2 to 14 kW. It is worth noticing that at a given frequency the particles contributing to constructive interference are those lying between two phase-matching curves offset by $\pm\lambda/4$ as shown, for example, in Fig. 4(a) for a frequency of 150 GHz. In this case we have verified that the spread in phase induced by the beam emittance does not affect the CSE spectrum and total radiated power.

By a simple analytical approach we have shown that energy-phase correlation in a bunched beam can significantly increase CSE. The design of a device suitable for the investigation of this effect has been studied and supported by numerical simulations. The suggested technique is particularly useful in broad-band millimeter-wave radiators like waveguide FELs operating at zero slippage, but can, in principle, be extended to shorter wavelength devices.

We would like to thank our colleagues G. Dattoli and A. Renieri for many stimulating discussions on this subject.

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[1] A. Doria, R. Bartolini, J. Feinstein, G.P. Gallerano, and R.H. Pantell, IEEE J. Quantum Electron. **29**, 1428–1436 (1993).
[2] K.W. Berryman, E.R. Crosson, K.N. Ricci, and T.I. Smith, Nucl. Instrum. Methods Phys. Res., Sect. A **375**, 526 (1996).
[3] P. Kung, H. Lihn, H. Wiedemann, and D. Bocek, Phys. Rev. Lett. **73**, 967 (1994).
[4] J.F. Schmerge, J. Lewellen, Y.C. Huang, J. Feinstein, and R.H. Pantell, IEEE J. Quantum Electron. **31**, 1166 (1995).
[5] D.A. Jaroszynski, R.J. Bakker, C.A.J. van der Geer, D. Oepts, and P.W. van Amersfoort, Phys. Rev. Lett. **71**, 3798 (1993).
[6] G.P. Gallerano, A. Doria, E. Giovenale, and G. Messina, Nucl. Instrum. Methods Phys. Res., Sect. A **358**, 78 (1995).
[7] A. Doria, G.P. Gallerano, and A. Renieri, Opt. Commun. **80**, 417 (1991).
[8] F. Ciocci, R. Bartolini, A. Doria, G.P. Gallerano, E. Giovenale, M.F. Kimmitt, G. Messina, and A. Renieri, Phys. Rev. Lett. **70**, 928 (1993).
[9] A.F.G. van der Meer, R.J. Bakker, C.A.J. van der Geer, D. Oepts, and P.W. van Amersfoort, Nucl. Instrum. Methods Phys. Res., Sect. A **331**, 282 (1993).
[10] R. Prazeres, J.M. Ortega, F. Glotin, D.A. Jaroszynski, and O. Marcouillé, Phys. Rev. Lett. **78**, 2124 (1997).
[11] Phase and Radial Motion in Electron Linear Accelerators, L.M. Young, Los Alamos National Laboratory Report No. LA-UR-96-1835, 1996.
[12] M. Borland, M.C. Green, R.H. Miller, L.V. Nelson, E. Tanabe, J.N. Weaver, and H. Wiedemann, in *Proceedings of the 1990 Linear Accelerator Conference, Albuquerque, NM* (SLAC Internal Report No. SLAC-PUB-5333, 1990).