Why the Naive Quark Model Can Yield a Good Account of the Baryon Magnetic Moments

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The chiral quark model suggests that the baryon quark sea is negatively polarized. This modifies the spin structure as given by the naive quark model and agrees with experimental data. However, for the magnetic moments, there is significant cancellation between the contributions from this sea spin polarization and the orbital angular momentum so that effectively the moments are given by the valence constituent quarks alone, as in the naive quark model. [S0031-9007(98)05726-3]

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Ever since the discovery [1] that the proton spin content is very different from that given by the naive quark model (NQM), one of the puzzles has been: why is the same naive quark spin structure capable of giving such a good account of the baryon magnetic moments? In this paper we shall suggest, in the context of the chiral quark model (χQM), a qualitative explanation.

The basic idea of χQM [2] is that the nonperturbative QCD phenomenon of chiral symmetry breaking (χSB) takes place at distance scale significantly smaller than that of color confinement. Thus in the interior of a hadron, but not so small a distance that perturbative QCD is applicable, the effective degrees of freedom are the constituent quarks and the χSB Goldstone bosons (GBs). Prior chiral quark model study has indicated that the various nucleon flavor and spin puzzles can be understood by the presence of a quark sea which is perturbatively generated by valence quark's emissions of internal GBs [3–6]. This model can naturally account for the $\bar{u}$-$\bar{d}$ asymmetry as measured by the deviation from the Gottfried sum rule [7] and by the Drell-Yan processes [8], as well as a strange quark content consistent with the various phenomenological determinations [9]. The axial coupling of GBs and constituent quarks can modify the spin content because the GB emission by a valence quark flips the quark spin direction:

$$q_{\pm} \longrightarrow q'_{\mp} + GB \longrightarrow q'_{\mp} + (\bar{q}'q)_0 . \quad (1)$$

The subscripts denote the helicity states. We shall call the three quarks (in S-wave state) of the NQM as the *valence quarks* and all the other quarks (and antiquarks) broadly as the *quark sea*. The processes in (1) lead to a quark sea $(q'\bar{q}'q)$ which is polarized (as given by $q'_{\mp}$) in the opposite direction to the baryon spin. (At the leading perturbative order, the antiquark $\bar{q}'$ and q in the sea are not polarized because they are produced through the spin-zero GB channels [10,11].) In this way, we find that the quark contribution to the baryon spin is substantially reduced from that of the NQM, in agreement with the phenomenological result obtained by several generations of deep inelastic polarized lepton-nucleon scattering experiments [1,12].

This reduction of the quark polarization also implies a significant decrease of the quark-spin contribution to baryon magnetic moments. It is then puzzling why the original quark model (without a polarized quark sea) can yield such a good description of the magnetic moments. Our χQM explanation is that the quark sea must also carry a significant amount of orbital angular momentum. In fact, angular momentum conservation implies that the final state quark q' and $(\bar{q}'q)$ in the GB emission process (1) must be in a relative P-wave state. This orbital angular momentum, which is parallel to the baryon spin, makes a positive contribution to the baryon magnetic moment and thus compensates the quark-spin's reduction.

When we separate the spin and the orbital angular momentum contributions, we are using the nonrelativistic approximation, which can provide us with an intuitive physical picture of the hadron structure. As we shall comment on at the end of the paper, existent chiral quark field theory calculations also support our explanation.

From the SU(6) wave function of NQM we can calculate the number of valence quarks with polarization $\sigma_Z = \pm 1$ (denoted by particle names with subscript $\pm$). In the case of the proton with valence quarks (uud), we have

$$u_{v+} = \frac{5}{3}, \quad u_{v-} = \frac{1}{3}, \quad d_{v+} = \frac{1}{3}, \quad d_{v-} = \frac{2}{3} . \quad (2)$$

The quark contribution to the baryon spin being the sum of the quark and antiquark polarizations $\Delta q = \Delta_q + \Delta_{\bar{q}} = (q_+ - q_-) + (\bar{q}_+ - \bar{q}_-)$, and because there are no antiquarks and strange valence quarks, we have

$$\Delta u_v = \frac{4}{3}, \quad \Delta d_v = -\frac{1}{3}, \quad \Delta s_v = 0 , \quad (3)$$

which makes up the total proton spin, $\Delta \Sigma_v = \Delta u_v + \Delta d_v + \Delta s_v = 1$. When it comes to the quark spin contribution to the baryon magnetic moment, $\mu(B) = \sum_q (\tilde{\Delta}q)_B \mu_q$ with $\tilde{\Delta}q = \Delta_q - \Delta_{\bar{q}}$ (as antiquarks have opposite charges). In the NQM with $\bar{q}_+ = \bar{q}_- = 0$ (thus $\tilde{\Delta}q_v = \Delta q_v$), we have, from Eq. (3):

$$\mu(p)_v = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d = \left(\frac{e}{2M} \right), \quad (4)$$

where we have used $\mu_u = -2\mu_d = e/3M$ reflecting the mass relation of $M_u = M_d \equiv M$. The results for octet baryons yield a good account of the measured moments with $\mu_d \simeq -0.9$ n.m. (nuclear magneton) corresponding to a set of constituent quark mass values close to those used in fitting other hadron properties [13].

Ever since the publication by EMC of their experimental result [1], it is known that the proton spin content is quite different from that given by the NQM of Eq. (3),

$$\Delta u_{\text{expt}} = 0.82 \pm 0.06, \qquad \Delta d_{\text{expt}} = -0.44 \pm 0.06,$$

$$\Delta s_{\text{expt}} = -0.11 \pm 0.06, \qquad \Delta \Sigma_{\text{expt}} = 0.27 \pm 0.11, \tag{5}$$

showing clearly that a good portion of the proton spin arises from something other than quark spins [14].

Besides the problem of understanding why the valence quarks can give by themselves a good account of the baryon magnetic moments, we have another related puzzle. Suppose we make the *ad hoc* assumptions that the magnetic structure is still given entirely by the quark spins and that antiquarks are not polarized, as done in Ref. [15], $\mu(B) = \sum_q (\Delta q)_B \mu_q$. Even though the baryon spin content is significantly different from that given by the valence quarks: $\Delta q_{\text{expt}} \neq \Delta q_v$, one finds that Δq_{expt} can also lead to a good description of $\mu(B)$. Namely, somehow, we get $\sum_q \Delta q_v \mu_q \simeq \sum_q \Delta q_{\text{expt}} \mu'_q$. This, however, requires a $\mu'_d \simeq 1.4$ n.m.—an approximately 50% shift of the effective quark moments and masses. Thus we have the puzzle that in some way both Δq_v and Δq_{expt} can yield a good account of the baryon magnetic moments. But, only for (the phenomenologically incorrect) Δq_v the fit leads to a set of correct quark masses.

We now discuss the χQM resolution of these puzzles. As explained in the introduction, we need to calculate the spin and magnetic moment contributions by the quark sea as generated by the internal GB emission processes of the type in (1). We shall be working, for simplicity, in a χQM with a flavor-U(3) symmetry broken down to SU(3) × U(1): the quark and GB form degenerate multiplets, but with distinctive couplings for the octet GBs and the singlet η' meson: $g_1/g_8 \equiv \zeta \neq 1$. (In fact from our prior study [4] we expect $\zeta \simeq -1$ in this symmetric limit.) The transition probability for the process of $q_\pm \rightarrow q'_\mp + GB$ is parametrized to be

$$P(u \rightarrow d + \pi^+) = P(u \rightarrow s + K^+) = a$$

$$P(u \rightarrow u + \pi^0) + P(u \rightarrow u + \eta) + P(u \rightarrow u + \eta')$$

$$= \frac{1}{3}(2 + \zeta^2)a. \tag{6}$$

For any initial state q, the total transition probability for $(q \rightarrow \text{all})$ is simply

$$P(q) = \frac{1}{3}(8 + \zeta^2)a. \tag{7}$$

All calculations of the various angular momentum and magnetic moment contents of the quark sea involve a "three-part convolution": the contributions by a single re-

action are to be multiplied by the transition probability of the reaction Eq. (6) and by the number of initial valence quarks of Eq. (2). To calculate the spin polarization of the sea, the quantity for an individual process in this convolution involves a count of ± 1 (in units of $\frac{1}{2}\hbar$) for the two helicity states multiplied by ± 1 for the creation or destruction of a particular quark flavor, etc. Keeping in mind that $\Delta_{\bar{q}} = 0$ because to this order $\bar{q}_+ = \bar{q}_-$ in the sea, we obtain

$$\Delta u_{\text{sea}} = -\frac{37 + 8\zeta^2}{9}a, \qquad \Delta d_{\text{sea}} = -\frac{2 - 2\zeta^2}{9}a,$$

$$\Delta s_{\text{sea}} = -a. \tag{8}$$

Their sum is the total spin polarization of the quark sea:

$$\Delta\Sigma_{\text{sea}} = -\frac{2}{3}(8 + \zeta^2)a = \Delta\sigma P(q)\Delta\Sigma_v. \tag{9}$$

Namely, it is the product of the helicity change per reaction regardless of quark flavor $\Delta\sigma = -2$, the total transition probability Eq. (7), and the number of initial valence quarks weighted by the spin directions (hence effectively the total valence quark polarization $\Delta\Sigma_v = 1$). By taking parameters such as $a \simeq 0.1$ and $\zeta \simeq -1$ one can then get a fair account [4] of the observed spin structure (5). This includes the reduction of the nucleon axial vector coupling g_A from $5/3$ to around 1.2. All these changes from the NQM values are interpreted as the renormalization effects due to the quark sea.

The sea quark spin contribution to the proton magnetic moment is given by

$$\mu(p)_{\text{spin}} = \Delta u_{\text{sea}}\mu_u + \Delta d_{\text{sea}}\mu_d + \Delta s_{\text{sea}}\mu_s$$

$$= -\frac{7 + 2\zeta^2}{3}a\left(\frac{e}{2M}\right) \equiv \kappa_{\text{spin}}\left(\frac{e}{2M}\right). \tag{10}$$

It is easy to check that for octet baryons in general, because of the SU(3) symmetric nature of the calculation, we have $\mu(B)_{\text{spin}} = \kappa_{\text{spin}}\mu(B)_v$. This explains why $\mu(B) = \mu(B)_v + \mu(B)_{\text{spin}} = (1 + \kappa_{\text{spin}})\mu(B)_v$ can be fitted with Δq_{expt} by a simple rescaling of the effective quark moments as $\Delta q_{\text{expt}} \simeq \Delta q_v + \Delta q_{\text{sea}}$.

This change of angular momentum $\Delta\sigma \times \frac{1}{2} = -1$ due to quark spin flip in reaction (1) must be compensated by a final-state orbital angular momentum. We shall describe this orbital motion of the χQM quark sea as being due to the rotational motion of the two bodies in (1). In their center-of-mass frame (i.e., the rest frame of the initial valence quark), the orbital angular momentum is simply given by $\mathbf{l} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{r}$ and $\mathbf{p}$ are the relative displacement and momentum vectors: $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$, with $\mathbf{r}_1 = \frac{m_2}{m_1 + m_2}\mathbf{r}$, etc. The hadronic matrix element of this operator can be evaluated, even without the explicit knowledge of the baryon wave function, because angular momentum conservation requires that

$$\langle l_Z \rangle = 1, \tag{11}$$

so as to compensate the quark spin change. The total orbital angular momentum of the sea can be calculated in the same way as the total spin polarization of Eq. (9):

$$\langle L_Z \rangle = \langle l_Z \rangle P(q) \Delta\Sigma_v = \frac{1}{3}(8 + \zeta^2)a. \qquad (12)$$

Thus, according to χQM, the proton spin is built up from quark spin $\Delta\Sigma = \Delta\Sigma_v + \Delta\Sigma_{\text{sea}}$ and orbital angular momentum:

$$\frac{1}{2}\Delta\Sigma + \langle L_Z \rangle = \frac{1}{2}. \qquad (13)$$

Namely, the NQM spin sum $\Delta\Sigma_v = 1$ is redistributed from the valence quarks to the spin and orbital angular momenta of the quark sea: $\Delta\Sigma_{\text{sea}}$ and $\langle L_Z \rangle$, which is constrained by the angular momentum conservation condition:

$$\frac{1}{2}\Delta\Sigma_{\text{sea}} + \langle L_Z \rangle = 0, \qquad (14)$$

as seen in Eqs. (9) and (12).

We now perform the three-part calculation of the orbital angular momentum contribution to the magnetic moment. The orbital moment of each process $\mu(q_\pm \rightarrow q'_\mp + GB)$ is

$$\mu(q_+ \rightarrow q'_-)_L = \frac{e_{q'}}{2M}\langle l_{qZ} \rangle + \frac{e_q - e_{q'}}{2\tilde{m}}\langle l_{GB,Z} \rangle, \quad (15)$$

where (l_q, l_{GB}) and $(M, \tilde{m})$ are the orbital angular momenta and masses of quark and GB, respectively. The one unit of angular momentum in (11) is shared by the two bodies:

$$\langle l_{qZ} \rangle = \frac{\tilde{m}}{M + \tilde{m}} \quad \text{and} \quad \langle l_{GB,Z} \rangle = \frac{M}{M + \tilde{m}}. \quad (16)$$

The result (15) is then multiplied by the probability for such a process to take place, to yield the magnetic moment due to all the transitions starting with a given valence quark:

$$[\mu(q_\pm \rightarrow)_L] = \pm\bigg[\mu(q_+ \rightarrow q'_-)_L + \mu(q_+ \rightarrow q''_-)_L$$
$$+ \frac{2 + \zeta^2}{3}\mu(q_+ \rightarrow q_-)_L\bigg]a$$
$$= \pm\frac{9M^2 + (\zeta^2 - 1)\tilde{m}^2}{3\tilde{m}(M + \tilde{m})}a\bigg(\frac{e_q}{2M}\bigg). \quad (17)$$

The last step is to multiply the valence-quark numbers, Eq. (2). Thus for a baryons $B = (q_1 q_1 q_2)$ we have $\mu(B)_{\text{orbit}} = \frac{4}{3}[\mu(q_{1+} \rightarrow)_L] - \frac{1}{3}[\mu(q_{2+} \rightarrow)_L]$. In particular,

$$\mu(p)_{\text{orbit}} = \frac{9M^2 + (\zeta^2 - 1)\tilde{m}^2}{3(M + \tilde{m})\tilde{m}}a\bigg(\frac{e}{2M}\bigg)$$

$$\equiv \kappa_{\text{orbit}}\bigg(\frac{e}{2M}\bigg). \quad (18)$$

Adding up the components $\mu(B) = \mu(B)_v + \mu(B)_{\text{spin}} + \mu(B)_{\text{orbit}}$ of Eqs. (4), (10), and (18), we have $\mu(p) =$

$(1 + \kappa_{\text{spin}} + \kappa_{\text{orbit}})(\frac{e}{2M})$. The general result for octet baryon is

$$\mu(B) = (1 + \kappa_{\text{spin}} + \kappa_{\text{orbit}})\mu(B)_v. \qquad (19)$$

This means that quark sea contributions can be absorbed by an overall rescaling of quark magnetic moments. Because we are performing a flavor SU(3) symmetric calculation, the magnetic moment D/F ratio is not altered. Consequently, all baryon moment *ratios* are unchanged from their SU(6) limit values, e.g., $\mu_p/\mu_n = -3/2$, etc. This necessarily requires that the quark sea modification be proportional to the original NQM values.

For the principal enigma of why can the valence quarks alone yield a good account of the magnetic moments, the χQM offers a simple explanation: the contributions from the orbital and spin angular momenta of the quark sea have opposite signs, Eqs. (10) and (18):

$$\kappa_{\text{spin}} = -\frac{7 + 2\zeta^2}{3}a,$$

$$\kappa_{\text{orbit}} = \frac{9M^2 + (\zeta^2 - 1)\tilde{m}^2}{3(M + \tilde{m})\tilde{m}}a. \qquad (20)$$

This, of course, is intimately connected to the fact that the orbital and spin alignments of the sea must be opposite to each other because of angular momentum conservation, Eq. (14). In particular, for ζ in the range of $(-1, 0)$, we can have

$$\kappa_{\text{spin}} + \kappa_{\text{orbit}} \simeq 0 \quad \text{for} \quad M \simeq 1.5\tilde{m}. \qquad (21)$$

The orbital contribution being dominated by the light GB processes, this cancellation should be indicative of the actual situation. This diminution means that even though Δq_v is significantly different from Δq_{expt}, for a magnetic moment calculation we can still use Δq_v if at the same time the orbital angular momentum contribution is ignored. This explains why the NQM can give a satisfactory account of the baryon magnetic moments even if its spin content prediction has been found to be incomplete.

We conclude with the following remarks:

(1) Previous discussions of the orbital angular momentum contribution to the baryon magnetic moment [16] have been concerned with the configuration mixing, between the S wave and possible higher orbital states, of the three valence quarks rather than the contribution by the orbital angular momentum of the quark sea. Our viewpoint is that valence quark configuration mixing might not be a major factor because the simple quark model is known to yield an adequate account of the baryon magnetic moments. In a subsequent remark, we shall comment on the issue of improving upon the NQM description.

(2) Much of the current discussions on the proton spin problem [17] has to do with a possible gluonic contribution, which is studied in terms of the Lagrangian (hence perturbative) degrees of freedom—in contrast to the nonperturbative QCD quantities of constituent quarks and internal GBs of the present work. We view these two

descriptions as complimentary approaches: the validity of one does not preclude the correctness of the other [5]. An analogy with the baryon mass problem is instructive. Even if one finds, via the energy-momentum trace anomaly, that most of the baryon mass is gluonic in origin [18], it is still very useful to have the nonrelativistic QM picture of the baryon mass being mostly the sum of its constituent quark masses. The additional mass of a constituent quark results from QCD interactions, hence gluonic in origin. (In χQM this gluonic interaction corresponds to the quark gaining a large mass when propagating in the chiral condensate of the QCD vacuum.) In the same manner, gluons can contribute to the baryon spin through the axial anomaly. The analogy suggests the possibility of viewing the renormalization effects due to the χQM quark sea as ultimately corresponding to the gluonic contribution.

(3) The field theoretical calculation of the chiral renormalization effects will be, to the leading order, that of the one-loop diagrams with intermediates states of quarks and GBs. The relativistic computation automatically includes both the sea quark-spin and orbital angular momentum contributions. In fact, such χQM calculations have been carried out [19,20] and both groups found the resultant anomalous magnetic moments of the constituent quarks to be small. This lends support to our contention that there must be significant cancellation among the spin and orbital angular momentum contributions.

(4) Our present χQM discussion suggests that to improve upon the NQM calculation of the baryon magnetic moments we can start with the valence quarks, and augment them with the small anomalous quark moments due to the chiral loop effects, and also include the "exchange current effects" due to the GB-exchanges among the valence quarks [21]. Indeed, the study in Ref. [20] has concluded that such a calculation does indeed yield a very satisfactory description of the magnetic moments.

The conclusion we wish to draw is that the spin and magnetic moment data are consistent with the χQM predictions: (A) a significantly polarized quark sea in the direction opposite to the baryon spin, $\Delta q_{\text{sea}} < 0$, and yet (B) the antiquarks in the sea are not significantly polarized, $(\bar{q}_+ - \bar{q}_-) = 0$, and (C) there should also be a sizable amount of orbital angular momentum which because of conservation law just cancels the quark polarization of the sea: $\langle L_Z \rangle = -\frac{1}{2}\Delta\Sigma_{\text{sea}}$. This diminishes the quark sea contribution and allows for a successful description of the baryon magnetic moments by the NQM.

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[13] With SU(3) breaking effects taken only in the quark moments $\mu_s \simeq 2\mu_d/3$ or $M_s \simeq 3M_d/2$, and taking $\mu_d \simeq -0.9$ n.m., we can obtain a fairly good fit of the measured moments (to a general agreement of within 13%, with the notable exception of Ξ^- which is off by about 20%). This implies the constituent quark masses of $M \simeq 350$ MeV and $M_s \simeq 525$ MeV. These values just match, within expected uncertainty, those obtained in quark model fittings (with spin-dependent contribution included) of the baryon masses; see, for example, [5] and references cited therein.

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