Combining Real and Virtual Higgs Boson Mass Constraints

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Within the framework of the standard model we observe that there is a significant discrepancy between the most precise Z boson decay asymmetry measurement and the limit from direct searches for Higgs boson production. Using methods inspired by the Particle Data Group we explore the possible effect on fits of the Higgs boson mass. In each case the central value and the 95% confidence level upper limit increase significantly relative to the conventional fit. The results suggest caution in drawing conclusions about the Higgs boson mass from the existing data. [S0031-9007(98)05624-5]

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Perhaps the most pressing issue in particle physics today is the mass scale of the quanta that break electroweak symmetry, giving mass to the particles in the theory, including the quark and lepton constituents of ordinary atomic matter. That scale determines whether the symmetry breaking force is weak or strong and it sets the energy scale future accelerators will need for detailed studies of the mass-generating mechanism. In general the issue can be resolved only by discovering the symmetry breaking quanta at a high energy collider. However, in particular theoretical frameworks, such as, for instance, the standard model, radiative corrections to already measured quantities can be used to constrain the mass of the symmetry breaking sector.

Interpreted in the standard model framework, beautiful data from LEP, SLAC, and Fermilab appear to favor a light Higgs boson with mass of order 100 GeV [1]. The conclusion emerges from the effect of *virtual* Higgs bosons, via radiative corrections, on precision measurements of the Z and W bosons. In addition, the four LEP experiments have searched for *real* Higgs bosons, with negative results that when combined are expected to imply a lower limit $m_H \geq 77$ GeV at 95% confidence level (C.L.) [2]. Taken together the experiments suggest a window between 80 and a few hundred GeV. The purpose of this Letter is to suggest that the window may in fact be substantially larger, in part because of well known inconsistencies within the precision data, but more because of equally significant inconsistencies between precision data and the direct searches whose magnitude has, with some noteworthy exceptions [3,4], gone largely unnoticed and/or unremarked.

The problem of how to combine inconsistent data has led to the breakup of many beautiful friendships. The mathematical theory of statistics provides no magic bullets and ultimately the discrepancies can be resolved only by future experiments. The Particle Data Group [5] (PDG) has for many years scaled the uncertainty of discrepant results by a factor I will call S_{PDG}, defined by $S_{\mathrm{PDG}} = \sqrt{\chi^2/(N-1)}$, where N is the number of data points being combined. They scale the uncertainty of the combined fit by the factor S_{PDG} if and only if $S_{\mathrm{PDG}} > 1$. This is a conservative prescription, which amounts to requiring that the fit have a good confidence level, ranging from 32% for $N = 2$ to greater than 40% for larger values of N. If the confidence level is already good, the scale factor has little effect; it has a major effect only on very discrepant data. The PDG argues (see [6]) that low confidence level fits occur historically at a rate significantly greater than expected by chance, that major discrepancies are often, with time, found to result from underestimated systematic effects, and that the scaled error provides a more cautious interpretation of the data. A few authors [3,7,8] have applied S_{PDG} to the asymmetry measurements of $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lepton}}$, as I will also do here.

With the top quark mass fixed at the value determined by CDF and D0 Collaborations, the most sensitive probe of m_H is currently the effective leptonic weak interaction mixing angle, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lepton}}$, measured in a variety of Z boson decay asymmetries. The extent to which the asymmetries currently dominate the estimate of m_H can be seen by comparing the conventional fit given below to the seven asymmetry measurements, $m_H = 104 \pm^{110}_{54}$ GeV, with the LEP electroweak working group [9] global fit to all data, $m_H = 115 \pm^{116}_{66}$ GeV. Because it more than suffices for the purposes of this paper, the analysis that follows is based on the asymmetry measurements alone. The PDG scale factor then increases the uncertainty but not the central value of the combined fit for $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lepton}}$ and m_H. (If the rescaled fit to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lepton}}$ is included in a global fit as in [3], the rescaling does affect the central value.)

The focus of this paper is on the discrepancies between precision measurements and the limit from the direct searches, which will be addressed by a method analogous to the PDG scale factor. Like the PDG prescription, the idea is to scale the error so that the precision measurement has a significant probability P to be consistent with the direct search limit. I consider $P = 0.32$, corresponding to the PDG's choice, as well as larger and smaller values. To account for uncertainty in the search limits, which may also be subject to unknown systematic errors, I consider a range of different lower limits on m_H, from a very conservative 50 GeV to a futuristic 90 GeV. In this approach both the central value and the uncertainty of the fit are affected. In addition I present fits using two other methods discussed by the PDG.

By giving full weight to a measurement that is in serious conflict with the direct search lower limit, the conventional method risks underestimating m_H. The alternative methods considered here provide a more conservative estimate of the upper limit on m_H but risk skewing the fit to large m_H. Taken together the results strongly suggest caution in drawing conclusions from the precision data about the value of the Higgs boson mass.

Precision data.—The relevant values of $\sin^2\theta_{\rm eff}^{\rm lepton}$ and the quoted experimental uncertainties are shown in Table I, from the preliminary values presented at the 1997 summer conferences [1,9]. For each value the table displays the corresponding value of m_H and the 95% C.L. upper ($m_{95}^<$) and lower ($m_{95}^>$) bounds (that is, the symmetric 90% confidence intervals). Also indicated is the probability for m_H to lie below 77 GeV. Gaussian distributions are assumed for $\sin^2\theta_{\rm eff}^{\rm lepton}$ and $\log(m_H)$.

The values of m_H are from the state of the art $\overline{MS}$ computation of Ref. [8]. To obtain the confidence intervals and probabilities the parametric error is combined in quadrature with the experimental errors. The parametric error is equivalent to $\pm 0.000\,30$ uncertainty in $\sin^2\theta_{\rm eff}^{\rm lepton}$—see [8]. It is dominated by roughly equal contributions from the uncertainties in the top quark mass, $m_t = 175 \pm 6$ GeV, and the fine structure constant at the Z mass, $\alpha^{-1}(m_Z) = 128.896 \pm 0.090$, in addition to other much smaller contributions, including $\Delta\alpha_{\rm QCD}(m_Z)$ and uncomputed higher order corrections. (There are also negligible extrapolation errors from Ref. [8], equivalent to $\leq 0.000\,03$ in $\sin^2\theta_{\rm eff}^{\rm lepton}$ for $75 < m_H < 600$ GeV. Even outside this range they have no real effect on the analysis, since the confidence levels and scale factors depend only on the relationship between $\sin^2\theta_{\rm eff}^{\rm lepton}$ and m_H for $m_H = m_H^{\rm limit}$. The worst case is then $m_H^{\rm limit} = 50$ GeV, close enough for any additional error to be negligible. The very large values of $m_{95}^>$ in the tables could be affected but they have no precise significance in any case.)

The six LEP measurements in Table I are each combined from the four LEP experiments, and in each case the combined fit has a good confidence level. The conventional maximum likelihood fit for the LEP measurements is shown in the first row of Table II. The chi-squared per degree of freedom is $\chi^2/N - 1 = 4.4/5$ corresponding to a robust 0.5 confidence level. The central value is $m_H = 240$ GeV and the 95% C.L. upper limit is 860 GeV. There is no entry for the $S_{\rm PDG}$ fit since $S_{\rm PDG} < 1$.

Combining all seven measurements (the conventional LEP + SLC fit in Table II) the central value decreases to 100 GeV and the 95% C.L. upper limit falls to 310 GeV, demonstrating the effect of the high precision and lower $\sin^2\theta_{\rm eff}^{\rm lepton}$ from A_{LR}. The chi-squared per degree of freedom now rises to $12.5/6$, with a marginal confidence level of 0.05. The PDG scale factor is then $S_{\rm PDG} = 1.45$. Using it, the combined uncertainty of the fit increases from $\pm 0.000\,23$ to $\pm 0.000\,33$ and the 95% C.L. upper limit on m_H increases modestly to 420 GeV.

Direct search limits.—In addition to discrepancies among the measurements of $\sin^2\theta_{\rm eff}^{\rm lepton}$, which are problematic whether we assume the standard model or not, Table I also reveals a second discrepancy that occurs specifically within the standard model framework. The most precise measurement, A_{LR}, implies a 95% C.L. *upper* limit on m_H of 77 GeV, while the direct searches at LEP are expected to provide a combined 95% *lower* limit also at 77 GeV [2]. (The individual 95% C.L. limits quoted by the four experiments range from 66 to 71 GeV [2].) The third most precise measurement, A_{FB}^l, also has significant weight (71%) below the direct search limit. (A_{LR} and A_{FB}^l are also the only measurements with $m_{95}^>$ below the TeV scale.)

This raises a difficult question: Within the standard model framework what role if any should the direct search limits play in extracting the implications of the precision data? There is no single "right" answer to the question. A maximum likelihood fit including both the precision data and the direct search data would replicate the conventional fit if the central value lies above the lower limit, $m_H^{\rm limit}$, from the direct searches. That is a defensible interpretation, since if the true value of m_H were near $m_H^{\rm limit}$ we would expect values of m_H obtained from measurements of $\sin^2\theta_{\rm eff}^{\rm lepton}$ to lie both above and below $m_H^{\rm limit}$. By underweighting downward fluctuations while leaving upward fluctuations at their full weight, we risk skewing the fit upward. Mindful of this risk, it is still instructive to explore the sensitivity of the fit to the weight ascribed to measurements that are *individually in significant contradiction* with the direct search limit.

Clearly the direct search results are not irrelevant. If, for instance, the only information available were the direct search limits and the A_{LR} measurement, we would conclude that the standard model is excluded at 90% C.L. Theorists would have flooded the Los Alamos server with papers on the death of the standard model and the birth of new theories W, X, Y, Z, ... In the actual situation the A_{LR} measurement causes the fit to m_H to

TABLE I. Values for $\sin^2\theta_{\rm eff}^{\rm lepton}$ from asymmetry measurements [1] with 1σ experimental errors. The corresponding Higgs boson masses, the 95% C.L. upper and lower limits, and the confidence level for $m_H < 77$ GeV are given for each measurement.

	$\sin^2\theta_{\rm eff}^{\rm lepton}$ (1σ)	m_H (GeV)	$m_{95}^>$, $m_{95}^<$	P (<77 GeV)
A_{LR}	0.230 55 (41)	16	3, 80	0.95
A_{FB}^b	0.232 36 (43)	520	100, 2700	0.03
A_{FB}^l	0.231 02 (56)	40	5, 290	0.71
A_τ	0.232 28 (81)	440	30, 6700	0.14
A_e	0.232 43 (93)	590	28, 13 000	0.14
Q_{FB}	0.232 20 (100)	380	14, 10 000	0.21
A_{FB}^c	0.231 40 (111)	83	2, 3000	0.49

TABLE II. Fits to the LEP and LEP + SLC data as described in the text. The Bayesian and frequentist fits assume $m_H^{\rm limit} = 77$ GeV.

Data set	Fit	$\sin^2\theta_{\rm eff}^{\rm lepton}$ (1σ)	m_H (GeV)	$m_{95}^<$, $m_{95}^>$
U	Conventional	0.231 96 (28)	240	67, 860
	$S_{\rm PDG}$			
LEP	$S_{\rm VR}(0.32,\ 77\ {\rm GeV})$	0.232 03 (29)	270	74, 1000
	Bayes	0.231 97	250	<880 (95%)
	Frequentist (1)	0.232 04 (28)	280	78, 1000
	Frequentist (2)	<0.233 04 (95%)		1900 (95%)
	Conventional	0.231 52 (23)	100	32, 340
	$S_{\rm PDG}$	0.231 52 (33)	100	26, 420
LEP + SLC	$S_{\rm VR}(0.32,\ 77\ {\rm GeV})$	0.231 98 (28)	250	69, 920
	Bayes	0.231 71	150	<500 (95%)
	Frequentist (1)	0.231 83 (23)	190	58, 610
	Frequentist (2)	<0.232 84 (95%)		<1300 (95%)

shift by more than a factor of 2, from 240 to 100 GeV, and the 95% upper limit to fall from the TeV scale to $\simeq$340 GeV. It is fully weighted in the conventional standard model fit despite a significant contradiction with the standard model.

If the discrepancy were even greater—say, for instance, a precision measurement implying $m_H = 10$ MeV with a 99.99% C.L. upper limit at 77 GeV—the clear response would be to omit that measurement from a standard model fit, although it could still be considered in a broader framework encompassing the possibility of new physics. On the other hand, A_{FB}^l, with 31% probability to be consistent with a 95% lower limit at 77 GeV, would surely be retained. The question is how to resolve the intermediate cases in which the discrepancy is significant but not so significant that the data should clearly be excluded.

Consider a prescription, analogous to the PDG scale factor, that interpolates smoothly between the extremes. Imagine a measurement x with experimental error δ_E and a quantity y that is related to x with an uncertainty $x \pm \delta_P$ (the parametric error). Suppose there is a lower limit on y at $y = y_0$ that translates to a lower limit on x at $x_0 \pm \delta_P$, such that the measurement x falls below the implied limit, $x < x_0$. The discrepancy between the measurement and the limit is then characterized by a Gaussian distribution centered at x with standard deviation $\sigma = \sqrt{\delta_E^2 + \delta_P^2}$, with a computable probability P for $x > x_0$. If P is less than a chosen minimal confidence level $P_{\rm VR}$ (VR for "virtual-real"), then δ_E is scaled by a factor $S_{\rm VR}$ chosen so that the Gaussian centered at x with standard deviation $\sigma' = \sqrt{(S_{\rm VR}\delta_E)^2 + \delta_P^2}$ has probability $P_{\rm VR}$ for $x > x_0$. If $x_0 - x$ is small enough, the scale factor has little or no effect. If x is many σ below x_0, $S_{\rm VR}$ will be large and the data point x will have reduced weight in a combined fit with other data. Intermediate cases will interpolate smoothly between the two extremes, depending on the values of $x - x_0$, σ, and $P_{\rm VR}$.

The value of $P_{\rm VR}$ is of course arbitrary. One plausible choice is $P_{\rm VR} = 0.32$, since that is the confidence level implicit in the PDG scale factor for $N = 2$. A plausible choice for the lower limit on m_H is $m_H^{\rm limit} = 77$ GeV. The resulting fits are shown in Table II. The fit to the LEP data is affected only modestly, with an increase of $\sim$10% in m_H. For the LEP + SLC fit, the central value of m_H and the 90% confidence interval increase significantly, to the values of the LEP fit. The scale factors are $S_{\rm VR}(A_{FB}^l) = 1.2$ and $S_{\rm VR}(A_{LR}) = 4.1$.

Table III displays the results of varying $P_{\rm VR} = 0.20$, 0.32, and 0.40 and $m_H^{\rm limit} = 50$, 60, 70, 80, and 90 GeV. Though not reported by the experimental groups, the confidence levels at 50 and 60 GeV are probably much tighter than 97%, both because the LEP II cross sections increase for smaller m_H and because the LEP I data contribute to the confidence level at those masses—the 95% C.L. lower bound from LEP I alone is $m_H > 66$ GeV. The value 80 GeV is close to the presently projected 95% combined limit of the four LEP experiments, while 90 GeV is the anticipated limit if no discovery emerges from currently planned LEP II running. Vacant entries in Table III indicate that the fits are unmodified, $S_{\rm VR} \leq 1$, and that the conventional fit, line 1 of Table II, applies. For the LEP data, the Higgs mass scale varies by no more than a factor of 1.5 from the conventional fit over the entire range of Table III. For the LEP + SLC data, the difference is a factor of 1.5 for $(P_{\rm VR}, m_H^{\rm limit}) = (0.20, 50\ {\rm GeV})$ and becomes as large as a factor of 4.

Other methods.—In this section I will briefly present results using methods discussed by the PDG [10] for combining measurements that conflict with a limit. They are no less arbitrary than the $S_{\rm VR}$ scale factor method discussed above.

Consider a collection of measurements $x_i \pm \delta_{E,i}$, $i = 1, \ldots, N$, some of which are nominally inconsistent with a lower limit at x_0. The "Bayesian" method is to

TABLE III. Fits to LEP and LEP + SLC data using the S_{VR} scale factor with various values of P_{VR} and m_H^{limit}. Each entry displays the central value of m_H and $m_{95}^{<}, m_{95}^{>}$, the 95% C.L. lower and upper limits, in GeV. Empty entries indicate that no measurement is far enough below threshold to be modified by the scale factor and that the conventional fit of Table II applies.

	LEP			LEP + SLC		
m_H^{limit}	$P_{\mathrm{VR}} = 0.20$	0.32	0.40	0.20	0.32	0.40
50				160	210	230
				41, 590	59, 740	65, 830
60			310	170	220	300
			82, 1100	47, 650	61, 770	80, 1100
70			360	190	220	350
			94, 1400	51, 680	62, 790	91, 1300
80		290	380	190	260	370
		77, 1100	100, 1500	54, 700	72, 970	96, 1400
90		320	390	200	300	380
		85, 1200	100, 1500	56, 710	79, 1100	100, 1500

combine all data points in the conventional way and to multiply the combined Gaussian distribution by a step function $C\theta(x - x_0)$, so that the distribution vanishes below x_0. C is a normalization factor to guarantee total unit probability. I have modified the usual Bayesian method to account for the fact that the lower limit is not absolute but has 95% confidence, by choosing C to give the distribution probability 0.95 for $x > x_0$. The 50% and 95% tiles of the resulting distributions are shown in Table II for $m_H^{\mathrm{limit}} = 77$ GeV.

Three "frequentist" prescriptions are also discussed by the PDG, of which two are considered here. For $x_i < x_0$ one prescription assigns $x_i \pm \delta_{E,i} \rightarrow x_0 \pm \delta_{E,i}$ when the limit x_0 is known exactly. Including the parametric uncertainty, I modify this to $x_i \pm \delta_{E,i} \rightarrow x_0 \pm \sigma_i$, where $\sigma_i = \sqrt{\delta_{E,i}^2 + \delta_P^2}$. The readjusted points are then combined as usual (including S_{PDG} if applicable) with the other measurements. An extremely conservative variation, intended only to obtain the 95% C.L. upper limit, replaces $x_i \rightarrow \min(x_i, x_0 + 1.64\sigma_i)$, so that 95% of the probability distribution for each measurement is above the limit x_0. The results are illustrated in Table II for $m_H^{\mathrm{limit}} = 77$ GeV.

In summary, the A_{LR} measurement is inconsistent at 95% C.L. both with the LEP asymmetry measurements and, in the standard model, with the Higgs boson search limits, while its precision causes it to have a profound effect on the combined standard model fit. The conflict with the search limits may diminish or disappear if there is new physics outside the standard model framework but is necessarily germane to a standard model fit. The analysis presented here is meant as a warning signal, a yellow if not a red flag, suggesting caution in drawing conclusions from the precision data about the mass of the standard model Higgs boson. Applying methods inspired by the Particle Data Group to these discrepancies, we find that the central value of m_H increases by factors from $\sim$1.5 to $\sim$4 while the 95% C.L. upper limit increases toward the TeV scale. Only future experimental results can resolve the discrepancies in the present experimental situation.

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