Unlocking Hidden Entanglement with Classical Information

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The notion of "hidden" entanglement is introduced, and it is shown that this is a property associated with every separable mixed quantum state of two subsystems. The hidden entanglement is explicitly quantified for a general class of separable mixed states of two spin-1/2 particles, and a formula is derived giving the maximum amount of entanglement that can be hidden. The process of "unlocking" hidden entanglement with classical information is explained, and the number of bits required to unlock each ebit of entanglement is evaluated. It is argued that the entanglement-unlocking process can be seen as the converse of quantum cryptography schemes that use EPR pairs. [S0031-9007(97)05274-5]

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In this Letter I consider mixed quantum states of two subsystems that are separable in the sense that their density matrices can be written exclusively in terms of product states. I introduce the notion of "hidden" entanglement, which refers to the fact that a mixed state may have been *prepared* with entangled states, even if it is separable in the above sense. I show that *any* separable mixed state of two subsystems may contain hidden entanglement. A formula is derived giving the maximum hidden entanglement for a general class of separable mixed states of two spin-1/2 particles. I explain how hidden entanglement can be "unlocked" with classical information, and evaluate how many bits of classical information are required to unlock each ebit of entanglement in the general case, for two spin-1/2 particles. This analysis points to an upper limit for the amount of entanglement that can be unlocked with each bit.

The two-subsystem mixed states I consider have density matrices that can be written in the form $\rho_S = \sum_{i=1}^{N} p_i |\xi_i\rangle_1 |\eta_i\rangle_2\, {}_2\langle\eta_i|_1\langle\xi_i|$, where the subscripts 1, 2 refer to subsystems 1 and 2. It is assumed that neither subsystem is in a pure state, i.e., at least two of the $|\xi_i\rangle$ and two of the $|\eta_i\rangle$ are distinct. Clearly, ρ_S cannot violate any Bell inequality, nor can it provide a quantum channel for teleportation [1] or superdense coding [2]. Hence the labeling of ρ_S as "local" is justifiable.

The statistical distribution of individual pure states that constitutes a given mixed state cannot consistently be interpreted as representing merely a lack of knowledge of the state of each system in the ensemble. One reason that this so-called "ignorance interpretation" is problematic is that any mixed state will allow an infinite number of decompositions. Furthermore, the fact that (as we shall see shortly) any mixed state density matrix can be interpreted as the partial trace of an entangled *pure* state of the given system plus an ancilla system, where the ancilla is traced out, can lead to contradictions if the ignorance interpretation is adopted [3]. Nevertheless, it may be that we are given an ensemble of systems, with a statistical distribution of preparation states, where we know the specific decomposition of states used in the preparation, even though we have no information regarding the state of any individual system in the ensemble. Such an ensemble should not, strictly speaking, be labeled as being in a mixed state, but we could describe it as being in a *pseudo*mixed state. It is not in a genuine mixed state because the statistical nature of the distribution of individual states corresponds only to a lack of knowledge—but its physical properties with respect to any measurement will be identical to those of a genuine mixed state with the same density matrix.

For example, suppose that we are given a mixed state of two spin-1/2 particles which is represented by the density matrix $\rho_{12} = \frac{1}{2}(|\uparrow_1\downarrow_2\rangle\langle\uparrow_1\downarrow_2| + |\downarrow_1\uparrow_2\rangle\langle\downarrow_1\uparrow_2|)$. While this state is obviously separable, and hence able to accommodate a local description of the subsystem correlations, it is also possible to write it as $\rho_{12} = \frac{1}{4}\{(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle)(\langle\uparrow_1\downarrow_2| + \langle\downarrow_1\uparrow_2|) + (|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)(\langle\uparrow_1\downarrow_2| - \langle\downarrow_1\uparrow_2|)\}$. This means that, in spite of its apparent locality, ρ_{12} may have been prepared using only the maximally entangled states $1/\sqrt{2}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle)$ and $1/\sqrt{2}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$. Suppose that we are told that ρ_{12} has indeed been prepared using only these entangled states. The state described by ρ_{12} then becomes a pseudomixed state, which we can interpret as containing *hidden* entanglement. Although we know that each individual two-particle system in the ensemble is maximally entangled, this entanglement is invisible in that it cannot be demonstrated by any Bell inequality violation, nor can it be used as a resource for quantum computation, teleportation, cryptography, or superdense coding. Furthermore, it is impossible to *distil* [4] any entanglement from this pseudomixed state. However, the hidden entanglement can be *unlocked* with classical information. In this elementary example we will require 1 bit of classical information to unlock 1 ebit from the pseudomixed state, because, if we are given one two-particle system from the ensemble, we will (without performing any measurements) require 1 bit of information to establish which of the pure states $1/\sqrt{2}(|\uparrow_1\downarrow_2\rangle \pm |\downarrow_1\uparrow_2\rangle)$ it is in; and each of these pure states contains 1 ebit of entanglement [5].

It is now shown that *any* separable two-subsystem mixed state may contain hidden entanglement. As before, we write the general separable two-subsystem density matrix as $\rho_S = \sum_{i=1}^{N} p_i |\xi_i\rangle_1 |\eta_i\rangle_2 \, {}_2\langle\eta_i| {}_1\langle\xi_i|$. We now introduce a fictitious ancilla system with basis states $|e_i\rangle_A$, so that ρ_S can be thought of as originating in an entangled pure state $|\Psi\rangle_{S+A}$ of the combined system consisting of the original two subsystems plus the imaginary ancilla, with the ancilla traced out. That is, if $|\Psi\rangle_{S+A} = \sum_{i=1}^{N} \sqrt{p_i}\,|\xi_i\rangle_1 |\eta_i\rangle_2 |e_i\rangle_A$, then $\rho_S = \mathrm{Tr}_A(|\Psi\rangle_{S+A} \times {}_{S+A}\langle\Psi|)$. In order to show that ρ_S can contain hidden entanglement, we first introduce an alternative basis $|e_i'\rangle_A$ for the ancilla, such that

$$|e_1\rangle = \alpha|e_1'\rangle + \beta|e_2'\rangle, |e_2\rangle = \beta^*|e_1'\rangle - \alpha^*|e_2'\rangle,$$
$$|e_3\rangle = \gamma|e_3'\rangle + \delta|e_4'\rangle, |e_4\rangle = \delta^*|e_3'\rangle - \gamma^*|e_4'\rangle,$$

etc. Hence

$$|\Psi\rangle_{S+A} = \sqrt{p_1}\,|\xi_1\rangle_1|\eta_1\rangle_2(\alpha|e_1'\rangle_A + \beta|e_2'\rangle_A) + \sqrt{p_2}\,|\xi_2\rangle_1|\eta_2\rangle_2(\beta^*|e_1'\rangle_A - \alpha^*|e_2'\rangle_A) + \dots .$$

Once again tracing out the ancilla, we find that our original density matrix ρ_S can be written

$$\begin{aligned}\rho_S = {} & (\sqrt{p_1}\,\alpha|\xi_1\rangle_1|\eta_1\rangle_2 + \sqrt{p_2}\,\beta^*|\xi_2\rangle_1|\eta_2\rangle_2) \\ & \times (\sqrt{p_1}\,\alpha^*\,{}_2\langle\eta_1|\,{}_1\langle\xi_1| + \sqrt{p_2}\,\beta\,{}_2\langle\eta_2|\,{}_1\langle\xi_2|) \\ & + (\sqrt{p_1}\,\beta|\xi_1\rangle_1|\eta_1\rangle_2 - \sqrt{p_2}\,\alpha^*|\xi_2\rangle_1|\eta_2\rangle_2) \\ & \times (\sqrt{p_1}\,\beta^*\,{}_2\langle\eta_1|\,{}_1\langle\xi_1| - \sqrt{p_2}\,\alpha\,{}_2\langle\eta_2|\,{}_1\langle\xi_2|) + \dots .\end{aligned}$$

This means that ρ_S may contain hidden entanglement, since it can be written in terms of the entangled states

$$(\sqrt{p_1}\,\alpha|\xi_1\rangle_1|\eta_1\rangle_2 + \sqrt{p_2}\,\beta^*|\xi_2\rangle_1|\eta_2\rangle_2),$$
$$(\sqrt{p_1}\,\beta|\xi_1\rangle_1|\eta_1\rangle_2 - \sqrt{p_2}\,\alpha^*|\xi_2\rangle_1|\eta_2\rangle_2),$$

etc. This argument can of course be extended to nonseparable mixed states, i.e., mixed states containing distillable entanglement. That is, a mixed state with distillable entanglement in its density matrix may contain further entanglement which is hidden. For such a state it may be possible to both distill and unlock entanglement.

In order to quantify the hidden entanglement associated with the above decomposition of ρ_S, we first normalize [6] the entangled states shown, and rewrite ρ_S as

$$\rho_S = (p_1|\alpha|^2 + p_2|\beta|^2)\,|\phi_1\rangle\langle\phi_1| + (p_1|\beta|^2 + p_2|\alpha|^2)\,|\phi_2\rangle\langle\phi_2| + \dots, \quad (1)$$

where $|\phi_1\rangle, |\phi_2\rangle, \dots,$ are the normalized states

$$|\phi_1\rangle = \frac{1}{(p_1|\alpha|^2 + p_2|\beta|^2)^{1/2}} \times (\sqrt{p_1}\,\alpha|\xi_1\rangle_1|\eta_1\rangle_2 + \sqrt{p_2}\,\beta^*|\xi_2\rangle_1|\eta_2\rangle_2), \quad (2a)$$

$$|\phi_2\rangle = \frac{1}{(p_1|\beta|^2 + p_2|\alpha|^2)^{1/2}} \times (\sqrt{p_1}\,\beta|\xi_1\rangle_1|\eta_1\rangle_2 - \sqrt{p_2}\,\alpha^*|\xi_2\rangle_1|\eta_2\rangle_2), \quad (2b)$$

etc. The amount of hidden entanglement E_H ebits associated with the above decomposition of ρ_S can then be quantified as $E_H = (p_1|\alpha|^2 + p_2|\beta|^2)E_1 + (p_1|\beta|^2 + p_2|\alpha|^2)E_2 + \dots,$ where E_i ebits is the entanglement of $|\phi_i\rangle$, defined in terms of the Shannon entropy of the modulus-squared coefficients in its biorthogonal expansion ("Schmidt decomposition") [5]. It may be possible to combine pairs of $|\xi_i\rangle$, $|\eta_j\rangle$ in other ways to obtain different decompositions of ρ_S in terms of entangled states—and from each such decomposition we can obtain an infinite number of alternative decompositions by varying $|\alpha|^2$, $|\beta|^2$, $|\gamma|^2$, etc. Clearly E_H will depend on the particular decomposition of ρ_S that is referred to. Even if we know that a particular set of entangled states has been used to prepare ρ_S, we cannot in general evaluate the E_i directly from the $|\phi_i\rangle$ as given by Eqs. (2a) and (2b), because the pairs $|\xi_1\rangle_1|\eta_1\rangle_2$, $|\xi_2\rangle_1|\eta_2\rangle_2$, etc. will not in general be biorthogonal. However, it will always be possible to find a biorthogonal expansion for each $|\phi_i\rangle$ [7], so that it will always be possible to calculate the E_i indirectly.

I now consider separable mixed states of two subsystems where each subsystem has a Hilbert space of dimension two. The classification of separable mixed states is notoriously difficult [8], and the analysis here will be restricted to the general class consisting of those mixed states of two two-state subsystems that can be written as a weighted sum of projections on four orthogonal product states. The remaining analysis in this Letter will be specific to mixed states of this kind. Any such state can be written as

$$\begin{aligned}\rho_L = {} & p_1|\uparrow_1\downarrow_2\rangle\langle\uparrow_1\downarrow_2| + p_2|\downarrow_1\nwarrow_2\rangle \\ & \times \langle\downarrow_1\nwarrow_2| + p_3|\uparrow_1\uparrow_2\rangle\langle\uparrow_1\uparrow_2| + p_4|\downarrow_1\searrow_2\rangle \\ & \times \langle\downarrow_1\searrow_2|.\end{aligned} \quad (3)$$

(Here the states indicated can refer to any two-dimensional variable, not necessarily spin). Note that no pair of projections in this density matrix is biorthogonal. For example, in the first two terms, $|\uparrow_1\rangle$ is orthogonal to $|\downarrow_1\rangle$, but $|\downarrow_2\rangle$ is not orthogonal to $|\nwarrow_2\rangle$.

As before, by introducing a fictitious ancilla we can rewrite ρ_L in terms of normalized entangled states $|\chi_i\rangle$:

$$\begin{aligned}\rho_L = {} & (p_1|\alpha|^2 + p_2|\beta|^2)\,|\chi_1\rangle\langle\chi_1| \\ & + (p_1|\beta|^2 + p_2|\alpha|^2)\,|\chi_2\rangle\langle\chi_2| \\ & + (p_3|\gamma|^2 + p_4|\delta|^2)\,|\chi_3\rangle\langle\chi_3| \\ & + (p_3|\delta|^2 + p_4|\gamma|^2)\,|\chi_4\rangle\langle\chi_4|,\end{aligned} \quad (4)$$

where

$$|\chi_1\rangle = \frac{1}{(p_1|\alpha|^2 + p_2|\beta|^2)^{1/2}} \times (\sqrt{p_1}\,\alpha|\uparrow_1\downarrow_2\rangle + \sqrt{p_2}\,\beta^*|\downarrow_1\nwarrow_2\rangle),$$

$$|\chi_2\rangle = \frac{1}{(p_1|\beta|^2 + p_2|\alpha|^2)^{1/2}} \times (\sqrt{p_1}\,\beta|\uparrow_1\downarrow_2\rangle - \sqrt{p_2}\,\alpha^*|\downarrow_1\nwarrow_2\rangle),$$

etc. [Alternatively, we could have obtained a different set of entangled states $|\chi_i'\rangle$ by pairing $|\uparrow_1\downarrow_2\rangle$ with $|\downarrow_1\searrow_2\rangle$ rather than with $|\downarrow_1\nwarrow_2\rangle$. This would have yielded the states

$$|\chi_1'\rangle = \frac{1}{(p_1|\alpha|^2 + p_4|\beta|^2)^{1/2}} \times (\sqrt{p_1}\,\alpha|\uparrow_1\downarrow_2\rangle + \sqrt{p_4}\,\beta^*|\downarrow_1\searrow_2\rangle),$$

etc.]

Note that the set of $|\chi_i\rangle$ will be orthogonal if and only if $p_1 = p_2$ and $p_3 = p_4$, in which case ρ_L becomes $\bar{\rho}_L = 2p_1(|\chi_1\rangle\langle\chi_1| + |\chi_2\rangle\langle\chi_2|) + 2p_3(|\chi_3\rangle\langle\chi_3| + |\chi_4\rangle\langle\chi_4|)$. Conversely, given any complete set of orthogonal entangled states, we can construct a separable mixed state, of the form $\bar{\rho}_L$, with any p_1 and p_3 satisfying $p_1 + p_3 = 0.5$.

In order to evaluate the entanglement of the individual states $|\chi_i\rangle$ in Eq. (4), we must first obtain the modulus-squared coefficients in the Schmidt decompositions of these states. To do this we write the states $|\nwarrow\rangle$, $|\searrow\rangle$, as $|\nwarrow\rangle = x|\uparrow\rangle + y|\downarrow\rangle$, $|\searrow\rangle = y^*|\uparrow\rangle - x^*|\downarrow\rangle$. A straightforward calculation then shows that the entanglement E_1 ebits of $|\chi_1\rangle$ is given by

$$E_1 = -(\lambda_1^+ \log_2 \lambda_1^+ + \lambda_1^- \log_2 \lambda_1^-), \tag{5}$$

where

$$\lambda_1^\pm = \frac{1}{2}\left\{1 \pm \left(1 - \frac{4p_1p_2|\alpha|^2|\beta|^2|x|^2}{(p_1|\alpha|^2 + p_2|\beta|^2)^2}\right)^{1/2}\right\},$$

and we can derive similar expressions for the entanglements E_2, E_3, E_4 ebits of $|\chi_2\rangle$, $|\chi_3\rangle$, $|\chi_4\rangle$. The total hidden entanglement E associated with ρ_L, when it has been prepared with the $|\chi_i\rangle$, is then given by

$$E = (p_1|\alpha|^2 + p_2|\beta|^2)E_1 + (p_1|\beta|^2 + p_2|\alpha|^2)E_2 + (p_3|\gamma|^2 + p_4|\delta|^2)E_3 + (p_3|\delta|^2 + p_4|\gamma|^2)E_4 . \tag{6}$$

(A different value for E would of course be obtained if the alternative hidden entangled states $|\chi_i'\rangle$ were used in the preparation of ρ_L). Note that we must have $0 < E \leq 1$. E cannot be zero given that neither subsystem is in a pure state and E will equal unity if and only if all of the nonvanishing $|\chi_i\rangle$ are maximally entangled.

Straightforward calculus shows that the *maximum* hidden entanglement that can be obtained from the decomposition ρ_L by varying $|\alpha|^2$, $|\beta|^2$, etc., occurs when $|\alpha|^2 = |\beta|^2 = |\gamma|^2 = |\delta|^2 = \frac{1}{2}$, and is given by $\max(E_{\max}, \tilde{E}_{\max})$, where

$$E_{\max} = -\frac{(p_1 + p_2)}{2}\left\{\log_2\left(\frac{1-\theta_{12}^2}{4}\right) + \theta_{12}\log_2\left(\frac{1+\theta_{12}}{1-\theta_{12}}\right)\right\} - \frac{(p_3 + p_4)}{2}\left\{\log_2\left(\frac{1-\theta_{34}^2}{4}\right) + \theta_{34}\log_2\left(\frac{1+\theta_{34}}{1-\theta_{34}}\right)\right\}, \tag{7}$$

with

$$\theta_{ij} = \left(1 - \frac{4p_ip_j|x|^2}{(p_i + p_j)^2}\right)^{1/2}.$$

$\tilde{E}_{\max}$ refers to the case where the states $|\chi_i'\rangle$ rather than the $|\chi_i\rangle$ are used; effectively this means swapping p_2 with p_4 and replacing $|x|^2$ with $1 - |x|^2$ in Eq. (7).

In the special case where the original product-state decomposition (3) of ρ_L contains two pairs of biorthogonal projections, i.e., $|x|^2 = 1$, we find that Eq. (7) simplifies to

$$E_{\max} = -\sum_{i=1}^{4} p_i \log_2 p_i + (p_1 + p_2)\log_2(p_1 + p_2) + (p_3 + p_4)\log_2(p_3 + p_4)$$

and that $\tilde{E}_{\max} = 0$.

As we have already mentioned, if a separable mixed state has been prepared using entangled states, and we know the particular decomposition of entangled states used in the preparation, so that the mixed state becomes a pseudomixed state, then the hidden entanglement contained in this pseudomixed state can be unlocked with classical information. In order to unlock all the hidden entanglement, we require sufficient information to establish which particular entangled state each pair of subsystems has been prepared in. If we are given the separable mixed state ρ_L described by Eq. (3) and are told that it has been prepared using the decomposition of entangled states $|\chi_i\rangle$ as in Eq. (4), then we will require Σ bits of classical information per pair of subsystems in order to establish the state that each pair was prepared in, where

$$\Sigma = -\{(p_1|\alpha|^2 + p_2|\beta|^2)\log_2(p_1|\alpha|^2 + p_2|\beta|^2) + (p_1|\beta|^2 + p_2|\alpha|^2)\log_2(p_1|\beta|^2 + p_2|\alpha|^2) + (p_3|\gamma|^2 + p_4|\delta|^2)\log_2(p_3|\gamma|^2 + p_4|\delta|^2) + (p_3|\delta|^2 + p_4|\gamma|^2)\log_2(p_3|\delta|^2 + p_4|\gamma|^2)\}. \tag{8}$$

[We have assumed here that p_1 and p_2 are both nonzero, and/or p_3 and p_4 are both nonzero. If p_1 *or* p_2 vanishes, then the first two terms on the right hand side of Eq. (8) should be replaced by $p_1 \log_2 p_1$ (for vanishing p_2) or $p_2 \log_2 p_2$ (for vanishing p_1). A similar modification should be made if p_3 or p_4 vanishes.]

Hence the number of bits of classical information required to unlock each ebit of entanglement is Σ/E, where Σ is given by Eq. (8) and E is given by Eq. (6). It seems that Σ/E is always ≥ 1. Although I do not have an analytical proof for the general validity of this inequality, I have not been able to find any set of parameters that violates it. In other words, it seems that we always need at least one bit to unlock each ebit. In the maximum-hidden-entanglement case (where $|\alpha|^2 = |\beta|^2 = |\gamma|^2 =$

$|\delta|^2 = \frac{1}{2}$), Σ simplifies to $\check{\Sigma} = 1 - (p_1 + p_2) \times \log_2(p_1 + p_2) - (p_3 + p_4)\log_2(p_3 + p_4)$, and the number of bits required to unlock each ebit is $\check{\Sigma}/E_{\max}$, where $E_{\max}$ is given by Eq. (7).

The question arises as to whether it is possible to obtain a higher yield of ebits per bit if we are supplied with only *partial* information with regard to distinguishing the hidden entangled states. For example, suppose that we are supplied with the pseudomixed state described by ρ_L with known decomposition of preparation states $|\chi_i\rangle$ as given by Eq. (4), and that we are given sufficient information to identify the $|\chi_1\rangle$ states, but that the $|\chi_2\rangle$, $|\chi_3\rangle$, and $|\chi_4\rangle$ states remain indistinguishable from each other. The yield of ebits per bit for unlocked hidden entanglement is then given by E'/Σ', where $E' = (p_1|\alpha|^2 + p_2|\beta|^2)E_1$, with E_1 given by Eq. (5), and

$$\Sigma' = -\{(p_1|\alpha|^2 + p_2|\beta|^2)\log_2(p_1|\alpha|^2 + p_2|\beta|^2) + [1 - (p_1|\alpha|^2 + p_2|\beta|^2)] \times \log_2[1 - (p_1|\alpha|^2 + p_2|\beta|^2)]\}.$$

Interestingly, it seems that we always have $E/\Sigma > E'/\Sigma'$. Again, I have not proved the general validity of this inequality analytically, but I cannot find any set of parameters that violates it. This is surprising, because if one of the $|\chi_i\rangle$ is significantly more entangled than the others, then one might have expected to be able to obtain a higher yield of ebits per bit by requiring only sufficient information to identify the pairs of subsystems corresponding to that particular $|\chi_i\rangle$.

Entanglement can be seen as a valuable resource for applications such as quantum computation [9], teleportation [1], cryptography [10–12], and superdense coding [2]. The fact that any separable mixed state can contain hidden entanglement means that entanglement can be disguised within any separable mixed state of our choice. Furthermore, there are infinite numbers of ways of disguising entanglement within a given separable mixed state. This means that it is possible to transmit a quantity of entanglement in such a way that the entanglement cannot be unlocked by any thief who tries to intercept the transmission, and that this transmission can be achieved via any separable two-subsystem mixed state whatsoever. The information needed to unlock the hidden entanglement can be sent via a classical communication channel, once the mixed state ensemble has arrived safely. For further security, the entanglement-unlocking information can be encrypted using a separate key which is generated and shared between sender and receiver using quantum cryptography. This means that if a thief intercepts the transmission of the original mixed state and replaces it with another mixed state with identical density matrix but containing no hidden entanglement, then she will still not be able to access the hidden entanglement even after the unlocking information has been sent.

The process of unlocking hidden entanglement can be understood as the converse of quantum cryptography techniques that use EPR pairs [10–12]. In the latter techniques it is possible to generate a secret shared classical bit by using up 1 ebit of entanglement; whereas in the entanglement-unlocking process it is possible to release 1 ebit of entanglement by supplying 1 classical bit. It could be argued that in EPR-pair cryptography the secret shared bit is *created* rather than unlocked, in that, according to standard quantum mechanics, it did not exist prior to the protagonists' disentangling measurements. However, if we adopt a deterministic interpretation of quantum mechanics (such as Bohm's theory [13]), then each bit generated in EPR-pair cryptography already exists prior to the protagonists' disentangling measurements, so that we can interpret the effect of these measurements as the unlocking of classical information, in an analogous way to that in which hidden entanglement can be unlocked as has been outlined here.

We can also see that a quantum ensemble, when it is in a pseudomixed state where there is hidden entanglement, can be labeled as "nonlocal" or "local," depending on whether or not we have access to the entanglement-unlocking information. If this information is irretrievably destroyed, then the ensemble in question changes from one with potentially nonlocal properties to one with only local properties, even though nothing whatsoever happens to the ensemble physically. This demonstrates that, although nonlocality is exclusively a quantum mechanical property, *classical* information can provide a bridge between local and nonlocal physical systems.

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