Damping of Low-Energy Excitations of a Trapped Bose-Einstein Condensate at Finite Temperatures

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We present the theory of damping of low-energy excitations of a trapped Bose-Einstein condensate at finite temperatures, where the damping is provided by the interaction of these excitations with the thermal excitations. We emphasize the key role of stochastization in the behavior of the thermal excitations for damping in nonspherical traps. The damping rates of the lowest excitations, following from our theory, are in fair agreement with the data of recent JILA and MIT experiments. The damping of quasiclassical excitations is determined by the condensate boundary region, and the result for the damping rate is drastically different from that in a spatially homogeneous gas. [S0031-9007(98)05601-4]

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After the discovery of Bose-Einstein condensation (BEC) [1–3], one of the major directions in the physics of ultracold gases has been the investigation of collective many-body effects. Especially interesting is the behavior of low-energy collective excitations of a trapped condensate. The JILA [4,5] and MIT [6,7] experiments show that these excitations are damped and provide us with interesting results on the temperature dependence of the damping rates and frequency shifts.

In this Letter we develop the theory of damping of excitations of a trapped condensate in the Thomas-Fermi regime at finite temperatures, where the presence of a thermal component is important. We confine ourselves to the damping of low-energy excitations, i.e., the excitations with energies $E_\nu \ll \mu$, where μ is the chemical potential, and consider temperatures $T \gg \hbar\omega$ (ω is the characteristic trap frequency) ranging almost up to the BEC transition temperature T_c. Thus far, theoretical and numerical investigations of elementary excitations of trapped Bose-condensed gases predominantly remained on the mean-field level [8–17]. The investigation of damping phenomena requires analysis beyond the ordinary mean-field approach [18]. It should be emphasized that the damping of low-energy excitations in a trapped Bose-condensed gas differs fundamentally from the damping of Bogolyubov excitations in an infinitely large spatially homogeneous gas. In the latter case, characterized by a continuum of excitations, any given excitation can decay into two excitations of lower energy and momentum. This damping mechanism, first discussed by Beliaev for $T = 0$ [19] and employed by Popov [20] at finite temperatures, proves to be dominant at $T \ll \mu$. In a trapped Bose-condensed gas the character of the discrete structure of the spectrum of low-energy excitations makes Beliaev damping impossible under conservation of energy. Therefore, irrespective of the relation between T and μ, the damping of excitations with energies

$$E_\nu \ll \mu, T \tag{1}$$

has to be provided by their interaction with the thermal excitations. The damping mechanism involves processes in which the low-energy excitation (ν) and the thermal excitation (γ) are annihilated (created) and another thermal excitation (γ') is created (annihilated):

$$\nu + \gamma \leftrightarrow \gamma'. \tag{2}$$

We will discuss the case where the thermal excitations γ, γ' are in the collisionless regime. Under the condition (1) the energies E_γ of these excitations are much larger than the energies E_ν of the low-energy excitations. Therefore, the damping mechanism governed by the processes (2) can be treated as Landau damping. For spatially homogeneous gases this mechanism was first discussed by Szepfalusy and Kondor [21,22].

It is worth noting that inside the condensate spatial region, at $T \lesssim \mu$ the density of occupied states of thermal excitations peaks at the energies $E_\gamma \sim T$, whereas for $T \gg \mu$ this happens at $E_\gamma \sim \mu$. As just the excitations with $E_\gamma \sim \mu$ give the main contribution to the damping rate, the collective character of the thermal excitations remains important even at $T \gg \mu$ (cf. [21]).

In a trapped Bose-condensed gas the damping of low-energy excitations is determined by the behavior of the wave functions and by the distribution of the level spacings of thermal excitations with energies $E_\gamma \lesssim \mu$, which depends on the trap symmetry. We emphasize that stochastization in the behavior of these thermal excitations plays a key role for damping in nonspherical traps. For quasiclassical ($E_\nu \gg \hbar\omega$) low-energy excitations the main contribution to the damping rate Γ_ν comes from the boundary region of the condensate, which makes Γ_ν completely different from that in a spatially homogeneous gas. The damping of the lowest excitations ($E_\nu \sim \hbar\omega$) is determined by the behavior of the excitations in the entire condensate region. For this case the damping rates following from our theory are in fair agreement with the data of the JILA [5] and MIT [7] experiments.

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Elementary excitations of a Bose condensate trapped in an external potential $V(\mathbf{r})$ are commonly defined within the Bogolyubov–de Gennes approach (see [24]) on the basis of the grand canonical Hamiltonian

$$\hat{H} = \int d\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r}) \Big[-\frac{\hbar^2}{2m}\Delta + V(\mathbf{r}) + \frac{\tilde{U}}{2}\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}) - \mu \Big] \hat{\Psi}(\mathbf{r}) \quad (3)$$

assuming a point interaction between atoms, with $\tilde{U} = 4\pi\hbar^2 a/m$, m the atom mass, and a the (positive) scattering length. The field operator of atoms $\hat{\Psi}$ is represented as the sum of the above-condensate part $\hat{\Psi}'$ and the condensate wave function $\Psi_0 = \langle \hat{\Psi} \rangle$. Omitting the terms proportional to $\hat{\Psi}'^3$ and $\hat{\Psi}'^4$ and using the generalized Bogolyubov transformation $\hat{\Psi}'(\mathbf{r}) = \sum_\nu \hat{b}_\nu u_\nu(\mathbf{r}) - \hat{b}_\nu^\dagger v_\nu^*(\mathbf{r})$, where $\hat{b}_\nu$, $\hat{b}_\nu^\dagger$ are annihilation and creation operators of elementary excitations, the Hamiltonian (3) is reduced to the diagonal form $\hat{H} = \hat{H}_0 + \sum_\nu E_\nu \hat{b}_\nu^\dagger \hat{b}_\nu$, if the functions u_ν, v_ν satisfy the equations

$$\left[\frac{-\hbar^2\Delta}{2m} + V(\mathbf{r}) - \mu\right]\begin{bmatrix} u_\nu \\ v_\nu \end{bmatrix} + \tilde{U}|\Psi_0|^2\left(2\begin{bmatrix} u_\nu \\ v_\nu \end{bmatrix} - \begin{bmatrix} v_\nu \\ u_\nu \end{bmatrix}\right) = E_\nu \begin{bmatrix} u_\nu \\ -v_\nu \end{bmatrix}. \quad (4)$$

The condensate wave function Ψ_0 is determined by the well-known Gross-Pitaevskii equation. In the Thomas-Fermi regime, where $\mu \approx n_{0m}\tilde{U}$ (n_{0m} is the maximum condensate density) greatly exceeds $\hbar\omega$, one has [25]: $\Psi_0(\mathbf{r}) = [(\mu - V(\mathbf{r}))/\tilde{U}]^{1/2}$ for $\mu \geq V(\mathbf{r})$ and zero otherwise. The low-energy excitations ($E_\nu \ll \mu$) are localized in the condensate spatial region, and in a harmonic potential $V(\mathbf{r})$ the functions $f_\nu^\pm = u_\nu \pm v_\nu$ are [16]

$$f_\nu^\pm = \left[\frac{2n_0(\mathbf{r})\tilde{U}}{E_\nu}\right]^{\pm 1/2}\left(\prod_i l_i\right)^{-1/2} W_\nu(r_i/l_i), \quad (5)$$

where $n_0(\mathbf{r}) = |\Psi_0(\mathbf{r})|^2$ is the condensate density, $l_i = (2\mu/m\omega_i^2)^{1/2}$ the characteristic size of the condensate in the ith direction, and ω_i the ith trap frequency.

Interaction between the excitations, caused by the terms proportional to $\hat{\Psi}'^3$ and $\hat{\Psi}'^4$ in Eq. (3), leads to damping. Below we will assume the inequality

$$(T/n_{0m}\tilde{U})\,(n_{0m}a^3)^{1/3} \ll 1, \quad (6)$$

which is fulfilled up to $T \approx 0.9T_c$ for the conditions of the JILA [6] and MIT [7] experiments. Just Eq. (6) ensures that the contribution to the damping rate of low-energy excitations from the $\hat{\Psi}'^3$ terms is the largest, i.e., the damping is actually caused by the interaction of the low-energy excitations with thermal excitations through the condensate and governed by the processes (2). The corresponding interaction Hamiltonian reads

$$\hat{H}_{\mathrm{int}} = \tilde{U}\int d^3r\, \Psi_0 \hat{\Psi}'^\dagger(\hat{\Psi}'^\dagger + \hat{\Psi}')\hat{\Psi}'. \quad (7)$$

Second, under condition (6) the damping rate can be found within the first-order perturbation theory in H_{int}:

$$\Gamma_\nu = \mathrm{Im}\sum_{\gamma\gamma'} \frac{1}{\hbar}\frac{|\langle\gamma'|\hat{H}_{\mathrm{int}}|\nu\gamma\rangle|^2}{E_\gamma - E_{\gamma'} + E_\nu + i0}(N_\gamma - N_{\gamma'}), \quad (8)$$

where $N_\gamma = [\exp(E_\gamma/T) - 1]^{-1}$ are equilibrium occupation numbers for the thermal excitations. The transition matrix element can be represented in the form

$$\langle\gamma'|\hat{H}_{\mathrm{int}}|\nu\gamma\rangle = \frac{\tilde{U}}{2}[3H_{\nu\gamma\gamma'} - (H_{\gamma'}^{\nu\gamma} - H_\gamma^{\nu\gamma'} - H_\nu^{\gamma\gamma'})], \quad (9)$$

where $H_{\nu\gamma\gamma'} = \int d^3r\, \Psi_0(\mathbf{r}) f_\nu^-(\mathbf{r}) f_\gamma^-(\mathbf{r}) f_{\gamma'}^{-*}(\mathbf{r})$ and $H_\nu^{\gamma\gamma'} = \int d^3r\, \Psi_0(\mathbf{r}) f_\nu^-(\mathbf{r}) f_\gamma^+(\mathbf{r}) f_{\gamma'}^{+*}(\mathbf{r})$.

Since energies of the thermal excitations $E_\gamma \gg \hbar\omega$, these excitations are quasiclassical and, similarly to the spatially homogeneous case, one can write $f_\gamma^\pm(\mathbf{r}) = (E_\gamma/\{\sqrt{E_\gamma^2 + [n_0(\mathbf{r})\tilde{U}]^2} - n_0(\mathbf{r})\tilde{U}\})^{\pm 1/2} f_\gamma(\mathbf{r})$. Then, using Eq. (5), from Eq. (9) we obtain

$$\langle\gamma'|\hat{H}_{\mathrm{int}}|\nu\gamma\rangle = \left(\frac{E_\nu\tilde{U}}{2\prod_i l_i}\right)^{1/2}\int d^3r\, \Phi_{\nu\gamma}(\mathbf{r}) f_\gamma(\mathbf{r}) f_{\gamma'}^*(\mathbf{r}), \quad (10)$$

where $\Phi_{\nu\gamma}(\mathbf{r}) = W_\nu(r_i/l_i)F_\gamma(\mathbf{r})$, and

$$F_\gamma(\mathbf{r}) = \frac{2E_\gamma^2 + [n_0(\mathbf{r})\tilde{U}]^2 - n_0(\mathbf{r})\tilde{U}\sqrt{E_\gamma^2 + [n_0(\mathbf{r})\tilde{U}]^2}}{E_\gamma\sqrt{E_\gamma^2 + [n_0(\mathbf{r})\tilde{U}]^2}}. \quad (11)$$

For the distribution of energy levels of the thermal excitations with a given set of quantum numbers $\tilde{\gamma}$ determined by the trap symmetry (in cylindrically symmetric traps $\tilde{\gamma}$ is the projection M of the orbital angular momentum on the symmetry axis) we will use the statistical Wigner-Dyson [26,27] approach which assumes ergodic behavior of the excitations. Then, the quantum spectrum of the thermal excitations is random and the sum in Eq. (8) can be replaced by the integral $\int dE_\gamma\, dE_{\gamma'} \sum_{\tilde{\gamma}\tilde{\gamma}'} g_\gamma g_{\gamma'} R_{\gamma\gamma'}$ in which $g_\gamma(E_\gamma)$ is the density of states for the excitations with a given set $\tilde{\gamma}$, and $R_{\gamma\gamma'}$ the level correlation function. In nonspherical harmonic traps $g_\gamma E_\nu \gg 1$ and, hence, $R_{\gamma\gamma'} \approx 1$. Then, putting $N_\gamma - N_{\gamma'} = E_\nu(dN_\gamma/dE_\gamma)$ and writing $(E_\gamma - E_{\gamma'} + E_\nu + i0)^{-1}$ as the integral over time $i\int_0^\infty dt \exp\{i(E_\gamma - E_{\gamma'} + E_\nu + i0)t/\hbar\}$, from Eqs. (8) and (10) we obtain

$$\Gamma_\nu = \frac{E_\nu^2\tilde{U}}{2\hbar^2\prod_i l_i}\,\mathrm{Re}\sum_{\tilde{\gamma}}\int g_\gamma \frac{dN_\gamma}{dE_\gamma}dE_\gamma \int_0^\infty dt\, \exp\left\{i\frac{(E_\nu + i0)t}{\hbar}\right\}\int d^3r\, d^3r'\, \Phi_{\nu\gamma}(\mathbf{r})\Phi_{\nu\gamma}^*(\mathbf{r}')K_\gamma(\mathbf{r},\mathbf{r}',t), \quad (12)$$

where the quantum-mechanical correlation function

$$K_\gamma(\mathbf{r},\mathbf{r}',t) = \sum_{\tilde\gamma'} \int g_{\gamma'}\, dE_{\gamma'} \exp\left\{ i\,\frac{(E_\gamma - E_{\gamma'})t}{\hbar}\right\} \times f_\gamma(\mathbf{r}) f_\gamma^*(\mathbf{r}') f_{\gamma'}^*(\mathbf{r}) f_{\gamma'}(\mathbf{r}')\,. \quad (13)$$

In our calculation of the damping rate Γ_ν we will turn from the integration over the quantum states of the quasiclassical thermal excitations to the integration along the classical trajectories of motion of Bogolyubov-type quasiparticles in the trap. Following a general method (see [28–30]), for $K_\gamma(\mathbf{r},\mathbf{r}',t)$ we can write

$$\sum_{\tilde\gamma} g_\gamma K_\gamma(\mathbf{r},\mathbf{r}',t) = \int \delta(\mathbf{r}'' - \mathbf{r})\delta(\mathbf{r}_{\rm cl}(t|\mathbf{r}'',\mathbf{p}) - \mathbf{r}') \times \delta(E_\gamma - H(p,\mathbf{r}''))\,\frac{d^3p\, d^3r''}{(2\pi\hbar)^3}\,, \quad (14)$$

where $H(p,\mathbf{r}) = \sqrt{(p^2/2m)^2 + 2n_0(\mathbf{r})\tilde U(p^2/2m)}$ is the Bogolyubov Hamiltonian, and $\mathbf{r}_{\rm cl}(t|\mathbf{r},\mathbf{p})$ the coordinate of the classical trajectory with initial momentum $\mathbf{p}$ and coordinate $\mathbf{r}$. Then Eq. (12) is reduced to the form

$$\Gamma_\nu = \frac{E_\nu^2 \tilde U}{2\hbar^2 \prod_i l_i}\,\mathrm{Re}\int dE_\gamma\,\frac{dN_\gamma}{dE_\gamma}\int_0^\infty dt\,\exp\left(i\,\frac{E_\nu t}{\hbar}\right)\int \Phi_{\nu\gamma}(\mathbf{r})\Phi^*_{\nu\gamma}(\mathbf{r}_{\rm cl}(t|\mathbf{r},\mathbf{p}))\delta(E_\gamma - H(p,\mathbf{r}))\,\frac{d^3r\, d^3p}{(2\pi\hbar)^3}\,. \quad (15)$$

We first consider temperatures $T \gg \mu$, where the main contribution to the integral in Eq. (15) is provided by the thermal excitations with energies $E_\gamma \lesssim \mu$. In this case the use of the statistical approach in nonspherical traps is justified by the fact that, as shown in [31], the motion of corresponding classical Bogolyubov-type quasiparticles is strongly chaotic at energies of order μ. The characteristic values of p and t in Eq. (15) are of order $[mn_0(\mathbf{r})\tilde U]^{1/2}$ and $\hbar/E_\nu$, respectively. For quasiclassical low-energy excitations ($E_\nu \gg \hbar\omega$) this time scale is much shorter than ω^{-1} and important is only a small part of the classical trajectory, where the condensate density is practically constant and $\mathbf{r}_{\rm cl}(t|\mathbf{r},\mathbf{p}) = \mathbf{r} + \mathbf{v}t$, with $\mathbf{v} = \partial H/\partial\mathbf{p}$. Representing $\Phi_{\nu\gamma}(\mathbf{r})\Phi^*_{\nu\gamma}(\mathbf{r}_{\rm cl}(t|\mathbf{r},\mathbf{p}))$ as $|f_\nu(\mathbf{r})|^2F^2(\mathbf{r})\cos(\mathbf{p}_\nu\mathbf{v}t/\hbar)$, where p_ν is a classical momentum of the Bogolyubov-type quasiparticle, from Eq. (15) we obtain

$$\Gamma_\nu = \int d^3r\,|f_\nu(\mathbf{r})|^2\Gamma_{\nu h}(\mathbf{r})\,. \quad (16)$$

Here $\Gamma_{\nu h}(\mathbf{r})$ is the damping rate of the excitation with energy E_ν in a spatially homogeneous condensate of density $n_0(\mathbf{r})$. For $E_\nu \ll n_0(\mathbf{r})\tilde U$ we have [32]: $\Gamma_{\nu h} = E_\nu 3\pi^{3/2}(n_0a^3)^{1/2}T/(4n_0\tilde U\hbar)$.

For a trapped gas the result of integration in Eq. (16) drastically depends on the trapping geometry. For example, in cylindrically symmetric harmonic traps $|f_\nu(\mathbf{r})|^2 \propto \{E_\nu^2 + [n_0(\mathbf{r})\tilde U]^2\}^{-1/2}$ and strongly increases near the boundary of the condensate spatial region, where $n_0\tilde U \lesssim E_\nu$. Just this region of distances determines the density of states g_M and the damping rate Γ_ν. Here the contribution to Γ_ν from the thermal excitations with E_γ of order E_ν is important, and one should also include the Beliaev damping processes $\nu \leftrightarrow \gamma + \gamma'$. This requires a generalization of Eqs. (10)–(15). Omitting the details of the calculation which will be published elsewhere, we present here the result for $M = 0$:

$$\Gamma_\nu \approx 9\left(\frac{E_\nu}{\mu}\right)^{1/2}\frac{T}{\hbar}\,\frac{(n_{0m}a^3)^{1/2}}{\ln(2\mu/E_\nu)}\,. \quad (17)$$

For the lowest excitations ($E_\nu \sim \hbar\omega$) the characteristic values of p in Eq. (15) are of order $(m\mu)^{1/2}$, and the result of integration can be represented in the form

$$\Gamma_\nu = A_\nu\,\frac{E_\nu}{\hbar}\,\frac{T}{\mu}\,(n_{0m}a^3)^{1/2}, \quad (18)$$

where A_ν is a numerical coefficient which depends on the form of the wave function of the low-energy excitation ν. In contrast to the case of $E_\nu \gg \hbar\omega$, the calculation of A_ν requires a full knowledge of classical trajectories of (stochastic) motion of Bogolyubov-type quasiparticles in the spatially inhomogeneous Bose-condensed gas.

The criterion of the collisionless regime for the excitations with energies $E_\gamma \sim \mu$ assumes that their damping time Γ_μ^{-1} is much larger than the oscillation period in the trap ω^{-1} and, hence, the mean free path greatly exceeds the size of the condensate. From Eq. (16) we find $\Gamma_\mu \sim (T/\hbar)\,(n_{0m}a^3)^{1/2}$ and obtain the collisionless criterion

$$(T/\hbar\omega)\,(n_{0m}a^3)^{1/2} \ll 1\,. \quad (19)$$

Because of collective character of the excitations Eq. (19) is different from the Knudsen criterion in ordinary gases.

Remarkably, both Eq. (19) and the assumption of stochastic behavior of thermal excitations with energies of order μ are well satisfied in the conditions of the JILA [6] and MIT [7] experiments, where the temperature dependent damping of the lowest quadrupole excitations in cylindrically symmetric traps has been measured at temperatures significantly larger than μ. The JILA experiment [6], where the ratio of the axial to radial frequency $\beta = \omega_z/\omega_\rho = \sqrt{8}$, concerns the damping of two quadrupole excitations: $M = 2$, $E_\nu = \sqrt{2}\,\omega_\rho$, and $M = 0$, $E_\nu = 1.8\omega_\rho$. Our numerical calculation of Eq. (15), with W_ν from [16], gives $A_\nu \approx 7$ for $M = 2$ and $A_\nu \approx 5$ for $M = 0$. This leads to the damping rate $\Gamma_\nu(T)$ which is in agreement with the experimental data [6] (see Fig. 1). In the MIT experiment [7], where $\beta = 0.08$, the damping rate has been measured for the quadrupole excitation with $M = 0$, $E_\nu = 1.58\omega_z$. In this case we obtain $A_\nu \approx 10$. The corresponding damping rate $\Gamma_\nu(T)$ monotonously increases with T and for the conditions of the MIT experiment [7] ranges from 4 s^{-1} at $T \approx 200$ nK to 18 s^{-1} at $T \approx 800$ nK, which is in fair agreement with the preliminary experimental data.

Importantly, under the condition (19) the damping rate Γ_ν of the low-energy excitations is much larger than the

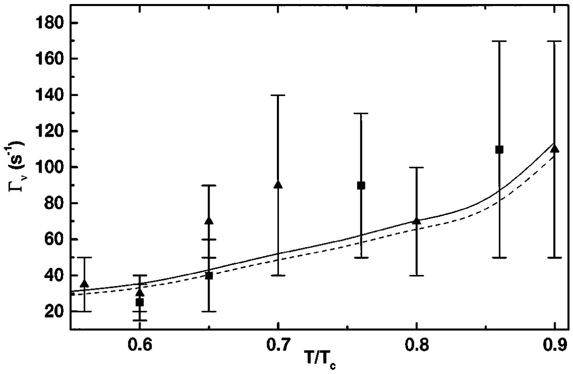

FIG. 1. The damping rate Γ_ν versus T for the JILA trapping geometry. The solid (dashed) curve and boxes (triangles) correspond to our calculation and the experimental data [6] for the excitations with $M = 2$ ($M = 0$), respectively.

damping rate Γ_T of the oscillations of the thermal cloud. This phenomenon was observed at JILA [6]. One can easily find that for $T \gg \mu$ the damping rate $\Gamma_T \sim n\sigma v_T$, where $n \sim (mT/2\pi\hbar^2)^{3/2}$ is the characteristic density of the thermal cloud, $\sigma = 8\pi a^2$ the elastic cross section, and $v_T \sim \sqrt{T/m}$ the thermal velocity. Accordingly, the ratio Γ_T/Γ_ν is just of order the left-hand side of Eq. (19).

In spherically symmetric traps at any excitation energies one has a complete separation of variables, which means that the classical motion of Bogolyubov-type quasiparticles is regular. The excitations are characterized by the orbital angular momentum l and its projection M, and for given l, M the level spacing $g_\gamma^{-1} \sim \hbar\omega$ can greatly exceed the interactions provided by the non-Bogolyubov Hamiltonian terms proportional to Ψ'^3 and Ψ'^4. In such a situation the discrete structure of the energy spectrum of thermal excitations becomes important, and one can get nonlinear resonances instead of damping. On the other hand, stochastization of motion of thermal excitations can be provided by their interaction with each other or with the heat bath. In this case the damping rate Γ_ν (18) follows directly from Eq. (12) by using the Dyson relation for the level correlation function [27] ($g_\gamma E_\nu \sim 1$) and f_γ from the WKB analysis of Eq. (4).

For $T \lesssim \mu$ the picture of damping of low-energy excitations changes, since Γ_ν will be determined by the contribution of thermal excitations with energies $E_\gamma \sim T$. In this case, the lower is the ratio T/μ, the more questionable is the assumption of ergodic behavior of the thermal excitations. But, even if the stochastization is present, at T significantly lower than μ the temperature dependent damping of the lowest excitations will be rather small. For cylindrically symmetric traps from Eqs. (12) and (15) one can find $\Gamma_\nu \sim (E_\nu/\hbar)(T/\mu)^{3/2}(n_{0m}a^3)^{1/2}$.

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