How Is the Coulomb Blockade Suppressed in High-Conductance Tunnel Junctions?

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We present a theory of the Coulomb blockade for a tunnel junction, with arbitrarily large tunnel conductance but with small channel transmissions, placed in a general electromagnetic environment containing no island. Our model, based on a self-consistent calculation of the complex admittance of the junction, predicts that high tunnel conductances wash out the Coulomb blockade and restore Ohmic behavior. We find quantitative agreement between predictions and measurements of the dc conductance of small metallic tunnel junctions in series with a resistor. [S0031-9007(98)05514-8]

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For tunnel junctions with tunnel conductance G_T much smaller than the conductance quantum $G_K = e^2/h \approx 38.7\ \mu\mathrm{S}$, the charge q transferred through the junction corresponds to an integer number of electrons. In the case of such a junction embedded in an electromagnetic circuit containing no island [see Fig. 1(a)], the conduction mechanism consists of single electron tunnel events in which one electron charge flows through the environment of the tunnel element. Single electron tunneling is an "inelastic" process in the sense that part of the electrostatic energy dissipated in the tunnel event is transferred to the electromagnetic environment and not to the quasiparticle degrees of freedom in the electrodes. These inelastic processes result in a reduction of the differential conductance of the junction at low voltage. The theory of this so-called environmental Coulomb blockade of tunneling has been worked out in the regime $G_T \ll G_K$, for arbitrary environment and temperature [1–3], and found in agreement with experiments [4,5]. On the other hand, when $G_T \gg G_K$ one expects that quantum fluctuations of q suppress the Coulomb blockade. In this Letter, we present an extension of the theory of the environmental Coulomb blockade which treats tunnel junctions with arbitrary conductance G_T. We compare the predictions of this theory with measurements of the dc conductance of small metallic tunnel junctions in series with a resistor.

The circuit we consider, shown in Fig. 1(a), consists of a pure tunnel element connected in series with an arbitrary linear electromagnetic environment. In this picture, the capacitance of the real tunnel junction has been incorporated into the impedance $Z(\omega)$ of the electromagnetic environment [3]. The Hamiltonian of the circuit is the following:

$$H = H_0 + \sum_{n=1}^{N} H_{Tn}\,,$$

with

$$H_0 = \sum_{\ell} \varepsilon_\ell c_\ell^\dagger c_\ell + \sum_{r} \varepsilon_r c_r^\dagger c_r + H_{\mathrm{env}}\,.$$

Here, ℓ and r denote quasiparticle levels in the left and right electrodes, H_{env} is the Hamiltonian of the electromagnetic environment, N is the total number of transverse channels, and $H_{Tn} = \Theta_n + \Theta_n^\dagger$ is the tunneling Hamiltonian of the nth channel with $\Theta_n = e^{i\varphi} \sum_{n,\ell,r} \tau_{n\ell r} c_{n\ell}^\dagger c_{nr}$. In this latter operator, the phase difference operator φ, conjugated with q, acts on the electromagnetic environment, and is related to the voltage drop v across the tunnel element by $v = (\hbar/e)\dot{\varphi}$. The current operator $I = \dot{q}$ through the tunnel element is thus $I = -(ie/\hbar)\sum_n(\Theta_n - \Theta_n^\dagger)$. In this paper, we calculate the junction conductance in perturbation theory with respect to H_T, for arbitrary values of $G_T = \sum_{n=1}^{N} g_n$, assuming as in the weak tunnel conductance theory [1–3] that the conductance $g_n = 4\pi^2 G_K \sum_{\ell r} |\tau_{n\ell r}|^2$ of any individual channel remains small. However, in contrast with this latter theory, we incorporate in the electromagnetic environment of any given channel of the junction the $(N-1)$ other ones. These other channels are

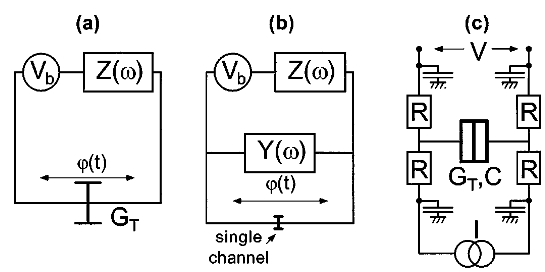

FIG. 1. (a) The circuit we consider consists of a pure tunnel element biased by a voltage source in series with an arbitrary impedance $Z(\omega)$. In this representation, the capacitance C of the real tunnel junction is incorporated in $Z(\omega)$. The pure tunnel element is characterized by its tunnel conductance G_T. (b) For junctions with a large number of weakly transmitting channels, the electromagnetic environment of any individual channel consists of the external impedance $Z(\omega)$ connected in parallel with the other channels. In our mean-field linear approximation, the other channels are replaced by the effective admittance $Y(\omega)$ of the tunnel element. (c) Four-probe measurement configuration of a junction used to test the predictions of the model. The resistors are microfabricated close to the junction and the capacitance of the leads connected to the external apparatus is so large that it can be considered in practice as a shunt to ground in the relevant frequency range. Our circuit thus implements circuits (a) and (b) with $Z(\omega) = R \parallel C$.

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described by an effective linear circuit whose frequency-dependent admittance is taken equal to the admittance $Y(\omega)$ of the whole tunnel element, neglecting $1/N$ corrections [see Fig. 1(b)]. This mean-field approximation leads to a self-consistent determination of $Y(\omega)$, which we now detail.

The linear response theory yields $Y(\omega) = i[S_I(\omega) - S_I(0)]/\omega$, where $S_I(\omega)$ is the current noise spectral density. This noise density is itself the Fourier transform of the current-current correlator function $\chi_{II}(t) = \langle [I(t), I(0)] \rangle \theta(t)$. The averages are evaluated by taking separate thermal equilibrium averages over the unperturbed quasiparticles and environmental degrees of freedom. This procedure is valid for the quasiparticles because each channel of the junction is weakly transmitting. The quasiparticle filling factors in the electrodes are thus Fermi functions at the inverse temperature β. The hypothesis of thermal equilibrium for the environment is exact at zero current but can be incorrect at finite current since energy is then injected in the environment. Exploiting the Gaussian character of the equilibrium phase fluctuations of a linear environment, one can write the average $\langle \exp i\varphi(t) \exp -i\varphi(0) \rangle$ as $\exp[ieVt/\hbar + J(t)]$, where V is the dc voltage across the junction and $J(t) = \langle [\tilde{\varphi}(t) - \tilde{\varphi}(0)]\tilde{\varphi}(0) \rangle$ is the phase-phase correlation function, $\tilde{\varphi}$ being the fluctuating part of φ [3]. Within these hypotheses, one obtains the following real-time expression for $Y(\omega)$ [6]:

$$Y(\omega) = G_T \left[1 + \int_0^{+\infty} \frac{2\pi}{(\hbar\beta)^2} \operatorname{csch}^2 \frac{\pi t}{\hbar\beta} \operatorname{Im} e^{J(t)} \times \cos \frac{eVt}{\hbar} \frac{e^{-i\omega t} - 1}{-i\omega} dt \right]. \quad (1)$$

The phase-phase correlation function $J(t)$ is itself related by the quantum fluctuation-dissipation theorem [3] to the impedance as seen from each channel of the junction:

$$J(t) = 2G_K \int_{-\infty}^{+\infty} \operatorname{Re}\left[\frac{1}{Y(\omega) + Z^{-1}(\omega)} \right] \times \frac{e^{-i\omega t} - 1}{1 - e^{-\beta\hbar\omega}} \frac{d\omega}{\omega}. \quad (2)$$

Note that, in our mean-field approximation, the total impedance across each channel of the junction is the proper parallel combination of the environmental impedance $Z(\omega)$ and of the admittance of the tunnel element itself. This writing of $J(t)$ generalizes that of Refs. [7,8], where G_T is used in place of $Y(\omega)$, corresponding to an Ohmic approximation of the tunnel element. In our formulation, the tunnel element admittance $Y(\omega)$ is obtained by solving self-consistently Eqs. (1) and (2). This procedure is similar to the self-consistent approach of Ref. [9], where a real frequency-dependent dissipation function is used in place of $Y(\omega)$. In our approach, $Y(\omega)$ is a complex quantity obeying Kramers-Kronig relations. Once $Y(\omega)$ is determined, the average dc current $I(V)$ is calculated as in the weak conductance case using the above phase-phase correlation function. In the limit where $G_T \rightarrow 0$, our self-consistent mean-field treatment of tunneling exactly coincides with the weak conductance theory [1–3]. In this limit, Eq. (1) generalizes at finite frequency the expression for the dc conductance of a tunnel junction in the real-time formalism [6,8,10]. In the opposite limit $G_T \rightarrow \infty$, our calculation predicts the suppression of the Coulomb blockade (i.e., Ohmic behavior of the junction), because the tunnel element shunts the external environment. In this regime, quantum fluctuations of $\tilde{\varphi}$ are suppressed and, correspondingly, q is completely delocalized, as expected. In the high temperature limit, our approach merges with the Ohmic approximation of the tunnel element. However, the hypothesis of linear behavior of the junction restricts the applicability of the model. Indeed, Göppert and Grabert [11] have recently shown, using the path integral formalism [12], that our approach is equivalent to replacing the exact periodic effective action of the tunnel junction by a self-consistent harmonic approximation [13] which minimizes the free energy. We thus expect that our calculation ceases to be valid when 2π phase jumps become important, i.e., at low temperature, for large environmental impedances [14].

We have tested the predictions of our model for the zero-voltage dc conductance $Y(\omega = 0)$ of single tunnel junctions embedded in an Ohmic environment. In our samples, the tunnel junction is connected to four nominally identical resistive leads [see Fig. 1(c)], to enable a four-probe measurement of the conductance of the sole junction. The samples are fabricated using standard e-beam lithography and multiple-angle shadow mask techniques [15]. The resistors consist of 10 μm long, 100 nm wide, and 5 nm thick Cr films evaporated at right angle with the substrate, and have nearly temperature-independent resistances of the order of 25 kΩ. In the same vacuum, we evaporate at an angle, a 15 nm thick Al layer which is then oxidized under a controlled O_2 pressure to grow the tunnel barrier. Finally, we evaporate a 20 nm thick Cu layer at the opposite angle to form the second electrode of the tunnel junction at an overlap with the oxidized Al layer. In this process the parasitic Al and Cu replicas of the resistors lay on the side of the resist and are removed during lift-off. The area of the tunnel junctions were nearly identical for the two samples we have measured (120 × 150 nm), corresponding to a number of channels $N \sim 10^6$. The samples are mounted in a copper shielding box anchored to the mixing chamber of a dilution refrigerator. The Al layer is driven in the normal state by applying a 1 T magnetic field. We measure the zero-voltage conductance of the sample by a lock-in technique near 70 Hz, through carefully filtered lines. The amplitude of the ac excitation is adapted to probe only the linear part of the characteristic. The variations of the zero-voltage conductance with temperature are shown in Fig. 2, for both samples. The comparison with theory involves the precise determination of the

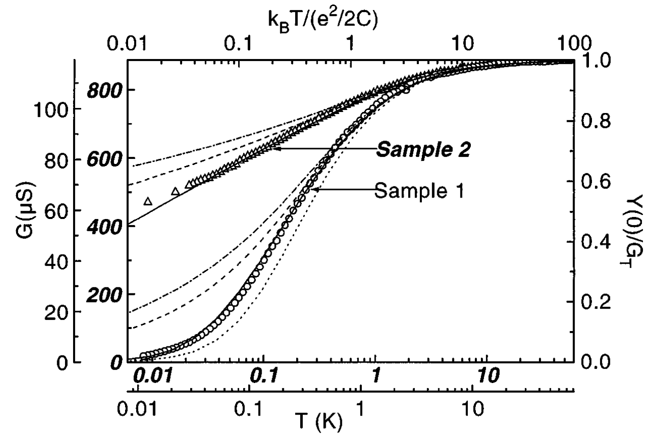

FIG. 2. Experimental zero-voltage conductance of our samples (bottom and left axes) and corresponding theoretical predictions, in reduced unit (top and right axes). The solid lines correspond to our self-consistent calculation, the dashed lines correspond to the Ohmic approximation of the tunnel element [6–8], the dash-dotted lines corresponds to the high temperature result of Ref. [17], and the dotted line is the prediction of the weak tunneling theory [1–3], nearly identical for the two samples. The values of the junction capacitances and tunnel conductances used for the comparison with theory are obtained from the high temperature data (see text).

sample tunnel conductance G_T, the junction capacitance C, and equivalent resistance R in series with the junction, the latter two defining the external impedance $Z(\omega) = R/(1 + iRC\omega)$. The resistance R is determined from the measured values of the four resistances. The junction capacitance and tunnel conductance are obtained by fitting the data between 2 and 70 K [16] with the exact high temperature expression of Ref. [17]. This gives for sample 1 (2), $R = 22.9$ kΩ (25.2 kΩ), $G_T = 119$ μS (886 μS), and $C = 1.13$ fF (1.54 fF). Using these values, we calculate the zero-voltage conductance $Y(0)$ at all temperatures, with no adjustable parameters. The functions $Y(\omega)$ and $J(t)$ are tabulated and calculated numerically by iterating Eqs. (1) and (2). The calculation is started by setting $Y(\omega) = G_T$ and converges within a few steps. The calculated dc conductance $Y(0)$ is in quantitative agreement with the measurements, as shown in Fig. 2. For comparison, we also plot the predictions of the weak conductance theory [1–3], the Ohmic approximation of the tunnel junction [6–8], and the exact high temperature result from the path integral formalism [17]. We see that the different predictions agree at high temperature [18], thus confirming our determination of the parameters G_T and C, but that at low temperatures our calculation predicts a Coulomb blockade intermediate between the weak tunneling model and the Ohmic approximation. We attribute the low temperature deviation between theory and experiment for sample 2 to spurious overheating of the resistors which, at low temperature, are almost thermally decoupled from the phonon bath [19]. Heating of the resistors is also the reason why we make no quantitative comparison at finite voltage. In principle, samples similar to ours constitute primary metrological thermometers at high temperature [20]. In view of our results, this kind of sample could possibly also be used as secondary thermometers on a much wider range of temperature, with self-calibration at high temperature.

To get further insight on the electrical behavior of tunnel junctions, in Fig. 3 we plot the calculated real and imaginary parts of the admittance of our two samples at different temperatures, for zero voltage. One sees that $Y(\omega)$ has a pronounced frequency dependence, except at very high temperature. The linear part of Im $Y(\omega)$ near zero frequency corresponds to an added capacitance $\Delta C = -iY'(0)$ which increases as T is decreased [6]. In Fig. 4 we plot the predicted temperature dependence of the dc conductance and of the relative capacitance increase of the junction $\Delta C/C$ at zero frequency for our samples. The model predicts that in a large temperature range the conductance varies logarithmically, while the capacitance increase follows a power law. Below a crossover temperature, the capacitance saturates and the system behaves as in the weak conductance regime but with a reduced charging energy $e^2/2(C + \Delta C)$. For sample 1 (2) where $G_T/G_K = 3.07$ (22.8), we find, as $T \to 0$, $\Delta C/C = 1.07$ ($\simeq$1500). This dramatic capacitance increase, which is the main consequence of increased tunnel conductance, was not captured in previous works which disregarded the effect of the reactance of the tunnel element on the dc conductance [6–9,13]. Indeed, calculating $J(t)$ with $Y(\omega) = [G_T^{-1} + (i\Delta C\omega)^{-1}]^{-1}$ already leads to a good estimate of the junction's conductance. In this approximation, the influence on a given channel of the other ones is mimicked by shunting the external environment by a fixed capacitance in series with a resistor. Note that a similar increase of the effective capacitance of high-conductance metallic junctions has been predicted [21] and observed [22] in single electron devices in which the Coulomb blockade is due to the charging energy of one or more almost isolated electrodes.

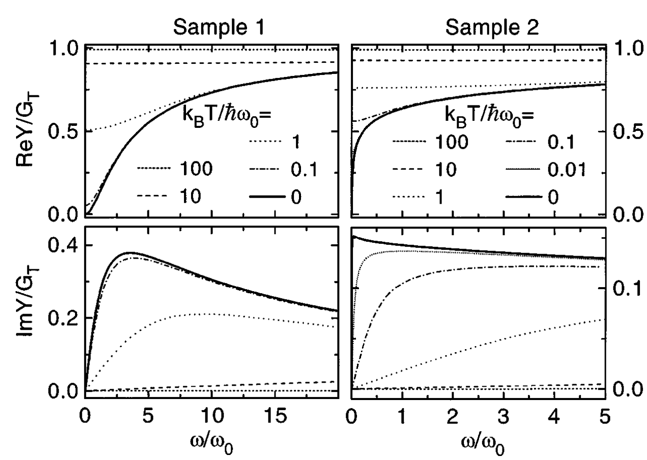

FIG. 3. Calculated real and imaginary parts of the admittance $Y(\omega)$ of the tunnel element for both samples. The angular frequency is normalized to $\omega_0 = G_K/C$.

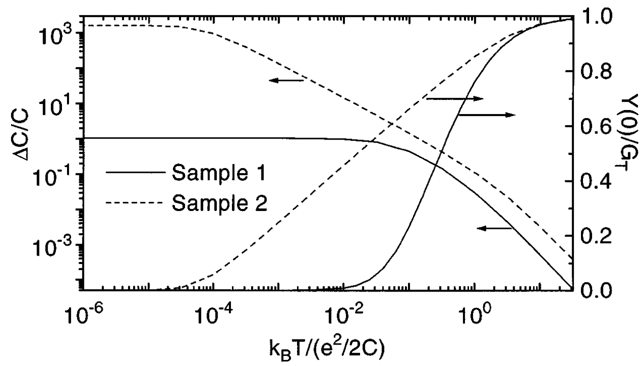

FIG. 4. Calculated conductance $Y(0)$ (right axis) and corresponding capacitance increase at low frequency (left axis), for both samples.

This capacitance increase results from the screening of the charge on the junction electrodes by virtual electron tunneling. The same effect also occurs in tunnel junctions with few conduction channels, such as quantum point contacts made in two-dimensional electron gases. In this case, the effective junction capacitance diverges as soon as one channel is perfectly transmitting [23].

In conclusion, we have derived a self-consistent equation for the complex admittance of a tunnel element of arbitrary tunnel conductance, in the linear response framework. Our model sheds a new light on the electrodynamics of a tunnel junction, and in particular, it shows that the capacitance increase occurring at low temperature in high-conductance tunnel junctions is the leading effect in the lifting of the Coulomb blockade. The experimental results at zero frequency in the case of an Ohmic environment agree quantitatively with the predictions.

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