Resonant Tunneling in an Aharonov-Bohm Ring with a Quantum Dot

Jian Wu,[1,2] Bing-Lin Gu,[1,2] Hao Chen,[3] Wenhui Duan,[2] and Yoshiyuki Kawazoe[4]
[1]*Center for Advanced Study, Tsinghua University, Beijing 100084, People's Republic of China*
[2]*Department of Physics, Tsinghua University, Beijing 100084, People's Republic of China*
[3]*Department of Physics, Fudan University, Shanghai 200433, People's Republic of China*
[4]*Institute for Materials Research, Tohoku University, Sendai 980-77, Japan*
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We investigate the resonant tunneling in an Aharonov-Bohm ring with a quantum dot embedded in one of its arms. A complete transmission mechanism, considering both the resonance of the dot and the interference effect, is presented. As a consequence, the experimental results, the phase features for conductance peaks, are well explained by a one-dimensional noninteracting model. New features concerning temperature effects are predicted. [S0031-9007(98)05304-6]

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The first measurement of the transmission phase through a mesoscopic ring was reported by Yacoby *et al.* in 1995 [1]. The authors devised a modified Aharonov-Bohm (AB) ring with a quantum dot embedded in one of its arms (see Fig. 1). Two distinguishing features were observed in this experiment: first, the phase of AB oscillations changes abruptly when the conductance of the AB ring reaches a peak; second, the AB oscillations at consecutive conductance peaks are in phase. Since then, many attempts [2–5] were made to explain those two features. One year later, the same group went on to measure the phase and magnitude of the reflection coefficient through a quantum dot by a new interferometry device [6]. While in a more recent experiment, they studied a modified interferometry system and observed similar features [7]. These experiments are of fundamental importance since they directly reveal the phase of electron transport, which is usually lost in measurements of transmission or reflection probabilities, and might provide additional information about a mesoscopic system. It has been shown theoretically that there should be a phase change of π for every conductance peak. However, the second feature that the AB oscillations at consecutive conductance peaks are in phase is not well understood yet [7].

It is the purpose of this Letter to present a complete transmission mechanism for such a system. This mechanism reveals the functions of both the dot and the ring. The system, a mesoscopic ring with a quantum dot embedded in one of its arms, can be divided into two parts: The small dot and its big complementary partner (CP) which is attached by two ideal leads. These two parts form a loop—an interferometry system. The states of the system can be divided into two sets: the states corresponding to the bound states of an isolated dot, and the preliminary states of the CP from which the electrons tunnel to the dot. The isolated CP can be considered as a quantum line with two stubs, which will cause small oscillations on the conductance, at both ends. The transmission character is basically dominated by the CP when there is no resonance in the transmission through the dot. However, while a bound state of the dot is close to the Fermi level, resonant tunneling occurs and the interference effect will appear. We found that the resonant tunneling through the whole system can be observed only when the phase shift introduced by the resonant state of the dot matches the transmission phase of the CP. In other words, if the phase shift introduced by the resonance of the dot causes constructive interference, the resonant peak of the conductance will appear. Otherwise, the resonance of the dot will not contribute to the total transmission because of the destructive interference.

In order to explain it more explicitly, we model the system by a one-dimensional noninteracting ring attached by two ideal leads from both sides. As Yeyati *et al.* did [2], we use a quantum well (dot potential ϵ_D) with two barriers (barrier heights ϵ_B) to simulate the dot. Then, the effective electrostatic potential on the ring is parametrized by the quantities ϵ_D, ϵ_B, and ϵ_0 (potential outside the dot), which is schematically represented in Fig. 1. Our aim is to investigate how the states of the dot affect the resonant tunneling of the system. We use a tight-binding (TB) representation of the electron states. To study the multienergy-level effects of the dot, a number of sites are assumed to be present in the well region. The effect of the magnetic flux ϕ is included by introducing the Peierls phase factor of the hopping integral t. Then the

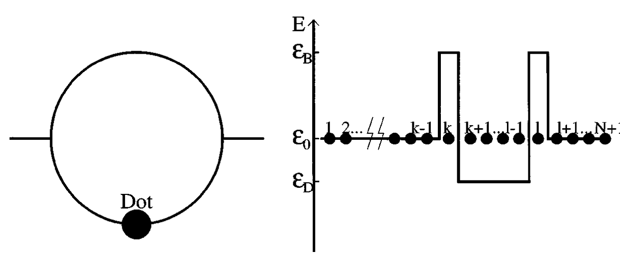

FIG. 1. Geometry of an Aharonov-Bohm ring with a quantum dot embedded in one of its arms (left) and the on-site energy in the tight-binding model (right).

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Hamiltonian of the N sites ring can be written as

$$\mathcal{H} = \sum_{j=1}^{N} (\epsilon_j |j\rangle\langle j| - e^{i\varphi}|j\rangle\langle j+1| - e^{-i\varphi}|j+1\rangle\langle j|), \tag{1}$$

where the TB hopping integral, $t = \hbar^2/2m^*a^2$, is taken to be unity as the energy unit, and $|N+1\rangle$ is identical with $|1\rangle$. As shown in Fig. 1, the on-site energy ϵ_j is equal to the effective electrostatic potential on the lattice site. In this Letter, $\epsilon_0 = 0$ and $\epsilon_B = 3$ if not specified. The phase factor is given by $\varphi = 2\pi\phi/N\phi_0$, where ϕ_0 is the elementary flux quantum $\phi_0 = hc/e$.

The conductance of the system is calculated by use of the real space Green's function (RGF) and temperature Kubo formula [8–10]. Here, only the Green's function G of the isolated sample, ring with a dot, is required in the calculation [10]. Moreover, the local density of states (LDOS) can be conveniently obtained from the RGF by

$$\mathrm{LDOS} = -\frac{1}{\pi}\,\mathrm{Im}\,G_{jj}\,, \tag{2}$$

where G_{jj} is the diagonal matrix element of the RGF [9]. Thus, the density of states (DOS) within the dot, and then the energy levels of the dot, can be obtained in sequence.

Now, let us focus on the two features of the experiment [1]. In Fig. 2, we plotted the conductance g and the phase β of AB oscillations versus dot potential ϵ_D at zero magnetic flux. Here β is defined to be the phase introduced by the magnetic flux at which the conductance has a maximum, and the temperature T is taken to be zero. We observe the abrupt phase change and in-phase AB oscillations at consecutive conductance peaks. Meanwhile, it is also interesting to note that there is another abrupt phase change of π between two peaks, just similar to the results mentioned in Ref. [7]. This should be observed, otherwise we would not have observed the in-phase peaks. In order to reveal the nature of this phase change, we calculated the DOS within the dot near the Fermi level and the LDOS of the dot at the Fermi level in the new phase change region. The results are given in Fig. 3. For the sake of comparison, the DOS within the dot near the Fermi level and the

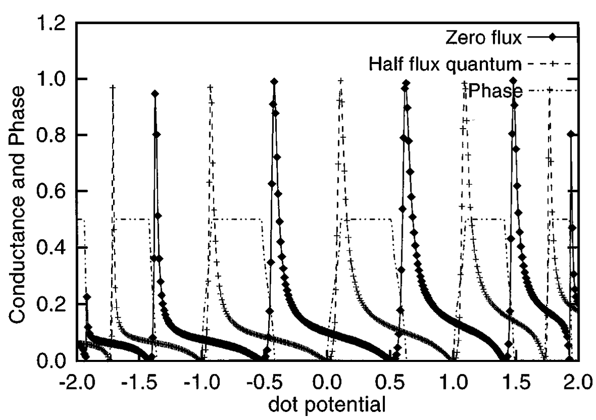

FIG. 2. Conductance (in units of e^2/h) as a function of the dot potential ϵ_D for different magnetic flux. The marks give the calculated values of the conductance. The dashed line without marks represents the phase (in units of 2π) of the AB oscillations.

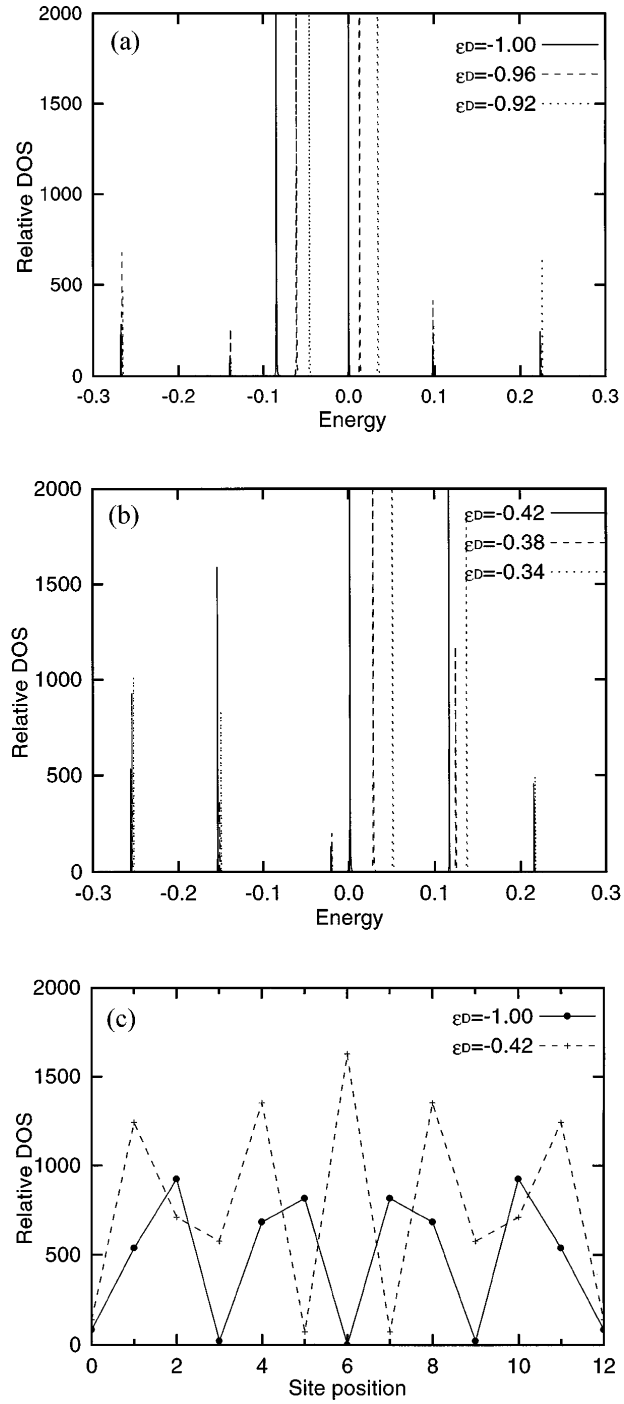

FIG. 3. (a) The relative DOS in an antiphase region (for different dot potential ϵ_D) as a function of energy. (b) The relative DOS in the adjacent in-phase region (for different dot potential ϵ_D) as a function of energy. (c) The relative LDOS at the Fermi level at the site positions. The calculated values are given as marks. The lines are given only as a guide.

LDOS of the dot at the Fermi level for an adjacent conductance peak are also given. By tracing the movement of the energy levels with increasing dot potential ϵ_D, we find also a bound state of the dot near the Fermi level [Fig. 3(a)] in the new phase change region. This is the same as the case for the adjacent conductance peak [Fig. 3(b)]. And the two bound states, the state near the Fermi level in the new phase change region and the state contributing to the adjacent conductance peak, correspond to the adjacent states in an isolated dot. They are shifted to the Fermi level by the well potential ϵ_D one by one. As shown in Fig. 3(c), they are antiphase, which is also a direct result of the quantum theory. If we notice the existence of the AB ring, an interferometer, it can easily be understood that the additional phase change of π in this resonant state will cause destructive interference between the two arms of the ring, and then will not contribute to the conductance. How are the things going on if we introduce another phase change of π? Motivated by this thought, the conductance g versus dot potential ϵ_D at half flux quantum is also calculated, and the result is given in Fig. 2. It can be seen that new peaks appear exactly in the new phase change region, and there are no peaks in the old region. This implies that the resonance of the dot does introduce phase shifts between two arms and results in interference. This result suggests an interesting experiment, which would check our conjecture.

Another interesting feature of our result is the finite width of the phase change region. Though previous authors alleged that there should be a sharp jump of zero width [2], a finite width was actually observed in the experiment [1]. In Ref. [2], Yeyati and Büttiker assumed that the conductance (as a function of flux) can be presented by a uniform functional form. However, it has been pointed out that the resonance of the system cannot be described by the concept of a "phase shift" [4] in the phase change region. What happens instead is that the conductance changes its functional form, just like the linear conductance in Ref. [4]. Obviously, the magnetic field will cause the shift of the energy levels. The phase change reaches a value of π only after the resonant state of the dot has passed through the Fermi level at all magnetic fields. Therefore, the effect of the flux on the energy levels results in the finite width of the phase change region. The scale of the transition is determined by the energy difference of two adjacent states of the CP, but not the difference of the bound states of the dot.

What is the transmission mechanism of such a system? The curve of the conductance resonant oscillations versus the plunger gate voltage on the dot showed that the resonance is related to the resonant states of the dot [7]. But in this interferometer system, the coherence effect caused by the ring cannot be ignored. The conductance peaks appear only when the phase shift introduced by the resonant state of the dot matches the transmission phase of the CP. Otherwise, the destructive interference will result in the decrease of the conductance (as observed in Fig. 2). In addition, the maximum conductance at a definite dot potential is given as the function of the dot potential ϵ_D in Fig. 4. It can be seen that once the additional phase due to the magnetic flux causes the phase matching, the resonant states of the dot will contribute to the conductance, and result in conductance peaks.

To complete our study, the temperature effects are calculated and shown in Fig. 5. It is well known that high temperatures are disadvantageous to quantum effects. With increasing temperatures the quantum oscillations of the resonant tunneling decrease gradually. What surprised us is the appearance of new peaks in the antiphase region. How can it occur? The answer is in Fig. 3. There are some energy levels which are close to the resonant state of the dot. By tracing these energy levels in different well potential ϵ_D, we found that these levels are affected little by the ϵ_D in most cases. In other words, these states belong to the tunneling states of the CP, which will contribute to the conductance. By comparing Figs. 3(a) with 3(b), we found five tunneling states of the CP near the Fermi level whose relative DOS is small in normal case. The original positions of states are, respectively, at $E = -0.27, -0.15, -0.02, 0.10$, and 0.22. Meanwhile, the resonant state of the dot, whose relative DOS is much larger than the others, moves from the Fermi level to higher energies with increasing well potential ϵ_D. When the resonant state of the dot passes through a tunneling state to an in-phase region, the latter will not be affected. In other words, the two states will form accidental degenerate states and the amplitude of the DOS will remain unchanged. However, when the resonant state of the dot is passing through a tunneling state [for example, the state at $-0.04 < E < -0.01$ in Fig. 3(a), and the state at $0.10 < E < 0.15$ in Fig. 3(b)] to the antiphase region, it will not be degenerate with the latter. (For convenience, we note this tunneling state as state P in the following context.) It appears that the resonant state and tunneling state repel each other. Meanwhile,

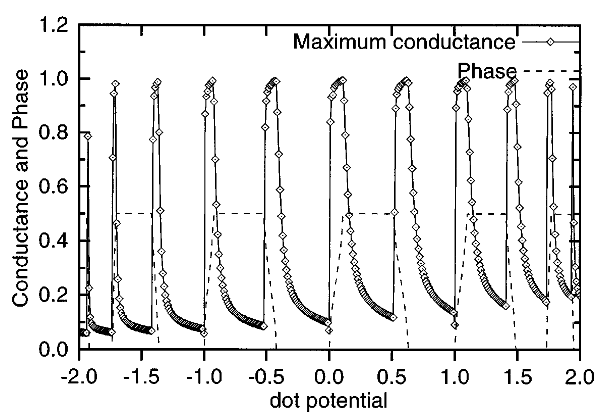

FIG. 4. The maximum conductance (in units of e^2/h) at a definite dot potential ϵ_D as the function of the dot potential. The phase of the AB oscillations is replotted contrastively.

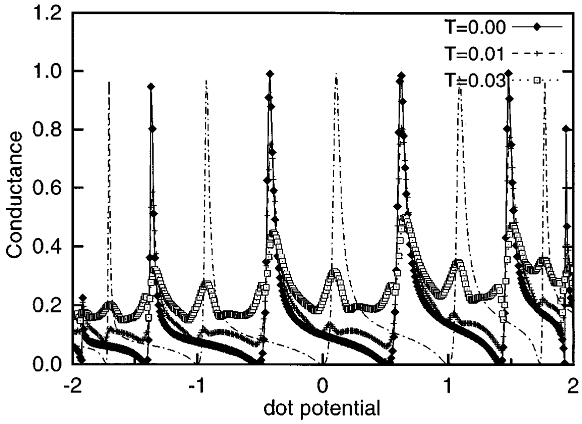

FIG. 5. Conductance (in units of e^2/h) as a function of the dot potential ϵ_D for different temperatures at zero magnetic field. The marks give the calculated values of the conductance. The conductance for $T = 0$ at half flux quantum is given as a dotted line for comparison.

the amplitude of the DOS for the tunneling state P increases greatly. This is very similar to the case of electrons at Bloch zone boundaries. Actually, these phenomena are caused by the same reason—periodicity. Moreover, the boundary condition $|N + 1\rangle = |1\rangle$ in the ring is much more restricted than the one for a lattice. The repelling effect can be understood by the degenerate perturbation theory. In this case, the phase shift by the resonant state of the dot is also favorable to the transmission state of the CP, from which the electrons tunnel to the dot to form the tunneling state P. As a result, the tunneling state P is enhanced greatly. This is just the case that the first order perturbation is nonzero in the degenerate perturbation cases. Consequently, the two states are separated away. As shown in Fig. 3(a), when the resonant state of dot is shifted to the Fermi level, the tunneling state P is repelled away from the Fermi level. With increasing ϵ_D, the resonant state is shifted away, and the tunneling state P is shifted back. Therefore, at finite temperatures, the contribution by the tunneling state P leads to an increase of the conductance with increasing ϵ_D. Simultaneously, as the resonant state of the dot is shifted away from the tunneling state P, the conductance decreases due to decaying resonant effect. The formation of new peaks are determined by the competition between the tunneling and resonant effects. With increasing temperature, the contributions from the tunneling state P become dominant and consequently the new peaks appear.

In conclusion, we have presented a complete mechanism for the interference phenomena in an AB ring with a quantum dot embedded in one of its arms. The electron transmission through such a system is characterized by both the resonance of the dot and the interference effect. The experimental results are well explained by a one-dimensional noninteracting resonant tunneling model of electron transmission. An interesting experiment is suggested to support our viewpoint. A new phenomenon, the temperature effect in such systems, is predicted.

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