Controlling Spiral Waves in a Model of Two-Dimensional Arrays of Chua's Circuits

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(Received 16 June 1997)

A model of two-dimensional arrays of Chua's circuits is numerically investigated. In a certain parameter region the spatiotemporal system has both synchronized oscillation and spiral wave attractors. Feedback pinnings are suggested to migrate the system from the spiral wave state to the coherent oscillation. The influences of the pinning density, forcing strength, and different pinning distributions on the driving effect are investigated. It is shown that some properly designed control schemes may reach very high control efficiency, i.e., killing a spiral wave consisting of a huge number of cells by injecting only very few cells. The wide applications of the approach are addressed. [S0031-9007(98)05390-3]

PACS numbers: 05.45.+b, 47.54.+r, 82.20.Wt

Spiral waves are observed in oscillatory and excitable media; they belong to the most intriguing spatiotemporal patterns in nonequilibrium systems. Recently, the investigation of spiral waves has attracted much attention [1–14] due to its great potential of applications. Spiral waves appear very commonly in nature; they are observed in biology systems such as cardiac muscle tissue, in chemical systems like Belousov-Zhabotinsky (BZ) reaction, and in hydrodynamic systems where spiral wave cores are responsible for the formations of defects and defect-mediated turbulence. Thus, the interest of spiral waves covers a wide range of fields. It is fascinating if one can find some effective approach to control spiral waves; this is just the main task of the present Letter. Spiral waves are often considered as harmful objects; they may cause some fatal diseases in biological bodies (cardiac disease is one of the most serious ones of this kind), heavy damages in storms, unwanted disorders in hydrodynamical systems, and so on. Thus, in this Letter we regard spiral waves as undesirable objects and seek possible effective control methods to kill them.

We numerically investigate the following model of coupled Chua's circuits:

$$\begin{aligned}
\dot{x}_{i,j} &= \alpha[y_{i,j} - x_{i,j} - g(x_{i,j})] + D[x_{i+1,j} + x_{i-1,j} + x_{i,j+1} + x_{i,j-1} - 4x_{i,j}],\\
\dot{y}_{i,j} &= x_{i,j} - y_{i,j} + z_{i,j},\\
\dot{z}_{i,j} &= -\beta y_{i,j} \qquad (i,j = 1,2,\ldots,N),\\
g(x) &= (1/2)[(s_1 + s_2)x + (s_0 - s_1)(|x - B_1| - |B_1|) + (s_2 - s_0)(|x - B_2| - |B_2|)],
\end{aligned} \tag{1}$$

where each site obeys the dynamics of single Chua's circuit, and nearest diffusive couplings are applied to x variable. Free boundary conditions $x_{0,j} = x_{1,j}$, $x_{n+1,j} = x_{N,j}$, $x_{i,0} = x_{i,1}$, and $x_{i,N+1} = x_{i,N}$ are used. Throughout the paper the parameters are fixed at $\alpha = 10$, $\beta = 0.334\,091$, $s_1 = 0.020\,706$, $s_2 = 15$, $s_0 = -0.921$, $B_1 = -1$, and $B_2 = 0.059\,148\,6$, at which each individual cell has a limit cycle attractor shown in Fig. 1(a). The diffusion coefficient is taken to be $D = 5$, at which the homogeneous oscillation synchronized to Fig. 1(a) is a stable state of the system, which is our aim state after killing spiral wave. Nevertheless, from an arbitrary initial state, the system can hardly approach this aim state due to the coexistence of a huge number of attractors. In particular, for certain preparations, one can easily find a spiral wave because of the high relaxational character of the cells. For instance, if we take the initial conditions shown in Figs. 1(b), 1(c), and 1(d) (for specific values of these initial distributions, see Ref. [2]), the system evolves to a spiral wave asymptotically, Fig. 1(e) for 48×48 and Fig. 1(f) for 100×100 lattices, respectively. This spiral wave is stable in the sense that it is insensitive to small noise impacts and to the slight change of the initial condition, and it persists forever unless some external forces drive the system away. Now our central task is to kill the spiral waves of Figs. 1(e) and Fig. 1(f) and migrate the system to the wanted homogeneous oscillation.

Various global and local injections have been suggested for controlling spiral wave [3,4]. Here we will develop a local control approach by injecting signals to few space units, and then propagating the control through coupling to free sites [15]. Now we feedback (1) with the aim state of Fig. 1(a) as

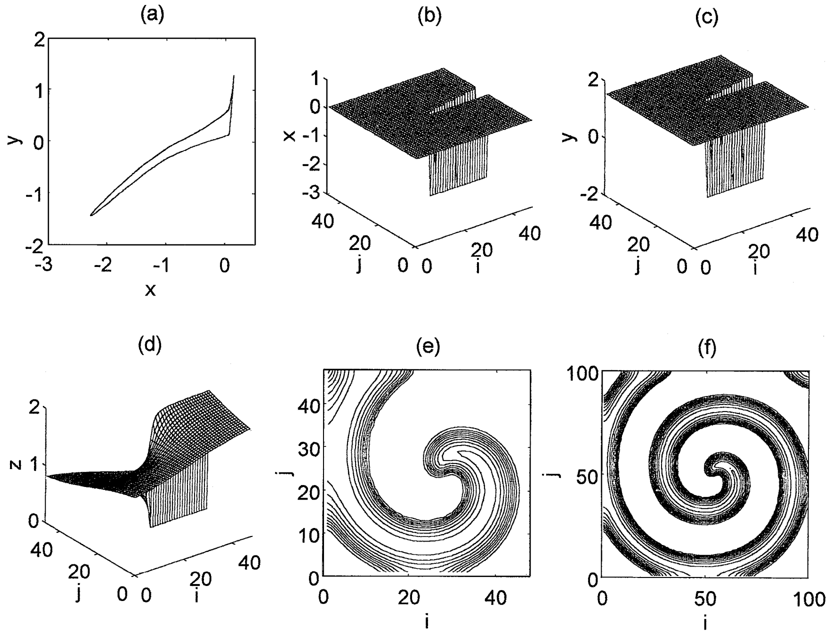

FIG. 1. (a) The limit cycle of a single cell of Eqs. (1), which will be used as the aim state for synchronization. (b), (c), and (d) The initial preparations of $x_{i,j}$, $y_{i,j}$, and $z_{i,j}$, respectively. (e) and (f) Contour diagrams of x variable (the same in all the following contours) of the asymptotic spiral wave state developed from the initial conditions of (b)–(d) for 48×48 lattice (e) and 100×100 lattice (f), respectively. $D = 5$ (the same D is also used below).

$$\dot{x}_{i,j} = M_{i,j} - \Theta_{i,j}\lambda[x_{i,j}(t) - \hat{x}],$$
$$\dot{y}_{i,j} = N_{i,j}, \qquad \dot{z}_{i,j} = O_{i,j}, \tag{2}$$

where M, N, and O are given in Eqs. (1), and $\hat{x}(t)$ is the periodic trajectory given in Fig. 1(a). λ represents the control strength. In testing the control efficiency, we first try some regular control schemes by setting $\Theta_{i,j}$ to $\Theta_{i,j} = 1$, $i = nI$; $j = mJ$; $n, m = 1, 2, \ldots$, and $\Theta_{i,j} = 0$ otherwise, where I and J are positive integers. In the following we will call the control with I and J as (I, J) control. For a (I, J) control the density of forced cells is roughly equal to $\rho = 1/(I \times J)$. We hope, of course, to use lower density ρ to make effective immigration. It is interesting to investigate how the efficiency of control depends on the arrangement of I and J. In Fig. 2(a) we take 48×48 lattice and plot the regions for successful control in $(1/\rho) - \lambda$ plane (controllable regions are to the upper left of the corresponding curves). The three solid curves from lower to upper represent the controls $(1, J)$, $(2, J)$, and $(3, J)$, respectively, while the dashed curve shows more uniformly (I, I) control. Some features of Fig. 2(a) are worth noting. First, the uniform (I, I) control is less effective. In the best case, we can kill the spiral wave by applying $(4, 4)$ control, and then the pinning density is $1/16$. We cannot destroy the spiral wave by $(5, 5)$ control for the control strength as large as $\lambda > 200$. The nonuniform controls (I, J) $I < J$ (or $J < I$) are more effective. The reason can be heuristically understood. Spiral waves can survive only in a moving phase. Small I's (or small J) build up strong control walls to cut the moving road of the wave, and then wipe the object away. In the best case, we can kill the spiral wave by applying $(3, 16)$ control, which corresponds to the efficiency of $\rho = 1/48$. Both $(4, 16)$ and $(3, 17)$ controls fail for arbitrary control strength ($\lambda > 200$ for our numerical tests). In the former case, there exist holes in the walls big enough for the spiral wave passing, and, in the latter case, there is a space between two control middle walls large enough for the core of the spiral wave to survive. Two characteristic parts in the solid curves of Fig. 2(a) are interesting. First, we observe that all curves have a part parallel to $1/\rho$ axis at certain critical λ, and the control efficiency can be greatly enhanced after λ passes these critical values. Second, the horizontal part can turn to the vertical part at certain turning points, represented by black dots in Fig. 2(a). The dots show both the minimal ρ and the minimal λ for the given I ($I = 1, 2, 3$ for our curves), and the hard turnings at the dots indicate that greatly increasing ρ (or λ) cannot compensate a slight decreasing of λ (or ρ). In Figs. 2(b), 2(c), and 2(d) we present three kinds of controls in 100×100 lattice failing in erasing spiral waves. In Fig. 2(b) we use $(5, 5)$ control, and the entire spiral wave remains with certain modifications. In Fig. 2(c), $(3, 35)$ control is applied, and the middle part of the spiral wave survives while the side parts of the spiral wave are wiped out. In Fig. 2(d) we put control on the heart of the spiral wave (i.e., $\Theta_{i,j} = 1, 35 \le i, j \le 65$, and $\Theta_{i,j} = 0$ otherwise), and the spiral wave tip survives, in the counterclockwise moving fashion, although it cannot enter the central controlled part.

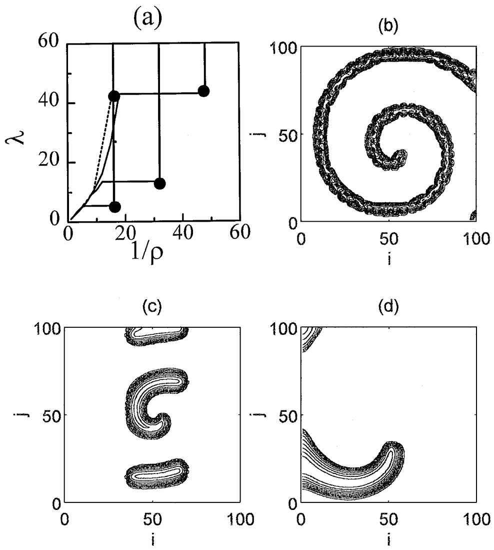

FIG. 2. (a) λ plotted vs $1/\rho = I \times J$. 48×48 lattice. Control can be successfully performed in the regions in the upper-left part of the curves. The solid curves from lower to upper represent $(1, J)$, $(2, J)$, and $(3, J)$ controls, respectively, and the dashed line shows (I, I) control. (b) 100×100 lattice, $\lambda = 200$. $(5, 5)$ control is applied. Spiral wave survives, (c) The same as (b) with $\lambda = 200$, $(3, 35)$ control is applied. A part of spiral wave survives in the middle strip while synchronized oscillations are realized in both left and right sides of the lattice. (d) The same as (b) with the core of the spiral wave controlled ($\Theta_{i,j} = 1, 35 \leq i, j \leq 65$, and $\Theta_{i,j} = 0$ otherwise). $\lambda = 200$. The spiral wave tip survives outside of the control region.

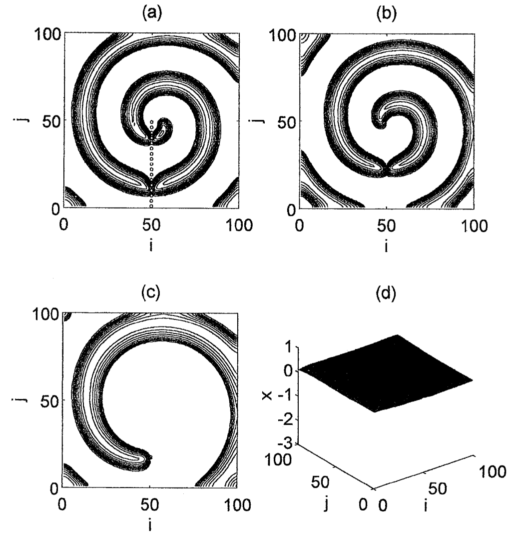

FIG. 3. Feedback controls are applied at $t = 0$ to 17 sites of Fig. 1(f) indicated by dots, i.e., $\Theta_{50,1+3n} = 1, n = 0-16$, and $\Theta_{i,j} = 0$ otherwise. $\lambda = 70$. (a) $t = 50$, (b) $t = 100$, (c) $t = 200$, and (d) $t = 350$. The spiral wave moves counterclockwise, and leaves the lattice from the lower-left boundary.

It is already noted in Fig. 2(a) that we can successfully kill the spiral wave and perform full synchronization by injecting only one equation among 144 equations when $(3, 16)$ control is applied. An interesting problem is whether we can further considerably enhance the control efficiency by designing more effective and more inhomogeneous pinning schemes. Since the spiral wave state is highly inhomogeneous, this possibility is obviously open. From Figs. 2(c) and 2(d) we can get some hints for doing this. From Fig. 2(c) we realize that for killing a spiral wave it is important to control its middle part. From Fig. 2(d) it is also clear that one can never successfully erase a spiral wave by controlling its core only. Therefore, an attracting idea may be to cut the moving channel of the spiral wave by injecting a middle line from a boundary to the core. For instance, we may use a control scheme $\Theta_{N/2,1+3n} = 1$, $n = 0, 1, 2, \ldots, K$, and $\Theta_{i,j} = 0$ otherwise. K should be large enough to spoil the tip of the spiral wave, and the wall with holes of width 2 sites can effectively stop the moving of the spiral wave. In Fig. 3 we use a 100×100 lattice, take $\lambda = 70$ and $K = 16$ [i.e., only 17 sites, indicated by the dots in Fig. 3(a), are injected], and plot the evolution of the spiral wave, starting from the spiral wave state of Fig. 1(f) at $t = 0$. We find that the spiral wave moves counterclockwise. Because of the control [the dots in Fig. 3(a)] the spiral wave cannot cross the lower half line at $i = 50$, and the object dies out on the lower-left boundary. Now by injecting only 17 equations a system consisting of $3 \times 100 \times 100$ equations can be fully under control, and the wanted state can be approached on purpose. After the synchronized state is approached, it can remain alive forever by lifting all the pinnings because the homogeneous oscillation is an attractor in our system.

Now we give a brief discussion on the conditions of successful control. First, a sufficiently large control strength is necessary. For instance, as λ is larger than the minimal value $\lambda \approx 68$, we can always make successful immigration from Fig. 1(f) to the homogeneous oscillation of Fig. 1(a) by using the control scheme of Fig. 3(a). However, by slightly reducing λ to $\lambda = 67$ the control completely collapses, and the entire spiral wave survives. An interesting point is that the collapse caused by slightly decreasing λ cannot be compensated by considerably increasing the number of injected sites. For instance, with $\lambda = 67$, we cannot erase the spiral wave even if we double the number of injected sites as $\Theta_{50,1+3n} = 1$, $n = 0, 1, 2, \ldots, 33$, and $\Theta_{i,j} = 0$ otherwise. This situation is similar to the horizontal parts of the solid curves in Fig. 2(a).

Second, a critical number of injected sites is also necessary. For instance, if we take $K = 15$, i.e., lift control only from one site, in the same time we increase the control strength up to $\lambda = 200$, the spiral wave exists in the asymptotic state. It is again striking that such enormously increasing control intensity cannot help to recover the controllability lost by lifting the pinning from a single site; the situation is similar to the vertical parts of the solid lines in Fig. 2(a). The reason can be intuitively interpreted as follows: By lifting the control at $i = 50$, $j = 49$, the spiral wave tip has enough space to survive, and the entire spiral wave can be developed and maintained from this tip, no matter how large λ is.

The key ingredient for erasing spiral wave is to kill the spiral wave tip. The spiral wave tip survives only in a rotating moving fashion, then for killing this tip it is of key importance to break its moving channel and to confine its rotating space. This idea leads to the above successful half pinning wall control of Fig. 3. In Fig. 4(a) we use $\lambda = 200$ and have the entire $i = 50$ line controlled except the three bottom and three top sites (i.e., $\Theta_{50,j} = 1$, $j = 4-97$, and $\Theta_{i,j} = 0$ otherwise). Although, comparing with Fig. 3(a), we have three times increased the control strength, and six times increased the number of controlled sites, the spiral wave tip still exists forever under the control. The tip moves counterclockwise, and crosses the control line through the top and bottom open channels (free sites). The moving tip can develop to a full spiral wave after we lift the control wall. If we shut one channel by having any one of the six free sites controlled, the spiral wave tip will eventually be annihilated, and the synchronized oscillation can finally be approached.

In Fig. 4(b), we use $\lambda = 200$ and $\Theta_{47,j} = 1$, $j = 1-100$, and $\Theta_{i,j} = 0$ otherwise. Now we control a whole line of sites while the line is slightly shifted to the side. In the asymptotic state we find half of a spiral wave in the right side, and the sites in the left side are practically synchronized to the aim oscillation. The reason why the half spiral wave can be maintained is clearly shown in Figs. 4(c) and 4(d) where the central tip of the spiral wave can survive after rotating to the right direction, and it continually generates half circles and keeps the half spiral wave alive. After moving the control line to the middle position (i.e., replacing $i = 47$ by $i = \mu$, $\mu \in [48, 60]$), the spiral wave tip does not have enough space to rotate to the right direction and generate new loops, and then this tip and the whole wave can be definitely cleaned away.

In conclusion, we have suggested an approach of controlling spiral wave by feedback pinnings. It is emphasized that the approach in this Letter is based on the characteristic features of spiral waves, not on the specific features of our particular systems, and thus this approach can be applicable to general problems of spiral wave control. We have tested this matter by considering other spiral wave systems, such as spiral waves in excitable media, and spiral waves in reaction diffusion systems [16], in the cases of tests our approach works perfectly, and then its wide applications can be expected. In this Letter, we consider controlling the spiral wave in a model system only; it would be significant to apply the presented method to real circuits, which will be our future work.

This work is supported by the National Nature Science Foundation of China and Nonlinear Science Project.

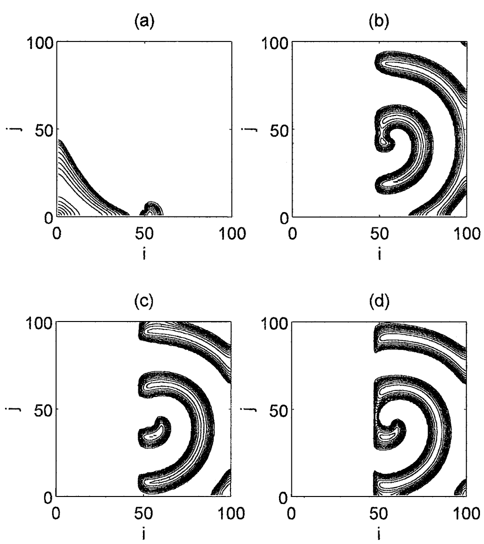

FIG. 4. (a) The same as Fig. 3 except changing $\lambda = 200$ and the control scheme to $\Theta_{50,j} = 1$, $j = 4-97$, and $\Theta_{i,j} = 0$ otherwise. The spiral wave tip survives forever, and moves counterclockwise by crossing the two top and bottom free channels. (b) The same as Fig. 3 except changing $\lambda = 200$ and $\Theta_{47,j} = 1$, $j = 1-100$, and $\Theta_{i,j} = 0$ otherwise. By slightly moving the control line to the left side, a half spiral wave survives in the right side asymptotically. (c), (d) The time evolution from the pattern (b) [(b) $t = 250$, (c) $t = 350$, and (d) $t = 500$]. All half circle waves are generated after the tip rotates to the right direction.

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