Regge Trajectories For All Flavors

Silvana Filipponi,[1,2] Giulia Pancheri,[3] and Yogendra Srivastava[2,4]
[1]*Physics Department, Harvard University, Cambridge, Massachusetts 02138*
[2]*Dipartimento di Fisica e INFN, Universita' di Perugia, Perugia, Italy*
[3]*Laboratori Nazionali di Frascati dell' INFN, Frascati, Rome, Italy*
[4]*Physics Department, Northeastern University, Boston, Massachusetts 02115*
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For a global analysis of mesonic resonances of all flavors, experimental data and phenomenological models are used to construct linear Regge trajectories. A satisfactory formula is found for the dependence of the intercept and slope on quark masses. We find agreement with data on charm production through exchange of our (spacelike) trajectories. Our results confirm the nonexistence of top meson and toponium. [S0031-9007(98)05398-8]

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Exactly how quarks are bound together to form hadronic bound states is not known. QCD is flavor independent, and it has been found difficult to compute the dependence of mesonic bound states (say) on the quark masses. In consequence, quite different approximation schemes and models exist which obtain the mass spectra for different flavors. There is as yet no overall picture providing an understanding of all the mesonic bound states.

In this Letter, we describe a different approach based on Regge trajectories to answer the generic question posed above: How do mesonic masses depend upon the quark masses? This formalism is particularly suited for strongly interacting systems since (i) Regge trajectories exist for all flavors, for large or small quark masses, and (ii) angular momentum becomes a continuous variable facilitating interpolation. Redundancy of data allows for self-consistency checks. As we show later, the same Regge trajectories constructed in the timelike region can be employed in the spacelike region (for exchange processes) as predictive tools. Our results *a posteriori* confirm this endeavor.

We made two crucial simplifying assumptions to obtain the Regge trajectories:

(i) All trajectories were assumed *linear* in $(\text{mass})^2$ of the hadronic state,

$$\alpha(s) = \alpha(0) + s\alpha'. \tag{1}$$

For light mesons (and baryons), we *know* this to be true experimentally.

(ii) The functional dependence of $\alpha(0)$ and α' on quark masses is via $(m_1 + m_2)$.

For light mesons, experimental data [1] and some models (Refs. [2,3]) give $\alpha'_{\text{light-light}} \approx 0.9\ \text{GeV}^{-2}$. We supplement this with experimental data [1] (for D and D_s mesons, charmonium and bottomium) and theoretical models (Refs. [4–6]) to obtain the following (purely phenomenological) simple formula giving a global description:

$$\alpha'(m_1 + m_2) = \frac{0.9\ \text{GeV}^{-2}}{[1 + 0.22(\frac{m_1+m_2}{\text{GeV}})^{3/2}]}, \tag{2}$$

where m_1 and m_2 are the corresponding constituent quark masses for that trajectory. Theoretical models are crucial inputs for heavy flavors, and quark masses refer to values actually employed therein. We show in Fig. 1, how well Eq. (2) compares with input data.

A similar analysis was performed for the intercept $\alpha_{\text{I}}(0)$, where the subscript I refers to the leading trajectory. Here we only consider mesonic systems for which the lowest physical state is at $J = 1$. A global description for these is given by

$$\alpha_{\text{I}}(m_1 + m_2; 0) = 0.57 - \frac{(m_1 + m_2)}{\text{GeV}}. \tag{3}$$

A comparison of Eq. (3) with input data is shown in Fig. 2. Only two points from a theoretical analysis [7] for the B_c system fall quite below our curve.

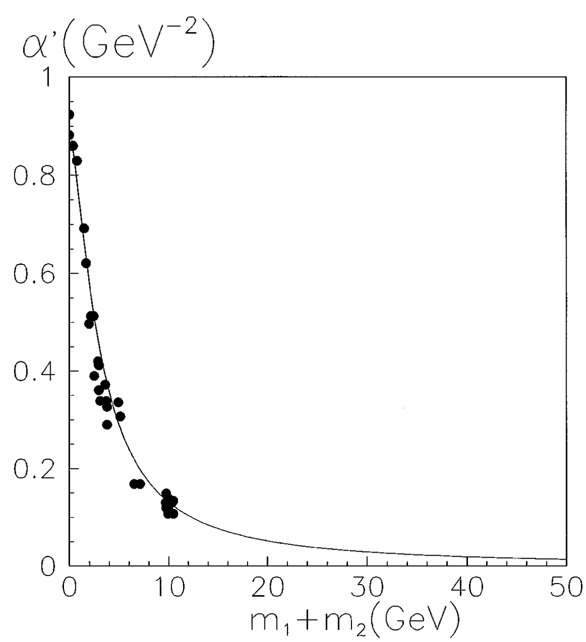

FIG. 1. Slope parameter of the Regge trajectories as a function of the sum of the constituent quark masses; input data compared with our analytic result Eq. (2).

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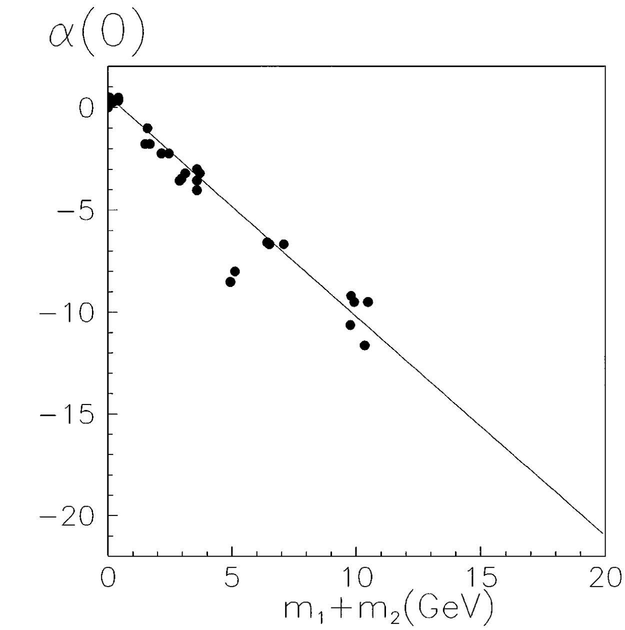

FIG. 2. The intercept parameter of the leading trajectory as a function of the sum of the constituent quark masses; comparison of Eq. (3) with input data.

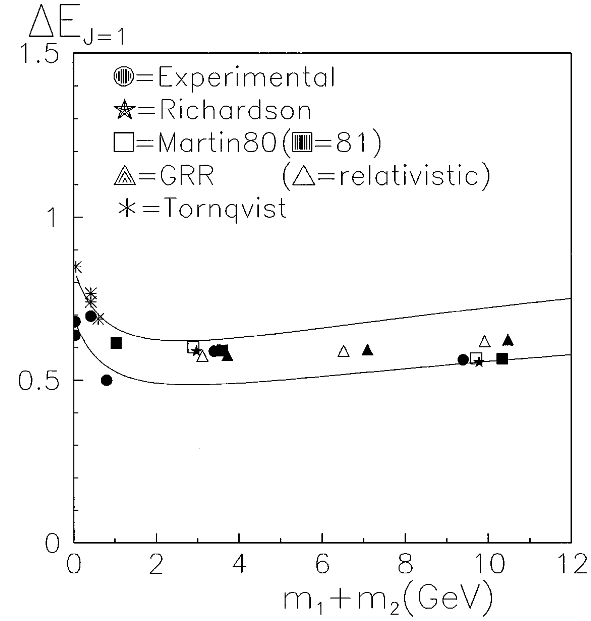

FIG. 3. Energy splitting for mesons of different flavors but the same angular momentum; our predictions compared to available data and some theoretical models.

We next consider secondary Regge trajectories and splitting between the energy levels for the same J of a given system. Calling $\alpha_{\mathrm{I,II}}(0)$ the leading and the secondary Regge trajectory intercept, we estimated the "distance"

$$\Delta\alpha(0) = \alpha_{\mathrm{I}}(0) - \alpha_{\mathrm{II}}(0) \tag{4}$$

from data (not very precise) and phenomenological models. We find a rather loose bound [8],

$$1.3 < \Delta\alpha(0) < 1.6\,. \tag{5}$$

The result becomes more interesting when transposed in terms of a physically more amenable quantity, viz., the energy splitting

$$\Delta E_J^{\mathrm{II\text{-}I}} = (E_J^{\mathrm{II}} - E_J^{\mathrm{I}}) \tag{6}$$

between states of the same angular momentum of a given system. Quite strikingly, it is found to be a constant (between 0.5–0.8 GeV) for all systems (composed of u, d, s, c, and b quarks). It is shown in Fig. 3. The approximate constancy of this energy difference for all J and all flavors gives us confidence in the generality of this result, and we expect it to be verified in all viable models.

Combining Eqs. (2) and (3), we are in a position to give a general expression for the leading mesonic Regge trajectory formed by any two quarks of masses m_1 and m_2

$$\alpha(m_1 + m_2; t) = 0.57 - \frac{(m_1 + m_2)}{\mathrm{GeV}} + \frac{0.9\ \mathrm{GeV}^{-2}}{[1 + 0.22(\frac{m_1+m_2}{\mathrm{GeV}})^{3/2}]}\, t\,. \tag{7}$$

In Fig. 4, we show the leading Regge trajectories for different flavors for spacelike and timelike regions ($-500 < t < 500\ \mathrm{GeV}^2$). In this figure, we have included the "top" and the "toponium" trajectories as well, even though, as we shall demonstrate later, top and toponium bound states cease to exist as physical states due to the fast weak decays of the top quark.

Equation (7) allows us to make "predictions" (or consistency checks) about the energy spectra of excited mesons for the D, D_s and B, B_s systems, which were not used as input data. Wherever data exist, reasonable agreement is found. A complete spectrum for mesons may be found in [9,10].

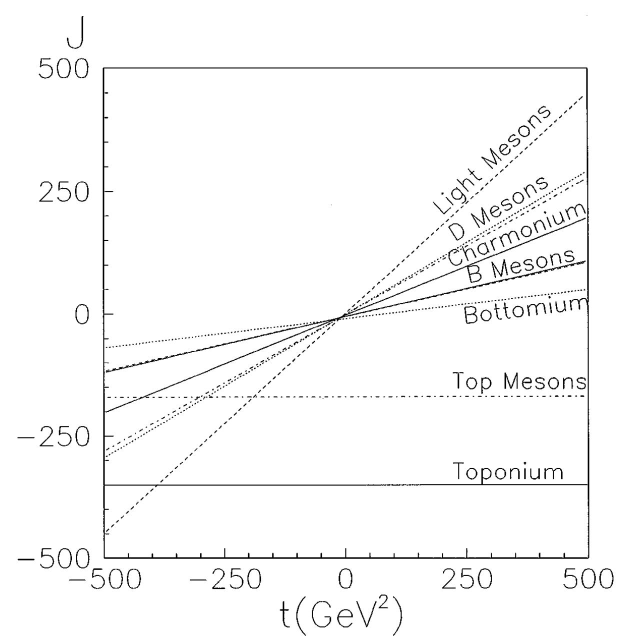

FIG. 4. Leading trajectories for different flavors for spacelike and timelike regions, as given by Eq. (7).

As a first application, we employ the analytic continuation into the spacelike region of these trajectories constructed through data in the timelike (resonance) region to discuss the inclusive production of heavy flavors through the di-triple Regge formalism. Data exist for D (Refs. [11–13]) and Λ_c (Ref. [14]) production reactions,

$$\pi + N \rightarrow D + X \quad \text{and} \quad \pi + N \rightarrow \Lambda_c + X \tag{8}$$

for a wide range of x_F. Assuming a factorized p_t dependence, experimental data have been parametrized through

$$\frac{d\sigma}{dx_F} \approx (1 - x_F)^n, \tag{9}$$

with $n \approx (3.7–3.9)$ (Refs. [11–13]), and $n \approx 3.5$ [14].

We invoke the di-triple Regge formula (Refs. [15,16]), valid for large M_X^2 and large (s/M_X^2),

$$\frac{d^2\sigma}{dM_X^2 dt} \rightarrow \frac{\gamma(t)}{s}\left(\frac{M_X^2}{s}\right)^{1-2\alpha(t)}, \tag{10}$$

where $\alpha(t)$ is the exchanged Regge trajectory. Neglecting the t dependence, we have in this region

$$\frac{d\sigma}{dx_F} \approx (1 - x_F)^{1-2\alpha(0)}. \tag{11}$$

For the reactions of Eq. (8), inserting a D Regge trajectory exchange, we have $\alpha(0) = -1.35$, leading to the exponent $n \approx 3.7$, in satisfactory agreement with the data. We stress that this is an important test since the spacelike extrapolation depends crucially on the slope parameter of the exchanged D trajectory (which is roughly half of that for the light system).

As a second application, consider the top system. It is generally believed that the top quark cannot form bound states either with another t quark or with a light one since the top quark would decay into a real W and b with a large width: The lifetime would be even smaller than the revolution time, thereby precluding the formation of mesonic bound states [17].

Our analysis confirms the above physical picture quite nicely. The energy splitting between the ground state and that lying on the second trajectory is as follows. For the toponium, we find $1.3 < \Delta E_{\text{toponium}} < 1.6$ GeV, in good agreement with Ref. [18]. For the top mesons, we find $1.1 < \Delta E_{\text{top}} < 1.3$ GeV. Neither can exist, since $\Delta E_{\text{toponium}} < 2\Gamma_t$ and $\Delta E_{\text{top}} < \Gamma_t$.

In conclusion, we have obtained a global unified description of linear Regge trajectories for all flavors, and their preliminary applications to spacelike as well as timelike regions seem encouraging. Further work in many directions is in progress. Inclusion of isospin, charge, and other quantum numbers can be incorporated perturbatively. So far we have also ignored the imaginary parts of the Regge trajectories $\alpha(s)$. A knowledge of $\text{Im}\alpha(s)$ in the timelike region is useful since it is directly related to the resonance widths. Through unitarity, it is also connected to the residue function $\beta(s)$, which when continued to the spacelike region is relevant for the Regge exchange amplitudes. If further work confirms the viability of this approach, an effective interaction should be constructed which is able to produce these results dynamically. We shall return to this question elsewhere. Here we shall limit ourselves to making one general observation. It appears that an interaction capable of generating linear Regge trajectories requires a linear mass dependence in the confining term in order to reproduce the $(m_1 + m_2)^{-3/2}$ behavior in the slope.

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