ρ-ω Mixing and Direct CP Violation in Hadronic B Decays

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(Received 28 May 1997; revised manuscript received 29 October 1997)

The extraction of Cabibbo-Kobayashi-Maskawa matrix element information from hadronic B decays generally suffers from discrete ambiguities, hampering the diagnosis of physics beyond the standard model. We show that a measurement of the rate asymmetry, which is CP violating, in $B^{\pm} \to \rho^{\pm}\rho^{0}(\omega) \to \rho^{\pm}\pi^{+}\pi^{-}$, where the invariant mass of the $\pi^{+}\pi^{-}$ pair is in the vicinity of the ω resonance, can remove the mod(π) uncertainty in $\alpha \equiv \arg[-V_{td}V_{tb}^{*}/(V_{ud}V_{ub}^{*})]$ present in standard analyses. [S0031-9007(98)05376-9]

PACS numbers: 11.30.Er, 11.30.Hv, 12.15.Hh, 13.20.He

Although CP violation in the neutral kaon system has been known since 1964 [1], it is not yet known whether the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and hence the standard model, provides a correct description of CP violation. The next generation of B-meson experiments will address both the empirical determination of the CKM matrix elements and the issue of whether a single CP-violating parameter, as in the standard model, suffices to explain them. Hadronic decays of B mesons will play an important role in elucidating the CKM matrix elements, and many clever methods have been devised to evade the uncertainties the strong interaction would weigh on their extraction [1]. Nevertheless, discrete ambiguities in the CKM matrix elements remain, for in $B^0 - \bar{B}^0$ mixing the weak phase ϕ enters as $\sin 2\phi$ [2]. It is our purpose to demonstrate that it is also possible to determine the sign of $\sin\phi$ through the measurement of the rate asymmetry in $B^{\pm} \to \rho^{\pm}\rho^{0}(\omega) \to \rho^{\pm}\pi^{+}\pi^{-}$, where the invariant mass of the $\pi^{+}\pi^{-}$ pair is in the ρ^0-ω interference region, so that the mod(π) ambiguity consequent to the $\sin 2\phi$ measurement is removed. This is necessary to test the so-called unitarity triangle associated with the CKM parameters α, β, and γ, for the standard model requires that these angles sum to π [3].

In $B^{\pm} \to \rho^{\pm}\rho^{0}(\omega) \to \rho^{\pm}\pi^{+}\pi^{-}$ decays, proposed by Enomoto and Tanabashi [4], CP violation is generated by the interference between a $b \to u$ tree amplitude and a $b \to d$ penguin amplitude, or their charge conjugates. The rate asymmetry, which is CP violating, arises exclusively from a nonzero phase in the CKM matrix, so that the CP violation is termed "direct." The latter's existence requires that at least two amplitudes contribute and that both a strong and weak phase difference exists between them [5]. If the internal top quark dominates the $b \to d$ penguin amplitude, then the weak phase in the rate asymmetry enters as $\sin\alpha$, where $\alpha \equiv \arg[-V_{td}V_{tb}^{*}/(V_{ud}V_{ub}^{*})]$ [3]. In $B^0 - \bar{B}^0$ mixing, in contrast, $\sin 2\alpha$ enters. Strategies to determine this latter quantity include the study of $B^0 \to \pi\pi$ [6], $B^0 \to \rho\pi$ [7], $B^0 \to \pi\pi$ and $B^0 \to \pi K$ [8], or the latter with B_s^0 decays as well [9]. The last two methods assume the top quark dominates the $b \to d$ penguin, although the non-negligible charm quark mass implies that the Glashow-Iliopoulos-Maiani (GIM) cancellation of the up and charm quark contributions to the $b \to d$ penguin is not perfect [10]. However, as the t quark penguin contribution is numerically larger [10], the sign of $\sin\phi$ suffices to determine that of $\sin\alpha$. For our purposes, then, ϕ is proportional to α.

Grossman and Quinn have suggested that the $B \to \pi\pi$ and $B \to \rho\pi$ analyses mentioned above can be combined to determine $\cos 2\alpha \sin\alpha$ and hence $\sin\alpha$ [2]. However, their analysis requires the use of the factorization approximation to estimate the sign of a ratio of hadronic matrix elements—the phase of this ratio is the strong phase. In the factorization approximation, the hadronic matrix elements of the four-quark operators are *assumed* to be saturated by vacuum intermediate states. This approximation can be justified in QCD in the limit of a large number of colors [11], and it finds phenomenological justification in a comparison with measured B-decay branching ratios [12]. We also use the factorization approximation, but the channel we propose has the important advantage that it permits a significant test of its applicability. This, we believe, is unique to the channel we study and is possible only because $e^{+}e^{-} \to \pi^{+}\pi^{-}$ data in the ρ^0-ω interference region provides additional hadronic information.

The CP-violating asymmetry in $B^{\pm} \to \rho^{\pm}\rho^{0}(\omega) \to \rho^{\pm}\pi^{+}\pi^{-}$ in the vicinity of the ω resonance is predicted to be more than 20% that of the summed decay rates with a branching ratio ($B^{\pm} \to \rho^{\pm}\rho^{0}$) of more than 10^{-5} [4,13,14]. Interfering resonances can generally both constrain and enhance the strong phase [15]. Here we can also extract the nonresonant strong phase; only its quadrant need be computed. To understand why the asymmetry is significantly enhanced by ρ^0-ω mixing,

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consider the amplitude A for $B^- \to \rho^- \pi^+ \pi^-$ decay:

$$A = \langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{T}} | B^- \rangle + \langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{P}} | B^- \rangle, \tag{1}$$

where $\mathcal{H}^{\mathrm{T}}$ and $\mathcal{H}^{\mathrm{P}}$ correspond to the tree and penguin diagrams, respectively. Defining the strong phase δ, the weak phase ϕ, and the magnitude r via

$$A = \langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{T}} | B^- \rangle [1 + r e^{i\delta} e^{i\phi}], \tag{2}$$

one has $\overline{A} = \langle \pi^+ \pi^- \rho^+ | \mathcal{H}^{\mathrm{T}} | B^+ \rangle [1 + r e^{i\delta} e^{-i\phi}]$. Here ϕ is $-\alpha$ if the top quark dominates the $b \to d$ penguin. Thus, the CP-violating asymmetry, A_{CP}, is

$$A_{CP} \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \frac{-2r \sin\delta \sin\phi}{1 + 2r\cos\delta\cos\phi + r^2}. \tag{3}$$

If we are to calculate A_{CP} reliably and hence determine $\sin\phi$ we need to know the strong phase, δ. Let t_V be the tree and p_V be the penguin amplitude for producing a vector meson V. In terms of the ρ and ω propagators, s_V^{-1} (with $s_V = s - m_V^2 + i m_V \Gamma_V$ and s the invariant mass of the $\pi^+\pi^-$ pair), and the effective ρ-ω mixing amplitude, $\tilde{\Pi}_{\rho\omega}$, we find

$$\begin{aligned} \langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{T}} | B^- \rangle &= \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} t_\omega + \frac{g_\rho}{s_\rho} t_\rho, \\ \langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{P}} | B^- \rangle &= \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} p_\omega + \frac{g_\rho}{s_\rho} p_\rho, \end{aligned} \tag{4}$$

where g_ρ is the $\rho^0 \to \pi^+\pi^-$ coupling. Maltman *et al.* [16] have considered the direct coupling $\omega \to \pi^+\pi^-$, not included in Ref. [4], as well as the "mixing" contribution $\omega \to \rho^0 \to \pi^+\pi^-$. Fortunately, this additional term can be absorbed into an energy-dependent, effective mixing amplitude, $\tilde{\Pi}_{\rho\omega}(s)$ [17]. The latter has recently been extracted [17] from the world data for the reaction $e^+e^- \to \pi^+\pi^-$ [18], for $\sqrt{s}$ near the ω mass. Assuming the SU(3) value of $1/3$ for the ratio of the ω to ρ photocouplings and adopting the form $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the best fit to the world data is $\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300\ \mathrm{MeV}^2$ and $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$ [17]. We have assumed that $\tilde{\Pi}_{\rho\omega}(s)$ is real in the resonance region [19]; relaxing this assumption yields $\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300\ \mathrm{MeV}^2$, with no change in the real part [17]. Note that if finite width corrections, of importance for the ρ, are included, the ratio of ω to ρ photocouplings decreases by about 10% [20] and $\tilde{\Pi}_{\rho\omega}(s)$ becomes more negative to the same degree. Such corrections do not impact the sign of this quantity, which is determined by the pion form factor in the vicinity of the ω. In contrast, Enomoto and Tanabashi adopt the model of Ref. [21] and make the above SU(3) assumption to find a real, s-independent $\tilde{\Pi}_{\rho\omega} = -0.63\Gamma_\omega m_\omega \approx -4100\ \mathrm{MeV}^2$ [4,22]. Using Eqs. (3) and (4) we find

$$r e^{i\delta} e^{i\phi} = \frac{\langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{P}} | B^- \rangle}{\langle \pi^+ \pi^- \rho^- | \mathcal{H}^{\mathrm{T}} | B^- \rangle} = \frac{\tilde{\Pi}_{\rho\omega} p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega} t_\omega + s_\omega t_\rho}. \tag{5}$$

Recalling the definitions of Ref. [4],

$$\frac{p_\omega}{t_\rho} \equiv r' e^{i(\delta_q + \phi)}, \qquad \frac{t_\omega}{t_\rho} \equiv \alpha e^{i\delta_\alpha}, \qquad \frac{p_\rho}{p_\omega} \equiv \beta e^{i\delta_\beta}, \tag{6}$$

one finds, to leading order in isospin violation,

$$r e^{i\delta} = \frac{r' e^{i\delta_q}}{s_\omega} \{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta}(s_\omega - \tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha})\}. \tag{7}$$

Note that δ_α, δ_β, and δ_q characterize the remaining unknown strong phases. In the absence of isospin violation, as characterized here by ρ^0-ω mixing, at least one of these phases would have to be nonzero in order to generate a rate asymmetry and hence CP violation. In the model of Bander *et al.* [5], these phases are generated by putting the quarks in loops on their mass shell and thus are referred to as "short-distance" phases.

The resonant enhancement of the CP-violating asymmetry is driven by $\tilde{\Pi}_{\rho\omega}/s_\omega$. We stress that β is essentially zero in $B^- \to \rho^-\pi^+\pi^-$ because the gluon in the strong penguin diagram couples to an isoscalar combination of $u\bar{u}$ and $d\bar{d}$. Thus it does not couple to an isovector ρ^0 in the absence of isospin violation. Hence, p_ρ as defined above is nonzero *only* if electroweak penguin diagrams, naively suppressed by $\alpha_{\mathrm{em}}/\alpha_s$, or isospin violating effects in the hadronic matrix elements which distinguish the $\rho^\pm$ and ρ^0 are included. Both effects were neglected in Ref. [4]. As $s \to m_\omega^2$, the asymmetry is maximized if $|\chi| = |\tilde{\Pi}_{\rho\omega}|/m_\omega\Gamma_\omega \sim O(1)$ and $\delta_q + \eta \sim \pm\pi/2$, where $\eta = -\arg s_\omega$. Using standard values for m_ω and Γ_ω [3] yields $|\chi| = 0.53$ and $\eta = -\pi/2$. Note that δ_q as estimated in the factorization approximation is $\gtrsim -\pi$ [4], so that $\delta_q + \eta \gtrsim -3\pi/2$ at the ω mass. Thus, the participation of the ω resonance, with its narrow width, strongly enhances the strong phase without the penalty of a severely small $|\chi|$.

The CP-violating asymmetry from Eqs. (3) and (7), then, is determined by $\tilde{\Pi}_{\rho\omega}$, m_ω, Γ_ω, and the short distance parameters α, δ_α, β, δ_β, r', δ_q, as well as ϕ, the weak phase which we wish to determine. The crucial issue is therefore how well the latter parameters can be determined. As the sign of $\sin\phi$ is of unique significance, our particular focus is on the short-distance phase information required to extract it without ambiguity. The sign of the CP-violating asymmetry in Eq. (3) is determined by $\sin\delta$ and $\sin\phi$. The sign of $\sin\delta$ is in turn determined by $\cos\delta_q\,\mathrm{Im}\,\Omega + \sin\delta_q\,\mathrm{Re}\,\Omega$, where $r\exp(i\delta) \equiv r'\Omega\exp(i\delta_q)/|s_\omega|^2$, noting Eq. (7). With $\chi \equiv \tilde{\Pi}_{\rho\omega}/(m_\omega\Gamma_\omega)$ as before, we find $|\chi| > \beta$, $\alpha \approx 1$, and $\mathrm{Im}\,\chi \ll \mathrm{Re}\,\chi$, so that for $s \approx m_\omega^2$,

$$r\sin\delta \approx \tilde{\Pi}_{\rho\omega} \frac{r'}{|s_\omega|^2} [(s - m_\omega^2)\sin\delta_q - m_\omega\Gamma_\omega\cos\delta_q]. \tag{8}$$

The sign of $\sin\delta$ at $s = m_\omega^2$ is thus given by $\tilde{\Pi}_{\rho\omega}$ and $\cos\delta_q$. The former is determined by e^+e^- data, but what of $\cos\delta_q$? We will use the factorization approximation to compute $\cos\delta_q$ and thus its sign, yet the above asymmetry can also be used to test its utility. As seen from Eq. (8), the shape of the asymmetry with the invariant mass of the $\pi^+\pi^-$ pair is primarily of Breit-Wigner form, as dictated by the $1/|s_\omega|^2$ factor, although it will be "skewed" if the coefficient of the $(s - m_\omega^2)$ term is nonzero. Thus, the sign of the skew of the asymmetry is driven by $\sin\delta_q$. We show below that the empirical s dependence of $\tilde{\Pi}_{\rho\omega}(s)$ about $s = m_\omega^2$ and the magnitude of its imaginary part at $s = m_\omega^2$ do not cloud this interpretation. Thus, we can extract $\tan\delta_q$. This does not fix the sign of $\cos\delta_q$, so that we use the factorization approximation to fix its sign and hence determine that of $\sin\phi$.

We have chosen to study just one of the channels considered in Ref. [4], namely, $B^\pm \to \rho^\pm\rho^0(\omega) \to \rho^\pm\pi^+\pi^-$, as it is of special character. In this case, the penguin amplitude to produce a ρ^0 meson is zero but for isospin violation, so that β is nonzero only when electroweak penguins and isospin violating effects which distinguish the $\rho^\pm$ and ρ^0 are included. As a consequence, we can associate the skew of the asymmetry with a single short-distance quantity, $\sin\delta_q$, and ultimately test the factorization approximation we apply.

In principle, the short-distance parameters can be computed by using the operator product expansion to construct an effective Hamiltonian at the b quark scale, $\mu \approx m_b$. It is usually written as a sum of tree and penguin contributions, as anticipated in Eq. (1). Following Ref. [23] we have, for example,

$$\mathcal{H}^{\rm eff} = \frac{4G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*\sum_{i=1}^{2}c_i(\mu)\mathcal{O}_i^{(u)} - V_{tb}V_{td}^*\sum_{i=3}^{10}c_i(\mu)\mathcal{O}_i^{(u)}\right\} + \text{H.c.}, \qquad (9)$$

with $\mathcal{O}_1^{(u)} = \bar{d}_{L\alpha}\gamma^\mu u_{L\beta}\bar{u}_{L\beta}\gamma_\mu b_{L\alpha}$ and $\mathcal{O}_2^{(u)} = \bar{d}_L\gamma^\mu u_L\bar{u}_L\gamma_\mu b_L$ [23]. Ten four-quark operators characterize the effective Hamiltonian; $i = 3,\ldots,6$ label the strong penguin operators, whereas $i = 7,\ldots,10$ label the electroweak penguins. The Wilson coefficients c_i are known through next-to-leading logarithmic order [23], yet consistency to one-loop order requires that the matrix elements be renormalized to one-loop order as well [1]. This renormalization procedure results in *effective* Wilson coefficients c_i' which multiply the matrix elements of the given operators at tree level. The effective Wilson coefficients develop an imaginary part if the quarks in loops are set on their mass shell; to compute them, we use the analytic expressions of Ref. [24] with a charm quark mass of $m_c = 1.35$ GeV. There is some sensitivity to k^2, the invariant mass of the exchanged boson, and thus we use two values of k^2, $k^2/m_b^2 = 0.3, 0.5$, covering the expected "physical" range [10].

We now turn to the evaluation of the matrix elements of this effective Hamiltonian. We use the factorization approximation, so that $\langle\rho_I^0\rho^-|\bar{d}_L\gamma^\mu u_L\bar{u}_L\gamma_\mu b_L|B^-\rangle = \langle\rho^-|\bar{d}_L\gamma^\mu u_L|0\rangle\langle\rho_I^0|\bar{u}_L\gamma_\mu b_L|B^-\rangle + (1/N_c)\langle\rho_I^0|\bar{u}_L\gamma^\mu \times u_L|0\rangle\langle\rho^-|\bar{d}_L\gamma_\mu b_L|B^-\rangle$. N_c is the number of colors, but here it is treated as a phenomenological parameter. Fits to measured branching ratios in $B \to D^*X$ decays indicate that the empirical value of the ratio $(c_1' + c_2'/N_c)/(c_2' + c_1'/N_c)$ is bounded by $N_c = 2$ and $N_c = 3$ [12]. Large N_c arguments justify the factorization approximation [11], yet $N_c = \infty$ yields a ratio of the wrong sign. Thus, we use $N_c = 2, 3$ as an empirically constrained gauge of the uncertainties inherent in the factorization approximation in what follows.

In the preceding paragraph, ρ_I^0 (and ω_I) denote isospin-pure states, for $\tilde{\Pi}_{\rho\omega}$ characterizes isospin violation in the ρ^0 and ω. Consistency requires that we compute all other sources of isospin violation to the same order. In particular, we need to estimate how isospin violation distinguishes $\rho^\pm$ from ρ_I^0 and ω_I in the hadronic matrix elements. This additional source of isospin breaking can be parametrized via

$$\frac{\langle\rho^-|\bar{d}_L\gamma^\mu u_L|0\rangle\langle\rho_I^0|\bar{u}_L\gamma_\mu b_L|B^-\rangle}{\langle\rho_I^0|\bar{u}_L\gamma^\mu u_L|0\rangle\langle\rho^-|\bar{d}_L\gamma_\mu b_L|B^-\rangle} \equiv 1 + \tilde{\varepsilon}. \qquad (10)$$

We use the model of Ref. [25] to evaluate the $B^- \to \rho^-, \rho_I^0$ transition form factors and find that $|\tilde{\varepsilon}|$ is no larger than 0.01. The resultant α, β, etc., as per Eq. (6), are, defining $\xi \equiv 4[(c_3' + c_4')(1 + 1/N_c) + c_5' + c_6'/N_c] + (c_9' + c_{10}')(1 + 1/N_c)$ and assuming t quark dominance, $\alpha\exp(i\delta_\alpha) = 1$,

$$\beta e^{i\delta_\beta} = \frac{3}{\xi}(c_9' + c_{10}')\left(1 + \frac{1}{N_c}\right) + \frac{2\tilde{\varepsilon}}{\xi}\left(c_4' + \frac{c_3'}{N_c} + c_{10}' + \frac{c_9'}{N_c}\right)\left[1 - \frac{3}{\xi}(c_9' + c_{10}')\left(1 + \frac{1}{N_c}\right)\right], \qquad (11a)$$

$$r'e^{i(\delta_q+\phi)} = -\left\{\frac{\xi + 2\tilde{\varepsilon}(c_3'/N_c + c_4' + c_9'/N_c + c_{10}')}{2(c_1' + c_2')(1 + 1/N_c)} - \frac{(c_1'/N_c + c_2')\tilde{\varepsilon}\xi}{2[(c_1' + c_2')(1 + 1/N_c)]^2}\right\}\frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}. \qquad (11b)$$

We use the parameters of Ref. [4] to evaluate V_{ij}. The CP-violating asymmetry, which follows from Eqs. (3), (6), (7), and (11), is shown in Fig. 1(a) as a function of the invariant mass of the $\pi^+\pi^-$ pair. The asymmetry is no less than 20% at the ω mass. The asymmetry really is driven by ρ^0-ω interference as the same asymmetries, now with $\text{Im}(c_i') = 0$, are shown in Fig. 1(b). In the absence of the short-distance phases, the asymmetry is symmetric about the ω mass. Figure 1(c) shows the sensitivity of the $N_c = 2$, $k^2/m_b^2 = 0.5$ asymmetry to the error in $\tilde{\Pi}_{\rho\omega}(m_\omega^2)$, and Fig. 1(d) shows the sensitivity of the same asymmetry to the allowed $\tilde{\Pi}'_{\rho\omega}$ and $\text{Im}(\tilde{\Pi}_{\rho\omega})$ contributions [17]. Both of the latter generate a slight skew to the shape of the asymmetry about the ω mass, yet

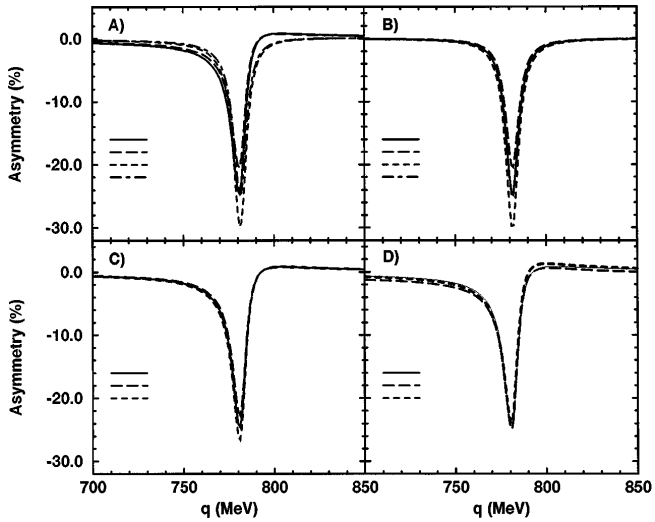

FIG. 1. The CP-violating asymmetry, Eq. (3), in percent, plotted versus the invariant mass q of the $\pi^+\pi^-$ pair in MeV for $[N_c, k^2/m_b^2]$. (a) The asymmetries with $\tilde{\Pi}_{\rho\omega} = -3500\ \mathrm{MeV}^2$ and $\tilde{\varepsilon} = -0.005$ are shown for $[2, 0.5]$ (solid line), $[3, 0.5]$ (long-dashed line), $[2, 0.3]$ (dashed line), and $[3, 0.3]$ (dot-dashed line). (b) The asymmetries of (a) are shown with $\mathrm{Im}(c_i^\prime) = 0$. (c) The $[2, 0.5]$ asymmetry of (a) is shown (solid line), with $\tilde{\Pi}_{\rho\omega} = -3200\ \mathrm{MeV}^2$ (long-dashed line) and with $\tilde{\Pi}_{\rho\omega} = -3800\ \mathrm{MeV}^2$ (dashed line). (d) The $[2, 0.5]$ asymmetry of (a) is shown (solid line), with $\tilde{\Pi}^\prime_{\rho\omega} = 0.027$ (long-dashed line) and with $\mathrm{Im}(\tilde{\Pi}_{\rho\omega}) = -300\ \mathrm{MeV}^2$ (dashed line).

these effects are sufficiently small for it to be meaningfully associated with the short-distance parameters. Our careful computation of the effects which would generate a nonzero β allows us to conclude that β, which ranges from 0.12–0.18, is indeed smaller than $|\chi| \sim 0.53$, so that the skew of the asymmetry constrains $\sin\delta_q$. *Indeed, a measurement of the shape of the asymmetry constrains whether any long-distance phase accrues in the breaking of the factorization approximation*—i.e., through $q\overline{q}$ pair creation in the full matrix elements.

We have computed the CP-violating asymmetry in $B^\pm \to \rho^\pm\rho^0(\omega) \to \rho^\pm\pi^+\pi^-$ decay and have found that the asymmetry, which is greatly enhanced through ρ^0-ω interference, is significantly constrained through $e^+e^- \to \pi^+\pi^-$ data in the $\rho^0(\omega)$ interference region. Indeed, the magnitude of the asymmetry would be preserved even if δ_q, the phase arising from the effective Wilson coefficients, were zero, though its *sign* does depend on $\cos\delta_q$. In the factorization approximation, $\cos\delta_q < 0$ for any $N_c > 0$ and k^2/m_b^2, as the magnitude of $\tilde{\epsilon}$ is set by that of isospin violation. Thus, for the decay of interest, the factorization approximation fails to predict the sign of $\cos\delta_q$ only if it is badly wrong. Fortunately, moreover, the measurement itself provides a significant test of the factorization approximation, for $\tan\delta_q$ can be extracted as well. This is a much more germane test of its utility than that afforded by empirical branching ratios. Thus, we are led to conclude that the CP-violating asymmetry in the above channel is large and robust with respect to the known strong phase uncertainties, admitting the extraction of the weak phase ϕ, or specifically $-\alpha$ [3], from this channel.

S. G. thanks A. Kagan, W. Korsch, and G. Valencia for helpful comments and references, and the University of Adelaide for hospitality during a visit when this work began. This work was supported by the DOE under DE-FG02-96ER40989 (S. G. and H. O. C.) and by the Australian Research Council.

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