Anthropic Considerations in Multiple-Domain Theories and the Scale of Electroweak Symmetry Breaking

V. Agrawal,[1] S. M. Barr,[1] John F. Donoghue,[2] and D. Seckel[1]
[1]*Bartol Research Institute, University of Delaware, Newark, Delaware 19716*
[2]*Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003*
(Received 2 October 1997)

One of the puzzles of the standard model is why the mass parameter μ^2, which determines the weak interaction scale, is closer to the quantum chromodynamics scale than to the grand unification or Planck scales. We consider a novel approach to this problem, based upon the idea that μ^2 takes different values in different domains of the Universe. The whole range of values for μ^2, from $+M_P^2$ to $-M_P^2$, is explored, and it is found that only for values in a narrow window is life likely to be possible. The observed value of μ^2 is fairly typical of the values in this window. [S0031-9007(98)05468-4]

PACS numbers: 12.15.−y, 98.80.Bp, 98.80.Cq

In our present theory of physics, there are only three parameters in the fundamental Lagrangian which are dimensionful. Two of these are associated with general relativity, i.e., the Planck mass $M_P^2 = G_N^{-1} = (10^{19}\ \text{GeV})^2$, and the cosmological constant, which is presently bounded to be $\Lambda \leq 10^{-120} M_P^4$. The third is the mass parameter in the Higgs potential of the standard model, μ^2, which leads to a vacuum expectation value for the Higgs field $v = \sqrt{-\mu^2/\lambda} = 246\ \text{GeV}$ $(\lambda \sim 1)$. The expectation value v is the origin of the masses of all of the quarks, leptons, and gauge bosons. A fourth mass scale does not appear in the Lagrangian, but enters indirectly as the energy at which the "running" strong coupling constant becomes of order unity. This quantum chromodynamics (QCD) scale is roughly 200 MeV. Because the QCD coupling varies only logarithmically with the energy, it is natural that the QCD scale is much smaller than the Planck mass. However, the smallness of the cosmological constant and the Higgs mass parameter are severe problems for our present understanding.

The Higgs vacuum expectation value is not only small compared to the Planck scale, $v \sim 10^{-17} M_P$, but it is also problematic because it receives large quantum corrections. If the standard model is the appropriate description up to some scale Λ_{SM}, then μ^2 receives radiative corrections of order Λ_{SM}^2. For the standard model to be valid to high energies $(\Lambda_{\text{SM}} \gg v)$, one requires a highly fortuitous cancellation of the bare parameter and its radiative corrections in order to produce a low physical value of μ^2. The puzzling smallness of μ^2 is often referred to as the "hierarchy problem," and the sensitivity to quantum corrections as the "fine-tuning problem" [1]. The smallness and fine-tuning of the cosmological constant are even more dramatic [2].

The problem of the Higgs mass parameter is one of the key issues in modern particle physics, and has led to the widespread expectation that new physics beyond the standard model must be present at energies $\Lambda_{\text{SM}} \sim 1\ \text{TeV}$. Prime candidates are supersymmetric theories [3] or theories without fundamental Higgs fields [4]. The search for this new physics is a prime goal of theoretical and experimental efforts.

However, there is the possibility of an entirely different explanation, in which one posits certain new cosmological features which would naturally imply "anthropic" [5] constraints on some parameters. In exploring theories of inflation, the possibility has emerged that different domains of the Universe could involve different values of the fundamental parameters. In such theories, typical of chaotic inflation [6], dynamical Higgs-like fields can get fixed at various vacuum expectation values, defining low-energy theories with different parameters. Our observed universe would be entirely within one such domain. The idea of multiple domains may be more general than chaotic inflation and may potentially be realizable in other contexts also [7]. With our present limited information, it is not any more scientific to assume that only one unique domain exists than it is to explore the possibility of multiple domains. The idea that multiple domains may exist takes the Copernican revolution to its ultimate limit—even our universe may not be the center of the Universe.

Within such a theory it is an obvious requirement that out of the ensemble of all domains we could only find ourselves in domains in which physical parameters are such as to allow the development of life—we will call these "viable" domains. This may drastically narrow the range of allowed values for the mass parameters. For example, Weinberg [7] has used this line of reasoning to argue that the anthropic need for the clustering of galaxies requires the cosmological constant to be smaller than a value which is close to the present bound. In this paper, we argue that under the assumption that life requires the complex elements to be formed in the Universe one has a constraint that allows only values of μ^2 close to the QCD scale and in a range near that found in our domain. If the multiple-domain cosmological theories are correct, this limited allowed range would plausibly provide an explanation for the observed small value of the mass scale of the standard model [8].

These considerations may also illuminate another problem posed by the standard model. Independent of any explanation for $|\mu^2| \ll M_P^2$, why should the weak scale and the QCD scales be similar? It is puzzling that, out of all of the available parameter space, the weak scale is intertwined with the QCD scale, i.e., quark and lepton masses (manifestations of the weak scale) appear at values both below and above the QCD scale, and to describe the physical world we need important inputs from both weak and QCD physics. Within the standard model, there is no need for these scales to be close, and we know of no explanation for this curious fact. Logically, the fine-tuning problem, the heirarchy problem, and this "intertwined scales" problem are all distinct. In the present context, even if a different mechanism accounts for the heirarchy and fine tuning problems, several of the arguments given below may apply to this question of intertwined scales.

We consider all values of μ^2 from $-M_P^2$ to $+M_P^2$, under the condition that all dimensionless parameters of the standard model are held fixed at the unification or Planck scale. Our results are displayed compactly in Fig. 1, and the rest of this paper is devoted to explaining this figure. The key ideas are relatively simple to present, and we provide more details in a longer paper [9]. We label the values of parameters found in our domain by a subscript zero, i.e., μ_0^2 and v_0.

The effect of the variable values of μ^2 and v is transmitted to the structure of the chemical elements largely through the quark and lepton masses, since these are linearly proportional to v, i.e., $m = m_0(v/v_0)$. The most important of these are the up and down quarks (with $m_u/m_d = 0.6$, $m_{d0} \sim 7$ MeV) and the electron ($m_{e0} = 0.5$ MeV). Despite the electromagnetic mass shift which enhances the proton mass [$(m_p - m_n)_{\text{EM}} \sim 1.7$ MeV], the neutron is heavier than the proton because of the larger down quark mass. The quark masses also play a role in the nuclear force, most importantly through the attractive long-range pion-exchange potential which has a range $r \sim 1/m_\pi$, with the pion mass squared roughly linearly proportional to the light quark masses, $m_\pi^2 \propto (m_u + m_d)$.

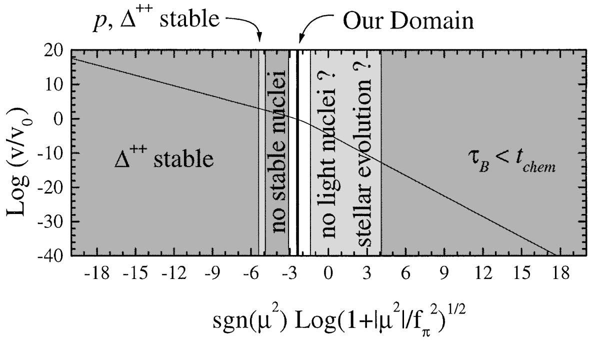

FIG. 1. This summarizes our arguments that $|\mu^2| \ll M_P$ is necessary for life to develop. For $\mu^2 < 0$ [$v \propto (-\mu^2)^{1/2}$], increasing $|\mu^2|$ increases the splitting between the light quark masses, leading to universes with but one or two species of stable nucleus (p or Δ^{++}), which we argue would not allow for viable chemistry. For $\mu^2 > 0$, quark chiral condensates lead to $v \propto f_\pi^3/\mu^2$, and quark and lepton masses become very small. Biochemical processes cannot occur until cosmologically late times, when baryons may have already decayed. Even if baryons are stable, the nature of nucleosynthesis or stellar evolution may make life improbable. What remains is a rather narrow range of $\mu^2 < 0$, which includes the physical value of our domain.

If we start close to the observed values, we note that smaller values of v appear to be allowed. As v becomes smaller, the nuclear binding becomes more effective (see the discussion below) and for v less than a critical value, which we we estimate to be $\sim 0.75 v_0$, the dineutron and diproton become bound. This has a large impact on the relative abundances of elements [10], but does not prevent the existence of complex nuclei. Stellar evolution is greatly affected. It is amusing to note that below $v/v_0 = 0.5$ the proton is heavier than the neutron and decays $p \to n e^+ \nu$. In such a domain there would be no hydrogen, and much of matter would consist of neutrons. However, deuterium and the complex elements would still exist and could have the potential to produce life of some form. We see no clear reason why domains with v somewhat less than v_0 would not be biologically viable.

For values of v larger than v_0, the elements will become increasingly unstable. The first key nucleus to become unbound will be the deuteron, which is just barely bound in nature. As the nuclear force becomes shorter range with increasing v, deuterium becomes unstable first to β-decay $d \to p p e \overline{\nu}$ and then to the strong decay $d \to p + n$. A weakly unstable d would be long lived enough to be effective in nucleosynthesis, but we estimate that for $v/v_0 \gtrsim 2$ (the precise value depends on the model used for the nucleon-nucleon potential) the deuteron is strongly unstable. This presents an obstacle to the formation of the elements, as both nucleosynthesis in the early universe and in the burning of stars requires a stable deuteron for the initial processes. Beyond this critical value of v/v_0, a domain would likely lack most of the elements required for life. However, even if there were a way to form the elements (e.g., via a three body process), a more severe problem develops at a value of v/v_0 at about 5. At values larger than this the neutron is heavier than the proton by more than the nucleon's binding energy in nuclei, so that even bound neutrons would decay to protons. (Of course, as N becomes less than Z in this way, the change in the nuclear fermi energies make $n \to p e^- \overline{\nu}$ less exothermic, but our understanding of nuclear structure indicates that nuclei with $Z \gg N$ are not bound anyway.) Such a domain would contain only protons, would not form complex nuclei, and would be chemically sterile, and therefore is probably not viable. This yields our first bound on μ^2 on the left side of Fig. 1. It is interesting to note that the existence of neutrons close enough in mass to the proton to be stable in nuclei appears to be a requirement for life to exist.

Domains with v/v_0 above 5 and below another critical value near 10^3 would appear as sterile "proton domains." In domains with v/v_0 above approximately 10^3 the only stable baryons would be Δ^{++} particles, which, being atomically equivalent to helium, would be even more chemically inert. This transition to "Δ domains" happens when the $d - u$ mass difference is large enough that the Δ^{++} (i.e., uuu) is lighter than the proton (uud) despite the QCD hyperfine energy which shifts the Δ's up in mass by about 300 MeV compared to the proton. We have estimated the nonrelativistic binding energy of six ultraheavy u quarks in a single object and find that almost certainly it would fission to two Δ^{++}'s. At the transition point between proton domains and Δ domains, there is a narrow range of v/v_0 where the electron mass would stabilize both p and Δ^{++}, but even this somewhat richer chemical environment seems unlikely to support life processes.

Where μ^2 has the opposite sign from that in our domain, the Higgs potential does not cause electroweak symmetry breaking; rather, the $SU(2)_L$ symmetry is broken by the chiral dynamics of QCD. As a result, the $W^\pm$ and Z^0 gauge bosons have small masses (~50 MeV), $v \sim f_\pi^3/\mu^2$ is tiny, and all the quarks and leptons are nearly massless. This leads to domains which are very different from our own, hard to analyze definitively, but with several features that appear to disfavor the possibility of life.

All energy scales in chemistry are set by the electron mass, which for $\mu^2 > |\mu_0^2|$ would be smaller by more than a factor of a billion.

Chemical binding energies would therefore be very small. It is clear that chemical life cannot emerge until the time t_{chem} when the temperature of the Universe drops below typical biochemical reaction energies; otherwise (to put it picturesquely) life would be fried by the primordial cosmic background radiation. For electron-dominated chemistry in a universe dominated by stable baryons, we estimate

$$t_{\text{chem}} \sim 10^{23}\ \text{yr} \left(\frac{\mu^2}{|\mu_0^2|}\right)^{3/2}. \tag{1}$$

This time scale could be reduced by a factor of up to 50 if the valence electrons were replaced by muons and/or τ leptons, which are effectively stable due to their small mass. In any event t_{chem} is a long time, and several factors relevant to the development of chemical life would be altered. For example, if life is to evolve it must do so before all the baryons decay, or before all stars reach the end of their evolutionary paths.

It is likely that baryons can decay. The unification of gauge couplings [11] suggests the existence of gauge bosons of mass 10^{16} GeV whose exchange leads to violation of baryon number. Even without this, it is plausible that Planck scale physics leads to baryon decay. We therefore parametrize the baryon decay rate as $\Gamma_B = m_p^5/M_X^4$, where M_X is assumed to lie between 10^{16} and 10^{19} GeV. In comparing t_{chem} to Γ_B we must include the thermalized energy from the decaying baryons, which modifies Eq. (1). The temperature at the epoch of baryon decay will be $T_{\text{rad}} \sim (\Gamma_B M_P)^{1/2}$. If T_{rad} is greater than some fraction (which in our universe is of order 10^{-3}) of the energy binding leptons to atoms, then life based on chemistry will be impossible. This constraint rules out the larger positive values of μ^2 as not being biologically viable, as shown for $M_X = 10^{16}$ GeV and electron chemistry in Fig. 1. This constraint could be much stronger if small v opens up new modes of baryon decay, such as sphaleron processes [12], which are suppressed in our world but may be allowed in a world with ultralight quarks and QCD-mass-scale weak bosons.

Even if baryons exist, one must ask if and how they would form nuclei appropriate for chemical life to evolve. Since all of the quarks are light, (a) the ground-state baryons will contain 27 members, including the neutron and proton, and (b) there will be a host of neutral mesons with masses less than a keV (for $\mu^2 > |\mu_0^2|$). Nuclear forces will be long range, although short-range repulsive forces would still lead to a saturation of nuclear density. The large number of nucleon species will produce lower fermi levels in nuclei. Since weak forces have a range of several fermis, in intermediate size nuclei (a few $< A <$ a few hundred) competition between electromagnetic and weak potential energy leads to $Z \sim A/4$. For larger nuclei $Z \ll A$. Given the uncertainties [9], it is unclear if there is a maximum nuclear size beyond which spontaneous fission occurs.

The long range of mesonic nuclear forces suggests that nucleosynthesis will proceed rapidly. However, in a thermal bath the effective mass of the mesons will be significant, and the range of nuclear forces will be reduced. Therefore, electrostatic coulomb and weak potentials may or may not provide an effective barrier to nuclear reactions in a plasma. If they do, then primordial nucleosynthesis will halt at modest charges and nuclear sizes. There will be ample fuel for stars and a plausible elemental mix for life. If not, then primordial nucleosynthesis will run away either to the equivalent of transiron elements or to superheavy nuclei with very low ratios of charge to mass. It is questionable if either of the last two scenarios would lead to biologically viable domains.

Even if nucleosynthesis produces an appropriate mix of elements, there is a question of stellar evolution and finding an environment and energy source for life to develop. With extremely light leptons, objects with mass less than a solar mass ($M < M_\odot$) will condense to (very large, low density) planets supported by nonrelativistic degenerate leptons. For $M > M_\odot$, as an object cools the leptons become relativistic before they become degenerate, and so such objects will condense to stars and burn nuclear fuel.

The cooling time during the preignition phases of stellar evolution will be dominated by photon diffusion at a time when the internal temperature is comparable to the electron mass (which maximizes the compton cross section). We estimate $t_{\mathrm{cool}} \approx 10^{17}\mu^2/\|\mu_0^2|$ yr. This is less than t_{chem}, but not by so much that stars may not be important as energy sources for life.

If electrostatic Coulomb barriers are effective in a plasma of charged leptons and neutral mesons, thermonuclear reactions will support the star at temperatures of $1-10$ keV. Because of the ultralight charged leptons, radiative opacities will be large. Therefore, given the small $W^{\pm}$ and Z^0 masses, such an object will cool by neutrino pair emission. We estimate nuclear burning lifetimes for $M \sim M_{\odot}$ of roughly a year, and much less for larger stars. This is very much less than t_{chem}.

Thus, within this crude treatment of stellar evolution, stars are expected to form slowly, and then burn nuclear fuel very quickly. But both time scales appear to be too small for there to be stars left when the temperature of the Universe will allow biochemistry. However, it is possible that other sources of energy may be available, e.g., gravitational energy of stars collapsing to the main sequence, “geothermal” energy, energy from radioactive decay, etc. It is therefore plausible, but by no means certain, that elemental and stellar evolutionary considerations exclude life in $\mu^2 > 0$ domains in the remaining area of Fig. 1.

In conclusion, in a universe which has a domain structure, and in which μ^2 has different values in different domains, it seems that life is unlikely to develop, except in those places where μ^2, and hence v, lies in a very narrow range. The observed value of v is typical of that range.

If μ^2 is negative, as in our domain, it seems that the whole range of values for v from M_P down to about $5v_0$ can be excluded. Any domain with $v/v_0 > 10^3$ (most of the range) would contain only sterile, heliumlike atoms whose nuclei were Δ^{++}. There would be essentially no reactions either chemical or nuclear. For $5 < v/v_0 < 10^3$ there would be no nuclei other than protons. Even for v/v_0 as small as 2, it is possible that the instability of the deuteron prevents effective nucleosynthesis and hence life. In essence, these arguments require the scales of the weak and QCD sectors of the standard model to be intertwined, and hence $-\mu^2 \ll M_P^2$.

If μ^2 is positive, then lepton masses are extremely small, as v is then set by QCD chiral symmetry breaking. Therefore, biochemical energies are also small, and the Universe may be so old, before it has cooled sufficiently to allow biochemical life, that baryons have all decayed away, or stars have ceased to form and burn.

Thus we see that the natural viability requirement present in multiple domain theories provides a plausible approach to the fine-tuning problem, the hierarchy problem, and the intertwined scales problem.

Finally, let us comment on the testablity of these ideas. If the weak scale is governed by anthropic considerations, there would be no need to invoke supersymmetry or technicolor or other structure at the weak scale to make the fine-tuning “natural” (though there could be other reasons to expect such structure). If no such structure is found, it would be a point in favor of anthropic explanations; indeed, there would be few if any alternatives. Direct tests of the idea are harder, as one cannot explore other domains of the Universe. But the theories which can produce such domains may eventually be testable through their other cosmological predictions. Moreover, if the hoped-for fundamental theory of particle interactions is found and tested, it will be possible to investigate theoretically whether it can give rise to domains and whether μ^2 can vary among them. For now, our conclusion must be modest: The observed value of the weak scale is typical of the biologically viable range.

---

[1] E. Gildener and S. Weinberg, Phys. Rev. D **13**, 3333 (1976); E. Gildener, Phys. Rev. D **14**, 1667 (1976).

[2] For a comprehensive review, see S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[3] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); Phys. Lett. **117B**, 287 (1982).

[4] S. Weinberg, Phys. Rev. D **13**, 974 (1976); S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979).

[5] B. Carter, in *Proceedings of the I.A.U. Symposium*, edited by M. Longair (Reidel, Dordrecht, 1974), Vol. 63; J. Barrow and F. Tipler, *The Anthropic Cosmological Principle* (Clarendon Press, Oxford, 1986); B.J. Carr and M.J. Rees, Nature (London) **278**, 605 (1979); A. Vilenkin, Phys. Rev. Lett. **74**, 846 (1995).

[6] A. Linde, Phys. Lett. **129B**, 177 (1983); Phys. Lett. B **175**, 395 (1986); Phys. Lett. B **202**, 194 (1988); Phys. Scr., **T15**, 169 (1987).

[7] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987); S. Weinberg, astro-ph/9610044; H. Martel, P. Shapiro, and S. Weinberg, astro-ph/9701099.

[8] R. Cahn [Rev. Mod. Phys. **68**, 951 (1996)] makes arguments similar to some of ours, but not in a multiple-domain framework nor to approach the gauge hierarchy problem.

[9] V. Agrawal, S.M. Barr, J.F. Donoghue, and D. Seckel, hep-ph/9707380.

[10] J.D. Barrow, Phys. Rev. D **35**, 1805 (1987), and references therein.

[11] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[12] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985); Phys. Lett. B **191**, 171 (1987); P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987); Phys. Rev. D **37**, 1020 (1988).