Sensitivity of *CPT* Tests with Neutral Mesons

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The sensitivity of experiments with neutral mesons to possible indirect *CPT* violation is examined. It is shown that experiments conventionally regarded as equivalent can have *CPT* reaches differing by orders of magnitude within the framework of a minimal *CPT*- and Lorentz-violating extension of the standard model. [S0031-9007(98)05411-8]

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Neutral-meson interferometry is a powerful tool for investigating the discrete symmetry *CPT*. This product of charge conjugation C, parity reflection P, and time reversal T is known to be an invariance of local relativistic quantum field theories of point particles in flat spacetime [1]. Among the various tests of *CPT* [2], the sharpest published bounds are obtained with the neutral-kaon system. As an example, the *CPT* figure of merit $r_K \equiv |m_K - m_{\overline{K}}|/m_K$ has recently been constrained to $r_K < 1.3 \times 10^{-18}$ at the 90% confidence level by the experiment E773 at Fermilab [3,4].

In neutral-meson interferometry, bounds on *CPT* violation are extracted using a phenomenological description of the meson time evolution. Denote by P^0 any of the possible neutral mesons K^0, D^0, B_d^0, B_s^0 produced using the strong interaction, and combine the Schrödinger wave functions of P^0 and its opposite-flavor antiparticle $\overline{P^0}$ into a two-component object Ψ. Then, the time evolution of Ψ is governed by a 2×2 effective Hamiltonian Λ through the equation $i\partial_t \Psi = \Lambda \Psi$. Off-diagonal components of Λ drive flavor oscillations between P^0 and $\overline{P^0}$.

Two possible kinds of *CP* violation can be studied within this formalism. The one usually considered involves T violation with *CPT* invariance and is controlled by a parameter ϵ_P. In the kaon system, for example, a nonzero value of ϵ_K is well established [2]. The other involves *CPT* violation with T invariance. It is controlled by a complex parameter $\delta_P \approx \Delta\Lambda/\Delta\lambda$, where $\Delta\Lambda \equiv (\Lambda_{11} - \Lambda_{22})/2$ is half the diagonal-element difference in Λ and $\Delta\lambda$ is the eigenvalue difference. In the kaon system, a bound on r_K constrains δ_K.

The parameter δ_P can be bounded experimentally whether or not a nonzero value has any theoretical basis. However, a framework for *CPT* violation based on conventional quantum field theory does exist. The idea is that apparent low-energy *CPT* and Lorentz breaking might arise spontaneously within a more fundamental theory that is otherwise *CPT* and Lorentz invariant [5]. Any apparent breaking at the level of the standard model would then merely reflect a feature of the vacuum rather than a fundamental property of the theory [6]. Potentially observable effects within this general framework have been studied in neutral-meson systems [7–10], in QED [11], and in baryogenesis [12].

Apparent Lorentz and *CPT* violation of this type can be incorporated in a general extension of the minimal $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ standard model that preserves gauge invariance and renormalizability [13]. In the underlying theory, the spontaneous breaking generates constant background expectation values as usual, but the fields involved are Lorentz tensors instead of Higgs scalars. In the standard-model extension, nonzero expectation values appear as coupling constants with Lorentz indices. For example, an expectation value a_μ would allow a *CPT*- and Lorentz-violating term $-a_\mu \overline{\psi} \gamma^\mu \psi$ for a fermion ψ.

The present work investigates the sensitivity of neutral-meson experiments to indirect *CPT*-violating effects produced in the standard-model extension [14]. Most of the theoretical considerations apply to any of the four neutral-meson systems. For definiteness, in the discussion of *CPT* tests some emphasis is placed on the E773 experiment mentioned above. The results are of immediate interest for *CPT* tests because at present the framework of the standard-model extension seems to be the only available consistent theoretical basis for a nonzero δ_P within conventional quantum field theory [15].

The first step is to obtain an explicit expression for δ_P within the standard-model extension. A key point is that the parameter δ_P must be C violating but P and T preserving. This is because the strong-interaction states P^0, $\overline{P^0}$ are eigenvectors of parity with the same eigenvalue, so the linear combinations forming the physical eigenstates P_S, P_L of Λ are parity eigenstates too. Parity is therefore preserved during the time evolution of a neutral-meson state, so any *CP* violation appearing in Λ is really C violation with P invariance.

In the Lagrangian $\mathcal{L}$ for the standard-model extension, the parameters controlling the Lorentz and *CPT* violation are assumed suppressed by the (small) dimensionless ratio of the relevant light energy scale to the Planck scale [13]. Thus, only contributions linear in these parameters could produce observable *CPT* violation in experiments with neutral mesons. Also, since $\Delta\Lambda$ is flavor diagonal, any term in $\mathcal{L}$ with both *CPT* breaking and flavor changing

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would affect δ_P at most as the square of a small parameter and hence can be disregarded.

Remarkably, an inspection shows that only one type of term in the standard-model extension is flavor diagonal while violating C but preserving P and T. For each quark field q it has the form $-a_0^q \overline{q}\gamma^0 q$, where a_0^q is the zeroth component of a background expectation value a_μ^q that varies with the flavor q (cf. the usual Yukawa couplings). Thus, at first order in perturbation theory the diagonal elements of Λ depend only on a_0^q.

This result is of particular interest both because no other CPT- and Lorentz-violating expectation values appear and because to date no other experiments sensitive to a_μ^q have been identified. Note that higher-order corrections from conventional interactions do not change the result. Pure strong or electromagnetic corrections preserve C, P, T and therefore at most can modify the magnitude of the contributions proportional to a_0^q. Any weak-interaction corrections violating C and P while preserving net flavor would be suppressed by several orders of magnitude. Also, possible CPT violation in the gauge sector is expected to be small [13] and in any case must appear as a higher-order correction here.

As a result of CPT violation, the parameter $a_0^{q_1}$ for the valence quark q_1 in the P^0 meson contributes with opposite sign to $a_0^{q_2}$ for the antiquark $\overline{q_2}$. This means that $\Delta\Lambda \propto \Delta a_0 \equiv (a_0^{q_2} - a_0^{q_1})$ at leading order [16]. The proportionality constant can be found in perturbation theory and is approximately one [7], so from the definition of δ_P one finds

$$\delta_P \approx i \sin\hat{\phi} \exp(i\hat{\phi}) \Delta a_0/\Delta m\,, \qquad (1)$$

where $\Delta m \equiv m_L - m_S$ and $\Delta\gamma \equiv \gamma_S - \gamma_L$ are the mass and decay-rate differences, respectively, between P_L and P_S, and where $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta\gamma)$. A subscript P is understood on all these quantities.

The result (1), valid at leading order in all Lorentz-violating parameters in the standard-model extension, holds for P^0 mesons at rest in the (oriented) inertial frame in which Δa_0 is specified. In considering the effects of rotations and boosts on this result, one must keep distinct transformations of the observer (laboratory) frame from changes of the momentum or orientation of a particle within a given observer frame. The former are conventional Lorentz transformations, and full covariance is maintained because the background fields are perceived as changed when the observer frame changes. In contrast, changes of the particle momentum or orientation leave unaffected the background values, producing (small) apparent changes in the intrinsic properties of the particle.

Equation (1) shows that the result of a CPT test with mesons at rest in the laboratory is explicitly rotationally invariant. However, no experiments with mesons at rest have been performed. An approximation is provided by the Cornell CLEO experiment, which involves (correlated) B mesons traveling at only about 6% of light speed. A measurement of δ_B in this experiment might therefore approximately bound Δa_0 via Eq. (1).

Most experiments involve relativistic mesons. This has implications for the CPT reach because the Λ formalism is obtained from nonrelativistic quantum mechanics, and so for a boosted meson δ_P is defined in the comoving frame. To obtain the analog of Eq. (1) valid for relativistic mesons, one can take advantage of the covariance of the standard-model extension under observer boosts. Suppose a particle at rest in the laboratory frame is described by a Lagrangian including the term $-a_\mu^q \overline{q}\gamma^\mu q$, as before. Then, another particle with momentum related to the first by an inverse boost $(L^{-1})^\mu{}_\nu$ is described in the same frame by $-a_\mu^q (L^{-1})^\mu{}_\nu \overline{q}\gamma^\nu q$, since the background value a_μ^q is fixed. The Lagrangian describing this boosted particle in a comoving observer frame is obtained by performing an observer boost with $L^\mu{}_\nu$, under which both the background values and the fields transform. The result is a term $-a_\mu^{q\prime} \overline{q}\gamma^\mu q$, where $a^{q\prime\mu} = L^\mu{}_\nu a^{q\nu}$. Thus, in the comoving frame in which the Λ formalism is valid and δ_P is defined, the CPT-violating physics is controlled by $a_0^{q\prime}$ instead of a_0^q. For a four-velocity $\beta^\mu \equiv \gamma(1, \vec{\beta})$, one has $a_0^{q\prime} = \beta^\mu a_\mu^q$ and hence [17]

$$\delta_P \approx i \sin\hat{\phi} \exp(i\hat{\phi}) \gamma (\Delta a_0 - \vec{\beta}\cdot\Delta\vec{a})/\Delta m\,, \qquad (2)$$

where $\Delta\vec{a} \equiv \vec{a}^{q_2} - \vec{a}^{q_1}$.

Since the expressions for δ_P in Eqs. (1) and (2) have the same phase, the real and imaginary parts of δ_P are scaled in the same way when a meson is boosted in the laboratory. However, Eq. (2) shows that there is an overall multiplicative factor of γ acting to enhance the CPT-violating effect. Moreover, the observed CPT violation for relativistic mesons depends not only on Δa_0 but also on the angle α between $\vec{\beta}$ and $\Delta\vec{a}$ and on the magnitudes β of $\vec{\beta}$ and Δa of $\Delta\vec{a}$. Since $|\beta\cos\alpha| < 1$, the factor involving Δa is always suppressed relative to Δa_0, although the combination $\Delta a_0 - \vec{\beta}\cdot\Delta\vec{a}$ may be larger or smaller than Δa_0. Note that if spontaneous Lorentz breaking generates only zero-component expectation values as seen in the laboratory frame, then $\Delta\vec{a} = 0$ for a meson at rest and so the value of δ_P is enhanced exactly by a factor γ for a boosted meson. If instead *no* pure zero-component expectation values are generated, then $\Delta a_0 = 0$ for a meson at rest, δ_P depends on $\gamma\beta\Delta a\cos\alpha$, and a meson boost is *necessary* to observe any effect. Finally, if $\Delta a > \Delta a_0$, then there is a hyperbola in β-$\cos\alpha$ space along which CPT violation is exactly canceled and near which significant suppression could reduce the CPT reach of certain experiments.

In an ideal case, increasing the statistics by a factor N would improve the CPT reach by a factor $\sqrt{N}$. However, the variation of δ_P with γ provides an alternative possibility. Suppose for simplicity that data at fixed α are obtained from an experiment in a regime where $\delta_P \propto \gamma$ holds to a good approximation. Then, the CPT reach of

the experiment can be doubled either by quadrupling the statistics or by doubling the boost.

One implication is that the bounds on δ_P reported from different experiments may be inequivalent. Experiments with comparable statistical precision can involve mesons with very different boosts and hence may have very different *CPT* sensitivity. In some experiments, the boost factor is large. For example, the B mesons in the proposed LHC-B experiment at CERN are expected to have $\overline{\gamma} \simeq 15$. Similarly, the E773 experiment at Fermilab involves kaons with mean value $\overline{\gamma} \simeq 10^2$. The figure of merit r_K bounded in this experiment is proportional to δ_K, so in the above scenario the attainable sensitivity to *CPT* violation is about 2 orders of magnitude better than the limit on r_K would naively suggest.

Many experiments with neutral mesons involve a distribution of momenta. If *CPT* violation were indeed detectable in such cases, then the predicted momentum dependence of δ_P would provide a striking signal. The presence of a momentum spectrum also has implications for the extraction of a *CPT* bound. Consider first an experimental asymmetry A that is directly sensitive to a linear combination of the real and imaginary parts of δ_P. An example is the fully integrated asymmetry A_f for uncorrelated neutral-B mesons described in Ref. [9]. In a regime where $\delta_P \propto \gamma$ holds and α is constant, δ_P and hence A both scale linearly with γ. The mean value $\overline{A}$ of the asymmetry is then proportional to $\overline{\gamma}$, determined by the form of the normalized meson-momentum spectrum. For example, $\gamma \propto p$ approximately for large momentum magnitudes p, so in this case the effective meson momentum determining the *CPT* reach is just the mean momentum $\overline{p}$ of the distribution [18].

Most experiments constrain δ_P either through fits to time-dependent asymmetries or from the measurement of other quantities. For example, the E773 experiment measures Δm, the K_S lifetime τ_S, and the usual two phases ϕ_{+-}, $\Delta\phi \equiv \phi_{00} - \phi_{+-}$ related to ratios of amplitudes for 2π decays: $A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-) \equiv |\eta_{+-}| \exp(i\phi_{+-})$, and similarly for $2\pi^0$ decays. Defining $\chi \equiv \phi_{+-} - \hat{\phi} + \Delta\phi/3$, a bound on $|\delta_K| \approx |\eta_{+-}||\chi|$ can be extracted using the established value [2] of $|\eta_{+-}|$, along with a bound on $r_K \approx 2\Delta m|\delta_K|/m_K \sin\hat{\phi}$. It can be shown that for small *CPT* violation a scaling of δ_K by γ_2/γ_1 occurring when the meson boost is changed from γ_1 to γ_2 would arise largely from a corresponding additive change to ϕ_{+-} by an amount $[(\gamma_2/\gamma_1) - 1]|\chi|$, while the other measured quantities remain essentially unchanged. In this case, *CPT* violation would manifest itself as a momentum dependence in the observed value of ϕ_{+-} [19].

Since precision constraints on δ_P require high statistics, experiments are performed over many days. For example, the E773 data were collected over several months [3]. Effects of the Earth's motion on the meson velocities and orientations must therefore be considered.

The parameter Δa_μ is constant in any special-relativistic inertial frame. In a frame in the solar neighborhood, the velocity of a laboratory on the Earth's surface is nonrelativistic, so any associated effects can be neglected. However, the rotation of the Earth about its axis introduces a time variation in the apparent orientation of $\Delta\vec{a}$ in an Earth-based laboratory [20]. If in an experiment the data are taken with time stamps, then it is possible in principle to search directly for such variations. The appropriate expression for δ_P is Eq. (2) with an apparent time dependence for the component of $\Delta\vec{a}$ lying in the Earth's equatorial plane.

Useful bounds can also be obtained without time binning. If, for example, the data are taken with a fixed orientation of $\vec{\beta}$ in the laboratory frame and are uniformly distributed in time over a rotation period, then the only net surviving *CPT*-violating effect arises from the component of $\vec{\beta} \cdot \Delta\vec{a}$ parallel to the Earth's rotational axis. As an example of this geometrical effect, if the plane of the experiment is tangent to the Earth at latitude ψ and χ is the angle made by $\vec{\beta}$ relative to the rotational north pole in this plane, then $\delta_P \propto \gamma(\Delta a_0 - \beta\cos\psi\cos\chi\Delta a_\parallel)$, where $\Delta a_\parallel$ is the component of $\Delta\vec{a}$ parallel to the Earth's rotational axis. If by mischance $\Delta\vec{a}$ is orthogonal to the Earth's rotational axis, any experiments performed over many rotation periods are sensitive only to $\delta_P \propto \gamma\Delta a_0$. The latter also holds for any $\Delta\vec{a}$ if the mesons are boosted solely east-west.

For the E773 experiment, $\beta\cos\psi\cos\chi \simeq 0.6$. An estimate of the attainable *CPT* bound on the quantities Δa_μ is then $|\Delta a_0 - 0.6\Delta a_\parallel| \approx m_K r_K/2\gamma \lesssim 10^{-20} m_K$. The scale of this bound is comparable to the ratio of the kaon mass to the Planck mass. Note that time binning could also be used to investigate the possibility of time variations, as suggested above. In any case, an improved bound may be feasible in the near future using data from the KTeV experiment at Fermilab or the NA48 experiment at CERN.

The E773 experiment involves collimated beams of uncorrelated boosted mesons, but other types of experiment also exist. Boosted mesons traveling in different directions can be produced in a collider. For example, uncorrelated B mesons with velocity orientation throughout most of the 4π range are accessible to CERN Large Electron-Positron Collider detectors. In experiments of this type, *CPT* studies allowing for angular dependence of δ_P by binning the data in angle slices could in principle allow separate extraction of bounds on Δa_0 and $\Delta\vec{a}$. The effects may be partially reduced by averaging due to the Earth's rotation. For example, for the special case of an integrated uniform meson distribution transverse to the beam, the angular dependence of the *CPT* violation is reduced by the time-averaged cosine of the angle between $\Delta\vec{a}$ and the beam. This effect could be avoided by time binning.

Another class of experiments involves correlated P_S-P_L pairs from quarkonium decays. The unboosted case (symmetric factory) produces primarily a line spectrum in

the momentum, so although a (small) γ scale factor may appear there is essentially no momentum dependence. However, the 4π coverage provides potential sensitivity to possible angular dependence. Note that the values of δ_P for the two mesons in each correlated pair are typically different because they travel in opposite directions. Also, the effect of the Earth's rotation must again be allowed for in the analysis. In contrast, the boosted case (asymmetric factory) produces a meson-momentum spectrum, so a combination of momentum and angular dependence can govern the CPT-violating effects.

This paper has considered the boost and orientation dependences appearing in the CPT-violating parameter δ_P within the context of the Lorentz-violating standard-model extension. A related analysis for the usual CP-violating parameter ϵ_P shows that it too could acquire contributions from Lorentz-violating terms in the standard-model extension that involve C and T violation but preserve P (and CPT). However, the effect would require second-order flavor-changing contributions, and hence it is expected to be suppressed relative to the contributions to δ_P discussed above. Moreover, unlike the case of δ_P, conventional contributions to ϵ_P from low-energy physics can arise, so any Planck-scale effect may be masked. It therefore appears unlikely that observable boost or orientation dependence for ϵ_P would arise in the context of the present framework [21].

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[1] The discrete symmetries C, P, T are discussed, for example, in R. G. Sachs, *The Physics of Time Reversal* (University of Chicago Press, Chicago, 1987).

[2] See, for example, R. M. Barnett *et al.,* Phys. Rev. D **54**, 1 (1996).

[3] B. Schwingenheuer *et al.,* Phys. Rev. Lett. **74**, 4376 (1995); R. A. Briere, Ph.D. thesis, University of Chicago, 1995; B. Schwingenheuer, Ph.D. thesis, University of Chicago, 1995.

[4] This bound holds for negligible direct CPT violation in decay amplitudes. See Ref. [3] and E. Shabalin, Phys. Lett. B **369**, 335 (1996).

[5] For example, this might occur in the context of string theory. See V. A. Kostelecký and S. Samuel, Phys. Rev. Lett. **63**, 224 (1989); **66**, 1811 (1991); Phys. Rev. D **39**, 683 (1989); **40**, 1886 (1989); V. A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Lett. B **381**, 89 (1996).

[6] An analogy is the behavior of an electron in a crystal, which reflects rotational (Lorentz) breaking although the underlying dynamics is invariant.

[7] V. A. Kostelecký and R. Potting, in *Gamma Ray-Neutrino Cosmology and Planck Scale Physics,* edited by D. B. Cline (World Scientific, Singapore, 1993); Phys. Rev. D **51**, 3923 (1995).

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[10] OPAL Collaboration, R. Ackerstaff *et al.,* Z. Phys. C **76**, 401 (1997); DELPHI Collaboration, M. Feindt *et al.,* DELPHI Report No. DELPHI 97-98 CONF 80, 1997.

[11] R. Bluhm, V. A. Kostelecký, and N. Russell, Phys. Rev. Lett. **79**, 1432 (1997); Phys. Rev. D (to be published).

[12] O. Bertolami *et al.,* Phys. Lett. B **395**, 178 (1997).

[13] D. Colladay and V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); IUHET Report No. IUHET 359, 1997.

[14] Direct CPT violation in decay amplitudes is neglected here because it is highly suppressed in the standard-model extension and would be unobservable [7].

[15] For the neutral-kaon system, it has been suggested that an unconventional quantum mechanics in which the Schrödinger equation is replaced with a density-matrix formalism might generate extra CPT-violating terms. However, the parameter δ_K is unaffected. See J. Ellis *et al.,* Phys. Rev. D **53**, 3846 (1996).

[16] The appearance of the *difference* Δa_0 in the physical observable is a general feature. The parameters a_μ^q can be shifted by a constant using a field redefinition, so only their differences are observable [13]. Note also that the velocities and spins of the valence quarks are irrelevant here because their expectations vanish in the wave function for the P^0 meson at rest.

[17] Although $\hat{\phi}$ and Δm are indirectly affected by the boost, this is at most second order in the small CPT-violating parameters and hence is unobservable. Note also that the appearance of the combination $\vec{\beta} \cdot \Delta\vec{a}$ in Eq. (2) is compatible with the requirement that δ_P be C violating but P and T preserving.

[18] Since the CPT reach improves linearly with γ but only as the square root of the statistics, it may be possible to *increase* the experimental sensitivity by examining only a (high-momentum) *subset* of the data available. For the simple case of a constant distribution in p in the regime where $\delta_P \propto p$, the gain is at most about 10%. For peaked distributions, such as the Malensek-type distribution of the E773 experiment, the loss of statistics at high momenta typically offsets any gain from the γ factor.

[19] The actual E773 experiment extracts ϕ_{+-} from an interference pattern involving also a regeneration phase ϕ_ρ, which is unaffected to leading order by CPT violation. Because of averaging effects, some sensitivity to CPT violation might be lost using a procedure for extracting ϕ_{+-} that disregards the momentum dependence, although it is unlikely that the net CPT-violating effect would vanish. Particular care is required in determining $\Delta\phi$ because the experimental acceptances differ for $2\pi^0$ and $\pi^+\pi^-$ decays, and so differences occur in the reconstructed momentum spectra used to extract ϕ_{00} and ϕ_{+-}.

[20] The variation is approximately diurnal, with a seasonal drift due to the orbital motion.

[21] Various authors have investigated the possibility that the usual observed CP violation might be boost dependent: J. S. Bell and J. K. Perring, Phys. Rev. Lett. **13**, 348 (1964); S. H. Aronson *et al.,* Phys. Rev. D **28**, 495 (1983).