## **Upper Critical Field Peculiarities of Superconducting YNi2B2C and LuNi2B2C**

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(Received 7 October 1997)

We present new upper critical field  $H_{c2}(T)$  data in a broad temperature region 0.3 K  $\leq T \leq T_c$  for LuNi<sub>2</sub>B<sub>2</sub>C and YNi<sub>2</sub>B<sub>2</sub>C single crystals with well characterized low impurity scattering rates. The absolute values for all *T*, in particular,  $H_{c2}(0)$ , and the sizable positive curvature (PC) of  $H_{c2}(T)$  at high and intermediate *T* are explained quantitatively within an effective two-band model. The failure of the isotropic single-band approach is discussed in detail. Supported by de Haas–van Alphen data, the superconductivity reveals direct insight into details of the electronic structure. The observed maximal PC near  $T_c$  gives strong evidence for clean limit type-II superconductors. [S0031-9007(98)05338-1]

PACS numbers: 74.60.Ec, 74.20. –z, 74.70.Ad

The discovery [1,2] of superconductivity in transition metal borocarbides has generated large interest due to their relatively high transition temperatures  $T_c \sim 15$  to 23 K and due to the relation between the mechanisms of superconductivity in these compounds, in cuprates, and in ordinary transition metals. Another highlight is the coexistence of magnetism and superconductivity in some of these compounds containing rare earth elements [3–5]. A study of the nonmagnetic compounds, such as  $LNi<sub>2</sub>B<sub>2</sub>C$ , with  $L = Lu, Y, Th, Sc$  [6], is a prerequisite for the understanding of their magnetic counterparts. Experimental data for LuNi<sub>2</sub>B<sub>2</sub>C [7] demonstrate, besides a maximal positive curvature (PC) of  $H_{c2}(T)$  near  $T_c$ , observed also for  $YNi<sub>2</sub>B<sub>2</sub>C$  [4,8,9], a weak *T*-dependent anisotropy within the tetragonal basal plane and a *T*-independent out-of-plane anisotropy of the upper critical field  $H_{c2}$ . Both anisotropies have been described [7] in terms of nonlocal corrections to the Ginzburg-Landau (GL) equations. In this picture the PC of  $H_{c2}^c$  ( $\vec{H}$ ) to the tetragonal *c* axis) is caused, almost entirely, by the basal plane anisotropy. However, it should be noted that the reported anisotropy of  $H_{c2}$  for YNi<sub>2</sub>B<sub>2</sub>C is significantly smaller than for  $LuNi<sub>2</sub>B<sub>2</sub>C$  [7,9,10], whereas its PC is comparable or even larger. Further explanations of the unusual PC of  $H_{c2}(T)$ , such as quasi-2D fluctuations [11], are excluded by the underestimation of  $H_{c2}(T)$  at low *T* [9] and the observed weak anisotropy. The quantum critical point scenario [12], as well as the bipolaronic scenario [13], can be disregarded because the slope of  $H_{c2}(T)$  decreases for  $T \rightarrow 0$  (see Fig. 1). Local density approximation (LDA) band structure calculations [14,15] predict a nearly isotropic electronic structure with rather complicated bands near the Fermi level  $E_F$ . However, in analyzing the superconductivity in terms of an isotropic *single*-band (ISB) Eliashberg model, the multiband character and the anisotropic Fermi surface have been widely ignored so far.

Here we present and analyze theoretically the new data of  $H_{c2}(T)$  in a broad interval 0.3 K  $\leq T \leq T_c$  for high purity  $LuNi<sub>2</sub>B<sub>2</sub>C$  and  $YNi<sub>2</sub>B<sub>2</sub>C$  single crystals. We show that typical features of both compounds, such as  $H_{c2}(0) \sim$ 8–10 T and the unusual PC of  $H_{c2}(T)$  for  $T \approx 0.5T_c$ , cannot in any way be explained consistently with the normal state properties within the ISB approach. Instead we propose a two-band model (TBM) approach. To clarify its relationship to the extended saddle-point model [16], which also predicts a PC, is beyond the scope of this Letter.

Platelet shaped  $LuNi<sub>2</sub>B<sub>2</sub>C$  and  $YNi<sub>2</sub>B<sub>2</sub>C$  single crystals with a mass of  $\sim$ 1 mg were grown by a high-temperature flux technique with  $Ni<sub>2</sub>B$  as flux. The values of  $T_c$ , 16.5 K, and 15.7 K, have been determined by low-field ac susceptibility  $\chi(T)$  with transition widths  $\Delta T_c =$ 0.2 K. The upper critical field  $H_{c2}^c(T)$  along the *c* axis, shown in Figs. 1 and 2, has been measured resistively for fixed *T* adopting the midpoint criterion,  $\rho(H_{c2}, T) = 0.5\rho(H = 0, T = 17 \text{ K}) \equiv 0.5\rho_n$ . The transition width  $\Delta H = [H(\rho = 0.9\rho_n) - H(\rho = 0.1\rho_n)]$ increases up to 0.75 T (1.5 T for  $LuNi<sub>2</sub>B<sub>2</sub>C$ ) at  $T < 0.5$  K starting from a nearly constant value of 0.3 T at  $T > 4$  K (8 K for LuNi<sub>2</sub>B<sub>2</sub>C). The low residual resistivity  $\rho(0) \approx \rho_n = 2.5 \mu \Omega$  cm and the ratio  $\rho(300 \text{ K})/\rho_n = 43$  (27 for LuNi<sub>2</sub>B<sub>2</sub>C), together with the observations of magnetoquantum oscillations [17–19], indicate a high quality and a low impurity content of our samples. This suggests that we are in the clean limit in terms of the traditional theory of type-II superconductors [20]. In this limit, one has to consider the electronic structure in more detail. We restrict ourselves to an effective two-band model [21] which, especially on the simple Bardeen-Cooper-Schrieffer (BCS) level, has a long history [22]. Because of the neglect of strong coupling effects, a BCS-like theory is not expected to describe real superconductors quantitatively. Such effects must be studied within the Eliashberg theory [23–27]. To calculate  $H_{c2}$ , we have solved numerically the corresponding linearized equations of Ref. [23],



FIG. 1. Experimental data for  $H_{c2}(T)$  of LuNi<sub>2</sub>B<sub>2</sub>C (magnetic field  $\hat{H}$  the *c* axis) compared with theoretical curves: (i) the isotropic single band model with Fermi velocity  $v_F = 2.76 \times$  $10<sup>7</sup>$  cm/s and various impurity scattering rates  $\gamma_{\rm imp}$ ; (ii) the two-band model with  $v_{Fi}$  ( $i = 1, 2$ ) in units of 10<sup>7</sup> cm/s.

$$
\tilde{\omega}_i(n) = \omega_n + \pi T \sum_{j,m} \left[ \lambda_{i,j}(m-n) + \delta_{mn}(\gamma_{\text{imp};i,j}) + \gamma_{\text{imp};i,j}^s \right] / 2\pi T \left[ \text{sgn}(\omega_m), (1) \right]
$$

$$
\tilde{\Delta}_i(n) = \pi T \sum_{j,m} [\lambda_{i,j}(m - n) - \mu^* \delta_{ij} \theta(\omega_c - |\omega_m|) + \delta_{mn} (\gamma_{\text{imp};i,j} - \gamma_{\text{imp};i,j}^s)/2\pi T] \times \chi_j(m) \tilde{\Delta}_j(m), \tag{2}
$$

$$
\chi(n) = (2/\sqrt{\beta_i}) \int_0^\infty dq \exp(-q^2)
$$
  
 
$$
\times \tan^{-1} \{q \sqrt{\beta_i}/[\tilde{\omega}_i(n)] + i \mu_B H_{c2} \operatorname{sgn}(\omega_n)]\},
$$
 (3)

$$
\beta_i = e H_{c2} v_{Fi}^2 / 2, \qquad (4)
$$

$$
\lambda_{i,j}(n) = \int_0^\infty d\omega \omega \alpha_{i,j}^2 F(\omega) / (\omega^2 + \omega_n^2). \qquad (5)
$$

The bands at  $E_F$  are labeled by *i*,*j*. Here  $\omega_n = 2\pi T(2n + 1)$  are the Matsubara frequencies,  $\alpha_i^2 F(\omega)$  and  $\tilde{\Delta}_i$  denote the spectral density and the superconducting order parameter of the *i*th band, respectively. In our approach, as in any two-band model, two gaps, below and above the BCS value of  $3.5k_BT_c$ , occur naturally. In general, interband coupling  $(i \neq j)$  mediated by phonons  $\left[ \alpha_{i,j}^2 F(\omega) \right]$  and impurities is important. Since there is no experimental evidence [28,29] for the presence



FIG. 2. Temperature dependence of  $H_{c2}(T)$  for YNi<sub>2</sub>B<sub>2</sub>C. Experimental points for the magnetic field  $\hat{H}$   $\parallel$  c axis. The Fermi velocities of the two-band model are given in units of  $10<sup>7</sup>$  cm/s. For the interaction constants, see the text.

of magnetic impurities in high quality samples, we neglect the magnetic scattering rate  $\gamma_{\text{imp}}^s$ . For the quantification of the nonmagnetic counterpart  $\gamma_{\text{imp}} \approx 2\pi T_D$ , the Dingle temperatures  $T<sub>D</sub>$ , measured by the de Haas–van Alphen (dHvA) effect, are very suitable [17–19]. The experimental values  $T_D = 2.8$  and 4 K reveal  $\gamma_{\text{imp}} = 18$ and 25 K for our  $YNi<sub>2</sub>B<sub>2</sub>C$  and  $LuNi<sub>2</sub>B<sub>2</sub>C$  single crystals, respectively, indicating that the clean limit is reached since  $\gamma_{\text{imp}} \leq 2\Delta_0 \approx 51$  K holds for both samples, where  $2\Delta_0$  denotes the smaller of the two gaps. Hence, the scattering by impurities can be neglected by setting  $\gamma_{\rm imp} = 0$ . In the weak coupling limit of an ISB case, Eqs.  $(1)$ – $(5)$  are equivalent to the standard theory [30]. Any anisotropy of  $H_{c2}$  can be described by a similar, but much more tedious, system of equations [26]. Since the measured anisotropy is relatively weak, it will be ignored for the sake of simplicity. Therefore, only  $H_{c2}^c(T)$ 's will be compared with those of our isotropic models.

The standard ISB model [27] describes *quantitatively* the renormalization of the physical properties of metals due to electron-phonon (el-ph) interaction. The input parameters are the density of states at  $E_F$ ,  $N(0)$ , the Fermi velocity  $v_F$ , the impurity scattering rate  $\gamma_{\rm imp}$ , the Coulomb pseudopotential  $\mu^*$ , and the spectral function  $\alpha^2 F(\omega)$  of the el-ph interaction. These quantities can be determined from a few experimental data: the normal state low-*T* electronic specific heat  $\gamma_s T$ , the plasma frequency  $\omega_{\text{pl}}$  inferred from the optical conductivity,  $H_{c2}(0)$ ,  $T_c$  and its isotope exponent  $\alpha$ , as well as the normal state low-T dc resistivity  $\rho(0) \approx \rho(T_c)$  which, similar to  $T_D$ , gives a direct measure of the sample purity. We adopt  $\mu^* = 0.1$ for the Coulomb pseudopotential and  $\hbar\omega_c = 600$  meV for the energy cutoff in Eq. (2). The total el-ph coupling For the energy cator in Eq. (2). The total ei-ph coupling<br>constant  $\lambda = 2 \int d\omega \alpha^2 F(\omega)/\omega$  can be estimated from the boron isotope effect  $\alpha_B \approx 0.2$  [31] and the phonon spectrum [32].

We first consider  $LuNi<sub>2</sub>B<sub>2</sub>C$ . To find a lower bound for  $\lambda$ , we accounted for only the high-energy carbon phonons centered at 50 meV and the boron branch at 100 meV. Fitting the experimental  $\alpha_B$  and  $T_c$  values, we obtained the partial coupling constants  $\lambda_{100} = 0.31$ ,  $\lambda_{50} = 0.22$ , and  $\lambda = \lambda_{100} + \lambda_{50} = 0.53$ , where the subscripts denote the corresponding phonon energies in meV. An upper bound of  $\lambda = 0.77$  has been found using the Lu phonons centered near 9 meV ( $\lambda$ <sup>9</sup> = 0.34) and the same *B* band  $(\lambda_{100} = 0.43)$  as in the earlier case. In the following, a wide averaged spectrum with  $\lambda = 0.65$  ( $\lambda_{100} = 0.37$ ,  $\lambda_{50} = 0.12$ ,  $\lambda_9 = 0.16$ ) will be used which reproduces the experimental values of  $\alpha_B$  and  $T_c$ .  $N(0) = 11.8 \text{ mJ/mol } k_B^2 \text{ K}^2$  has been estimated from the experimental value [8] of  $\gamma_S = 2\pi^2 \times$  $k_B^2(1 + \lambda)N(0)/3 = 19.5 \text{ mJ/mol K}^2$ . The value of  $v_F = 2.76 \times 10^7$  cm/s follows from the experimental value [33] of the plasma frequency  $\hbar \omega_{\text{pl}} =$  $4\pi e^2 v_F^2 N(0)/3 = 4.0$  eV. The analogous values for YNi<sub>2</sub>B<sub>2</sub>C are  $\lambda = 0.637$ ,  $N(0) = 11.1$  mJ/mol  $k_B^2$  K<sup>2</sup>,  $v_F = 3 \times 10^7$  cm/s, and  $H_{c2}(0) = 2$  T, where the data of Refs. [8] and [34] have been used.

We solved Eqs.  $(1)$ – $(5)$  with these parameter sets for two types of spectral densities  $\alpha^2 F(\omega)$ : (i) a wide spectrum and (ii) a single Einstein mode peaked at  $\hbar\omega_E$ 42.4 meV chosen to yield the experimental  $T_c = 16.5$  K for LuNi<sub>2</sub>B<sub>2</sub>C using the same value of  $\lambda = 0.65$  as in the first case. The results are shown in Fig. 1. Note that, in the intermediate coupling regime under consideration, as expected,  $H_{c2}(T)$  is insensitive to details of the shape of  $\alpha^2 F(\omega)$  [27] and, in the clean-limit case, it is also insensitive to the actual value of the small scattering rates. Comparing the  $LuNi<sub>2</sub>B<sub>2</sub>C$  data with the ISB curves, one clearly realizes strong deviations. In particular, there is a discrepancy of about 3 between experimental and ISB model values of  $H<sub>c2</sub>(0)$ . For YNi<sub>2</sub>B<sub>2</sub>C the discrepancy even reaches a factor of 5. Therefore it makes no sense to discuss any details of the shape of  $H_{c2}(T)$ , such as the PC, resulting in deviations of the  $H_{c2}(T)$  curves of the order 10% to 20%, until the reason for the large failure that accounts for the magnitude of  $H<sub>c2</sub>(0)$  has been elucidated. This serious difficulty was circumvented in previous studies as follows. First, frequently, the quantity  $h_{c2}(T) =$  $-H_{c2}(T)/[T_c(dH_{c2}/dT)_{T=T_c}]$ , describing the shape of the  $H_{c2}(T)$  curve, was considered, but the *absolute* values of  $H_{c2}(T)$  were not discussed at all [7]. Second, since, within the ISB model,  $H_{c2}(0)$  is a monotonically *increasing* function of the impurity content, in principle, large  $H_{c2}(0)$  values might be obtained. To check this approach [9] we calculated the impurity scattering rate  $\gamma_{\rm imp}$  which is required to increase  $H_{c2}(0)$  up to the LuNi<sub>2</sub>B<sub>2</sub>C value of 7.6 T. Thus we obtain  $\gamma_{\rm imp} \approx 427$  K, which would lead to  $\rho(0) \approx 17 \mu\Omega$  cm which strongly deviates from the experimental value  $\rho(0) \approx 2.5 \mu\Omega$  cm. In this context, we note that our data and those of Ref. [35] show dependencies just opposite to those predicted by the ISB model:  $H_{c2}(0)$  and  $T_c$  *increase* when  $\gamma_{\text{imp}}$  decreases. Third, a

further parameter, the clean limit coherence length, has been introduced in Ref. [35]. However, this results in the overdetermination of the model parameter set, and the consistency of the two values  $v_F$  obtained using (i) a cleanlimit coherence length and (ii) normal state data has not been checked. Thus the ISB approach fails to explain simultaneously the three values  $\gamma_s = 19.5 \text{ mJ/mol K}^2$ ,  $h\omega_{\text{pl}} = 4$  eV,  $H_{c2}(0) = 7.6$  T in the clean limit, and the four values  $\gamma_S$ ,  $\omega_{\text{pl}}$ ,  $H_{c2}(0)$ , and  $\rho = 2.5 \mu \Omega$  cm in the dirty limit. In addition, the ISB model is also unable to explain the small gap values  $2\Delta_0/k_B T_c < 3.5$  observed in microwave [36], tunneling [10], and dHvA [37] measurements. Furthermore, the PC of  $H_{c2}(T)$  near  $T_c$  and the extended quasilinear behavior of  $H_{c2}(T)$  down to  $T \sim 1$ to 2 K cannot be described within the ISB. The ISB model with a *single*  $v_F$  contradicts the dHvA data which clearly show the presence of six different sections  $F_{\alpha}, \ldots, F_{\eta}$ [18,19] with roughly *two* or *three* groups of  $v_F$ 's.

Turning to our TBM, we solved Eqs.  $(1)$ – $(5)$ . For the sake of simplicity, the same phonon spectrum as in the ISB case discussed above has been adopted. We achieve an excellent agreement with the  $LuNi<sub>2</sub>B<sub>2</sub>C$  data (see Fig. 1) for  $\lambda_1 = 0.51$ ,  $\lambda_2 = \lambda_{21} = 0.2$ ,  $\lambda_{12} = 0.4$ ,  $\mu_1^* =$  $\mu_2^* = 0.1$ ,  $v_{F2} = 3.7 \times 10^7$  cm/s,  $v_{F,1} = 0.96 \times$  $10^7$  cm/s. For YNi<sub>2</sub>B<sub>2</sub>C, we used the following set:  $\lambda_1 = 0.5, \ \lambda_2 = \lambda_{21} = 0.2, \ \lambda_{12} = 0.4, \ \mu_1^* = \mu_2^* = 0.1,$  $v_{F2} = 3.8 \times 10^7$  cm/s,  $v_{F1} = 0.85 \times 10^7$  cm/s, and  $N(0) = 11 \text{ mJ/mol } k_B^2 \text{ K}^2$ . Reproducing  $T_c = 15.6 \text{ K}$ , the adopted values of  $N(0)$  agree well with the LDA value [15] of 9.5 mJ/mol  $k_B^2$  K<sup>2</sup>. The plasma frequency  $\hbar\omega_{\text{pl}} = 4.4 \text{ eV}$  is in accord with  $\hbar\omega_{\text{pl}} = 4.25 \text{ eV}$  obtained in Ref. [34]. From the obtained  $v_F$ 's it is concluded that  $\omega_{\text{pl}}$  is related mainly to the second weakly coupled band. Then transport, optical, and tunneling data exhibit mainly the properties of that band, whereas the strongly coupled band remains almost hidden. The calculated value of the penetration depth at 4.2 K, 100 nm, is in agreement with the data of Ref. [35]. Our  $\gamma_s = 17.2 \text{ mJ/mol K}^2$ should be compared with  $\gamma_s = 18.2 \text{ mJ/mol K}^2$  reported in Ref. [8]. Finally, we arrive at  $H_{c2}(0) = 9.4$ to 9.9 T, in good agreement with our experimental value  $H_{c2}(0) = 10.6 \pm 0.2$  T. The experimental  $H_{c2}(T)$  curve of  $YNi<sub>2</sub>B<sub>2</sub>C$ , together with the results of the TBM, are shown in Fig. 2. The values  $v_F \approx 4.2 \times 10^7$  cm/s and 0.7 to  $1.3 \times 10^{7}$  cm/s of the extremal orbits  $F_{\beta}$ and  $F_{n1/2}$ , respectively, derived from earlier dHvA data [18,19] on the same  $YNi<sub>2</sub>B<sub>2</sub>C$  single crystal do not deviate much from the parameters  $v_{F2} \approx 3.8 \times 10^7$  cm/s and  $v_{F1} \approx 0.8$  to 0.96  $\times$  10<sup>7</sup> cm/s introduced empirically in our approach. The remaining deviations might be due to natural differences between  $v_F$ 's on extremal orbits seen in the dHvA experiments and the corresponding *effective* quantities of our TBM which contains implicit information on the whole Fermi surface.

Within our TBM, the PC of  $H_{c2}(T)$  depends mainly upon the strength of the interband coupling  $(\lambda_{12}, \lambda_{21})$  and to a lesser extent upon the ratio of the Fermi velocities and

the intraband coupling strength  $(\lambda_1, \lambda_2)$ . These findings may also be interpreted in terms of the flat Ni-derived band near  $E_F$  and of the dispersive bands crossing  $E_F$ seen in the LDA-band structure [14]. The strong mixing of these bands might be viewed as the microscopic origin for significant interband coupling. Further work is required to clarify this point. For  $T < 4$  K,  $H_{c2}(T)$ is very sensitive to the actual  $v_{F,2}/v_{F,1}$  ratio. This is illustrated by the additional curves shown in Fig. 2. The variation of  $v_{F1}$  and  $v_{F2}$  results in slightly higher and lower  $H_{c2}(0)$  values, respectively. Our empirical parameter sets for  $LuNi<sub>2</sub>B<sub>2</sub>C$  and  $YNi<sub>2</sub>B<sub>2</sub>C$  differ almost only in their  $v_{F1}$  values which roughly scale with the Ni-Ni distance as  $d_{\text{Ni}-\text{Ni}}^{-5}$ . In this context the study of  $Sc(Th)Ni<sub>2</sub>B<sub>2</sub>C$  having much reduced (increased) Ni-Ni distances is of interest.

Thus, both compounds can be well described within an effective two-band model, provided there are at least two groups of electrons having (i) significantly different Fermi velocities, (ii) strong coupling in the small- $v_F$ band, as well as (iii) sizable coupling between the small $v_F$  and large- $v_F$  band. This case differs from the situation considered in Refs. [24] and [26]. There, the stronger coupling is in the large- $v_F$  band and the curvature of  $H_{c2}(T)$  near  $T_c$  is *negative*. A PC would appear only at intermediate *T* if the interband coupling and the impurity scattering are both weak. In contrast, in this region,  $H_{c2}(T)$  shows almost no curvature in our model. In other words, the result of Refs. [24] and [26] can be understood as an average over two weakly coupled superconductors, the first with a high  $H_{c2}(0)$  but a low  $T_c$  and the second with a small  $H_{c2}(0)$  but high  $T_c$ . In our case, the isolated small- $v_F$  subsystem would have high values of  $\lambda$ ,  $H_{c2}(0)$ , and  $T_c$ . The values of  $H_c(0)$  and  $T_c$  of the coupled system are reduced by the second large- $v_F$  subsystem with weak interaction parameters. In this case, which to our knowledge has not been considered so far, the PC of the resulting  $H_{c2}(T)$  near  $T_c$  becomes a direct manifestation of that interband coupling. In our TBM, the PC of  $H_{c2}(T)$ , as well as  $H_{c2}(0)$ , and  $T_c$  is suppressed by growing impurity content, and the PC vanishes upon reaching the dirty limit with  $T_c \approx 11$  K.

In summary, we have shown that the superconductivity of pure nonmagnetic borocarbides should be described within a multiband picture. The isotropic TBM gives a reasonable starting point toward the understanding of the mechanism of superconductivity in these compounds. Combined studies of quantum oscillations (dHvA) and  $H_{c2}(T)$  unified by Eliashberg analysis are found to be valuable supplementary tools which are used to elucidate the specific role of subgroups of electrons having small  $v_F$ 's and strong coupling to bosons. These electrons may be readily overlooked by other experimental techniques.

We thank O. Dolgov, D. Rainer, E. Maksimov, H. Braun, N. Schopohl, and M. Golden for discussions. This work was supported by the INTAS-93-2154 grant, the SFB 463, and the Deutsche Forschungsgemeinschaft.

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