Electromigration-Induced Breakup of Two-Dimensional Voids

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The motion and shape evolution of a void in a two-dimensional current carrying conductor is studied numerically. A circular void is linearly stable, but becomes unstable beyond a finite threshold deformation amplitude which decreases with increasing void radius. If the void is initially elongated along the current direction it expels small, stable daughter voids, while for elongations perpendicular to the current an invagination occurs which splits the void in two. The behavior near threshold is linked to the non-normality of the eigenmodes of the linearized problem. Perfectly circular voids can also be destabilized by their mutual long-ranged electromagnetic interaction. [S0031-9007(98)05418-0]

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Electromigration along grain boundaries and interfaces [1] is a key factor determining the reliability of integrated circuits [2]. As miniaturization progresses and current densities increase, a detailed understanding of failure mechanisms is needed to safely extrapolate from an accelerated testing environment to actual operating conditions. Arzt and collaborators [3] have shown that a new type of failure appears when the linewidth of metallic interconnects becomes comparable to or smaller than the grain size of the film. In this "bamboo" regime grain boundaries no longer provide connected diffusion paths along the conductor line. Instead, failure occurs due to *intragranular* voids which nucleate at the edges of the line, migrate in the current direction, and finally collapse into a slit which disconnects the conductor. This observation has motivated recent theoretical work on the electromigration-induced motion and shape evolution of voids [4–8]. Since the conductor lines consist of thin metal films, two-dimensional modeling is appropriate.

In this Letter we present a detailed study of the conceptually simplest case of an insulating void in an infinitely extended, isotropic, homogeneous two-dimensional conductor. This idealization removes several complications present in real conductor lines—grain boundaries, edges, and crystalline anisotropy—but retains the fundamental physics of surface diffusion, capillarity, and long-ranged electromagnetic interaction, which underlies the dynamics of voids. We observe a surprisingly rich range of morphological transitions, reminiscent of the behavior of liquid droplets [9] or bilayer vesicles [10]. This suggests that void dynamics, apart from its practical importance, holds considerable promise as a model system for shape evolution far from equilibrium.

The basic solution of interest is a circular void moving at a constant velocity inversely proportional to its radius [11]. The motion proceeds in the direction of the applied current: The momentum transfer from the conduction electrons (the "electron wind" [1]) induces mass transport along the inner edge of the void, which therefore translates against the direction of electron flow. The circular solution is found to be linearly stable [6–8]; however, it is destabilized beyond a finite threshold perturbation amplitude [6] which depends on the ratio of the void radius to a characteristic length scale l_E determined by the balance of capillarity and electromigration [4,12]; larger voids move more slowly and are less stable.

Through the numerical solution of the full, nonlinear moving boundary value problem [12], we show that typically the void disintegrates at long times. Two main routes are observed. If the initial deformation is an elongation in the current direction, a protrusion develops at the leading end of the void, which subsequently pinches off in a kind of budding transition [10] and forms a separate daughter void. Since the daughter is smaller, it moves more rapidly and runs ahead of the mother void. This process can be repeated several times. If, on the other hand, the void is initially elongated perpendicular to the current, an invagination develops which eventually splits the void horizontally. The invagination scenario occurs also in the final stage of the budding process, after several daughter voids have been expelled. A typical example is shown in Fig. 1.

Model.—We adopt a continuum description of surface electromigration developed in a previous paper [12]. The void shape changes through the mass current J along the inner void surface. The current has two contributions, from capillary smoothening and from electromigration, which can be written in the form

$$J = \sigma[\gamma \partial_s \kappa(s) + qE(s)]. \tag{1}$$

Here σ and γ denote the adatom mobility and the surface tension, respectively, both of which are assumed to be isotropic, s is the arc length along the (one-dimensional) surface, and κ its curvature. The microscopic features of the electromigration force [1] are lumped into a constant effective charge density q which multiplies the local tangential electric field $E(s)$. Conservation of the void area (the two-dimensional volume) implies that the inner surface translates with a normal velocity v_n given by

$$v_n + \partial_s J = 0. \tag{2}$$

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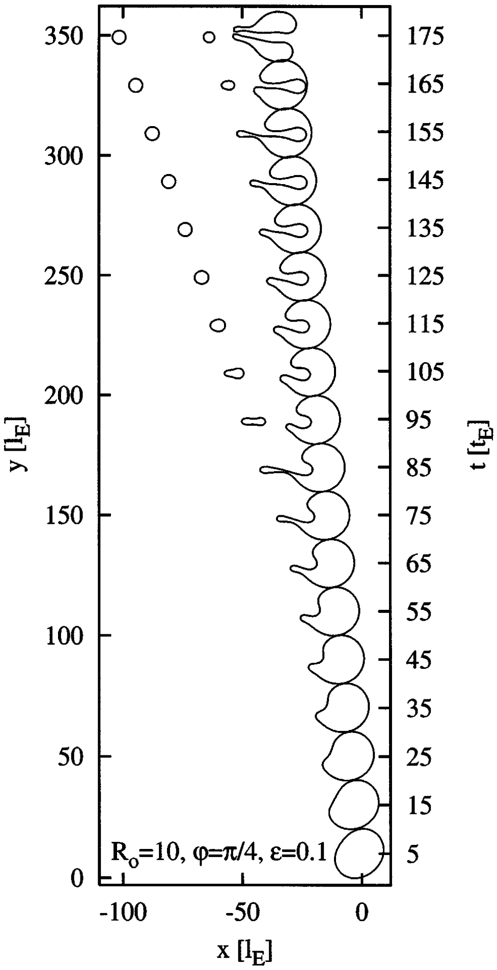

FIG. 1. Shape evolution of a void with dimensionless radius $R_0 = 10$, orientation $\varphi_0 = \pi/4$, and deformation amplitude $\epsilon = 0.1$. The void is shifted vertically in time in order to give a better visualization. Two budding incidents occur before the invagination leads to a splitting of the void.

The central theoretical difficulty lies in the nonlocal influence of the void shape on the electric field [12]. To compute $E(s)$, the Laplace equation for an electric potential U has to be solved in the infinite domain outside the void, subject to the boundary conditions that the normal electric field vanishes at the void surface and a constant electric field E_0 is imposed far away from the void. The existence of this potential is guaranteed within a quasistatic treatment of the electrodynamics, which is justified due to the separate time scales on which electrodynamic and diffusive processes take place [12]. Dimensional analysis shows that [4]

$$l_E = \sqrt{\gamma/(qE_0)} \tag{3}$$

is the only characteristic length scale in the problem. The natural time scale is then given by $t_E = l_E^4/(\sigma\gamma)$. After rescaling lengths with $1/l_E$, time with $1/t_E$, and the electric field with $1/E_0$, the only parameter left over in our model, apart from initial conditions, is the dimensionless void area, which will be expressed through the dimensionless radius R_0 of an equivalent circular void. The electromigration force dominates over the surface tension for voids large compared to l_E, i.e., for $R_0 \gg 1$.

Numerically the solution of (2) is achieved by first solving the Laplace equation using a boundary element method, extracting the tangential electric field, and then iterating a finite difference analog of (2) by a variable-order variable-step Adams method. The void is typically modeled by 100–200 surface points. A breakup procedure is triggered if two points belonging to different surface segments get closer than half the distance between neighboring points along the surface. Merging of two voids can be treated in a similar manner, but will not be considered here.

Shape evolution.—In our numerical analysis the electric field is oriented along the x axis. Assuming $qE_0 > 0$, the voids move in the negative x direction. As initial configuration we choose a circular void subject to a perturbation of the form

$$R(\varphi) = R_0 \frac{1 + \epsilon \cos[k(\varphi - \varphi_0)]}{\sqrt{1 + \epsilon^2/2}}. \tag{4}$$

ϵ determines the magnitude of the shape imperfection, k its wave number, and φ_0 its orientation relative to the direction of the applied electric field. Throughout this paper we present results for $\epsilon = 0.1$ and $k = 2$; the initial shape is then very close to an ellipse with eccentricity ϵ. The denominator in (4) ensures that the perturbed void has the same area as a circular void with radius R_0.

For small void radius R_0 the shape imperfection decays exponentially, restoring a circular void which moves with velocity $-2/R_0$ [11]. Increasing R_0 the behavior changes, depending on the orientation of the void. For voids elongated along the field direction we observe the formation of a bump at the leading end of the void (the "lemon" shape), whereas for voids elongated perpendicular to the field an invagination forms at the same end (the "crescent" shape). For intermediate void orientations a mixed shape occurs. In all cases the voids are still stable and eventually relax exponentially to the circular shape. Increasing R_0 further beyond a critical radius R_0^*, the deformations no longer decay. Instead, the void continues to distort and eventually breaks up, the lemon expelling a bud and the crescent splitting along the field direction (Fig. 1).

Figure 2 illustrates the time evolution using the circumference of the void as a measure of the deformation. The upper three curves end when the void breaks up. The critical radius R_0^* for the onset of the instability is seen to depend on the orientation of the void: Voids elongated along the field direction ($\varphi_0 = 0$) are more stable than those with the perpendicular orientation $\varphi_0 = \pi/2$.

Linear stability and non-normality.—As can be seen from Fig. 2, for void sizes close to but below R_0^*, the deformation initially *increases* in time before the exponential decay sets in. This nonmonotonic behavior

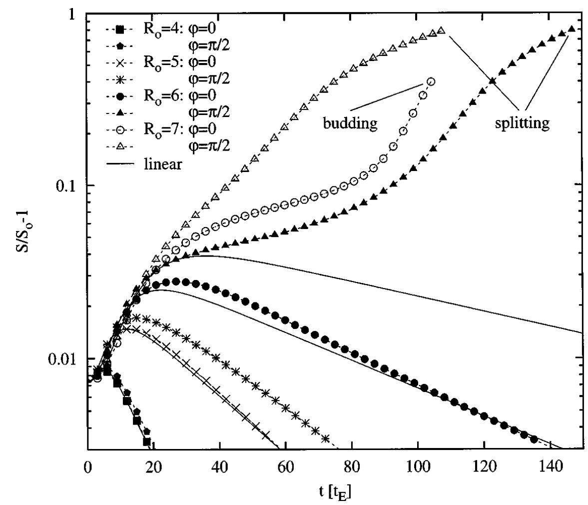

FIG. 2. Time evolution of the circumference S as a measure of deformation. S_0 denotes the circumference of a circle with the same area. The solid lines show the prediction of the linear dynamics which is independent of the orientation φ_0. For this perturbation strength the critical radius R_0^* is between 5 and 6 for $\varphi_0 = \pi/2$, and between 6 and 7 for $\varphi_0 = 0$.

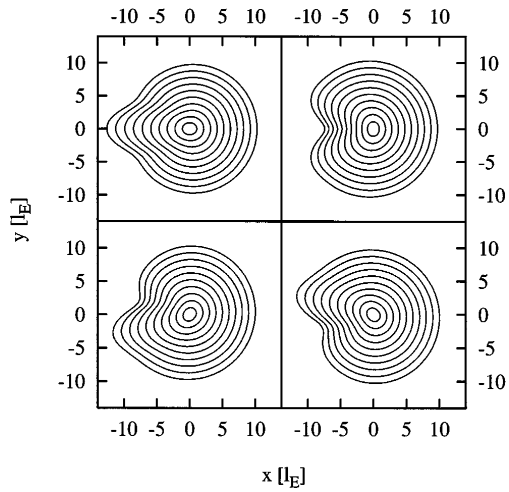

FIG. 3. The most slowly decaying eigenmodes obtained from the linear stability analysis for different values of $R_0 = 1-10$ added with a fixed amplitude of $\epsilon = 0.1$ to the circular solution. The upper frames show the (symmetric) cosine modes, the lower ones the sine modes [see Eq. (5)]. Between the left and right frames the perturbations differ in the sign.

is already inherent in the linearized dynamics [7], and hints at the non-normality of the eigenvalue problem. The linear stability analysis [6–8] starts from a perturbation expansion of the form

$$R(\varphi) = R_0\left\{1 + \epsilon \sum_{k>1}[a_k \cos(k\varphi) + b_k \sin(k\varphi)]\right\}, \quad (5)$$

where a_k and b_k are real coefficients, normalized such that $\sum(a_k^2 + b_k^2) = 1$. The $k = 0$ and $k = 1$ modes can be safely omitted, since they correspond to an increase of the void radius and a translation, respectively, which can be absorbed by a change of coordinates. The breaking of rotational symmetry by the electric field causes a coupling of mode k to the modes $k \pm 1$, leading to a nontrivial eigenvalue problem. Numerically it is easily shown that all eigenvalues are negative [6–8], indicating linear stability. The eigenvalues are twofold degenerate due to the independence of symmetric and antisymmetric perturbations. For long times the linear dynamics is dominated by the modes with the largest eigenvalue (smallest decay rate) $\omega_0 = -12/R_0^4$. Figure 3 shows the symmetric and antisymmetric eigenmode for different values of R_0 and a fixed perturbation strength $\epsilon = 0.1$. Depending on the sign of the perturbation, with increasing R_0 the symmetric mode approaches either the lemon shape or the crescent shape, while the shape of the antisymmetric mode corresponds to a void with initial orientation $\varphi_0 = \pm\pi/4$; compare to Fig. 1.

For small voids the effect of electromigration is small, and the coupling between the k modes is weak. The eigenvectors are then essentially given by the k modes themselves, and they are therefore orthogonal. By computing the scalar products of eigenmodes it can be shown that they become increasingly nonorthogonal with increasing R_0. As a consequence, a perturbation different from an eigenmode, such as that given by Eq. (4), may grow by a considerable amount before it eventually decays, in agreement with the data shown in Fig. 2. This mechanism for transient instability in linearly stable systems with a non-normal eigenmode structure has previously been described for hydrodynamic flows [13]. In the present context the important conclusion is that an initially increasing void deformation is not a reliable sign of instability, not even on the linear level.

Beyond breakup.—Looking at the budding events in Fig. 1, an obvious question concerns the size of the daughter voids. Systematic investigation shows that the size of the first daughter is an increasing function of the initial radius R_0 of the mother which, however, appears to saturate for large R_0; in the original units this implies that the size of the daughters expelled from large mother voids is proportional to l_E.

Subsequently expelled daughter voids become progressively smaller. The second daughter will therefore eventually catch up with the first, and initiate a "collision." An example of this situation is shown in Fig. 4, where two circular voids of comparable size have been started at a separation large compared to their radii, and slightly misaligned along the field direction. As the leading larger void is approached by the smaller one both deform, the former adopting a lemon shape while the latter becomes crescent shaped. The further evolution follows the two scenarios observed for single voids—the lemon expels a bud and the crescent splits horizontally. In this manner the number of voids constantly increases. In principle, there is no difficulty in handling an arbitrary number of voids numerically; however, if the total number of

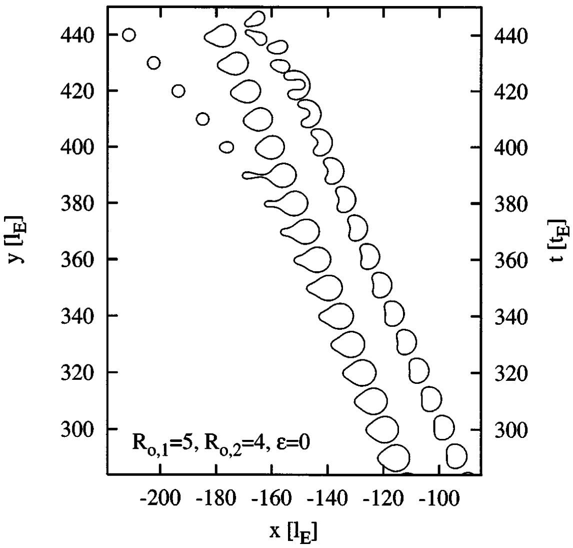

FIG. 4. Final approach of two initially circular voids with radii $R_{0,1} = 5$ and $R_{0,2} = 4$. Initially the voids were separated horizontally by $5(R_{0,1} + R_{0,2})$ and vertically shifted by $R_{0,2}/10$.

surface points is fixed, in the end too few points per void are left to accurately describe the further evolution. If, on the other hand, the number of points is increased the computing time increases quite dramatically.

We note, finally, that the collision scenario described above depends crucially on the size ratio of the two voids. For example, choosing $R_{0,1} = 5$, $R_{0,2} = 1$, the small void is found to run into the larger one and merges with it.

In conclusion, our work shows that the ultimate fate of a distorted void in an isotropic, infinitely extended two-dimensional conductor is simple: Either it relaxes back to the circular solution of Ho [11], or it breaks up into two or more voids. In contrast to earlier, less extensive numerical calculations by Suo and collaborators [6], we do not find any noncircular, stable translating void shapes, nor do we observe that a void spontaneously collapses into a slit oriented perpendicular to the current. In fact, the shape distortion never significantly increases the extension of the void perpendicular to the current flow.

To determine precisely which property of real conductor lines—strip geometry, crystal anisotropy, or grain boundary disorder—is responsible for the failure mechanism identified by Arzt *et al.* [3], these additional effects will have to be included in our model. At this point we remark that, while the multiple breakup of voids has been seen experimentally in conductor lines [14], the phenomenon appears to be less common than in our simulations. Future experiments which more closely approach the idealized conditions of this paper—e.g., using extended, single crystal films [15] in which voids are deliberately created—could test our prediction of electromigration-induced breakup as the dominant pathway of shape evolution.

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