Dynamical Scaling at the Quantum Hall Transition: Coulomb Blockade versus Phase Breaking

D. G. Polyakov[1,*] and K. V. Samokhin[2,†]

[1]*Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany*
[2]*Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, United Kingdom*

(Received 26 June 1997)

We argue that the finite-temperature dynamics of the integer quantum Hall system is governed by two independent length scales. The consistent scaling description of the transition makes crucial use of two temperature-critical exponents, reflecting the interplay between charging effects and interaction-induced dephasing. Experimental implications of the two-scale picture are discussed. [S0031-9007(97)05262-9]

PACS numbers: 73.40.Hm, 71.30.+h, 73.23.Hk

Scaling treatment of the Anderson metal-to-insulator transition is central to understanding of the integer quantum Hall (QH) effect [1]. The plateau transitions are understood as isolated critical points separating two localized phases, so that the localization length ξ diverges only at a discrete set of the critical energies E_c. While a reliable analytical theory is sorely missing, the scaling ideas have long served to correlate the results of experiment and of numerical simulation. The observed *dynamical* scaling, however, still presents a *puzzle* which has defied a convincing explanation for almost a decade, starting from the very first experiments [2]. On the experimental side, the scaling has been probed by tuning through the transition at different temperatures (by varying the Landau level filling factor) and observing how fast the critical singularities are rounded off with increasing T. The experimental data tell us that the long-distance cutoff L_h scales as $T^{-1/z}$ with the dynamical critical exponent $z = 1$. Specifically, the dissipative dc conductivity g (in units of e^2/h) has the scaling form $g = g_c F(L_h/\xi)$, where $F(0) = 1$, $F(\infty) = 0$, and $g_c \sim 1$. The traditional use of z in this context is related to the common belief [3] that at criticality the only relevant temporal scale is $\tau \sim T^{-1}$.

It can be readily seen, however, that despite the simplicity of this experimental picture, it implies the *inadequacy*, in describing the QH critical point, of the usual theoretical framework [3] based on the assumption that the system at criticality can be characterized by just one temporal scale T^{-1}. Indeed, the peculiarity of the Anderson transition in two dimensions—the nonvanishing g_c—means that the QH system at the critical point is *diffusive*, so that the irreducible dynamical susceptibility is a function of ω/q^z with $z = 2$ [4]. It follows that if there are only two scales (L_h and $\tau \propto L_h^z$) at play, they must be related via the diffusion law ($z = 2$). It has become customary to refer to the Coulomb interaction between electrons as the source of the "anomalous" $z = 1$. However, the long wavelength diffusion coefficient $D = h^{-1} g_c/(\partial n/\partial \mu)$ is finite in the interacting QH system as well, since for disordered electrons the thermodynamic density of states (DOS) $\partial n/\partial \mu$ does not exhibit any singular behavior when the Coulomb interaction is turned on, and we assume that the critical conductivity g_c also remains finite [5]. Likewise, the screening properties of the integer QH metal can be described in terms of the usual random-phase approximation response. In fact, the *only* peculiarity of the QH metallic phase, as compared to a weakly disordered conventional metal, is a fractal dispersion of the diffusion coefficient at large q^2/ω [4]. Thus the attempt to explain the cutoff $L_h \propto \tau^{1/z}$ by introducing $\tau \sim T^{-1}$ and setting $z = 1$ [3] is confronted by the fact that electron dynamics at the critical point is diffusive ($z = 2$).

Another recent attempt to substantiate the observed dynamical scaling relates [6] the apparent degradation $z = 2 \to z = 1$ to the linear vanishing of the one-particle DOS $\rho_1(\omega) \propto |\omega|$ at the Fermi level ($\omega = 0$). This fault with dimension counting underlines the common misconception of the problem once more. First, it is misleading to insert the one-particle DOS in the renormalization group machinery in place of $\partial n/\partial \mu$. Moreover, there is every reason to question the very assumption that $\rho_1 \propto |\omega|$ at the *metallic* critical point. We argue below that in actual fact $\rho_1(\omega)$ vanishes at the QH transition faster than any power of ω.

Apart from the purely scaling arguments, there is controversy about the physical mechanism of the cutoff. Again, if one follows [3] and identifies the cutoff with the interaction-induced dephasing length L_ϕ, one encounters the difficulty in trying to connect the T^{-1} behavior of L_h with the usual dependence $L_\phi \propto T^{-1/2}$, which merely reflects the diffusive character of transport of interacting particles and should be valid at the QH critical point as well. Hence the concept [3] of the quantum-classical crossover controlled by the dephasing length appears to be inadequate to the physics of the QH transition. Note, however, that the discarding of L_ϕ is not quite trivial since $L_\phi \ll L_h$ in the low-T limit, which means that the *shorter* of the two length scales is irrelevant.

In this paper, we attempt to sort out the problem of the dynamical scaling. Our findings are as follows: The scaling description of the integer QH transition

for *interacting* electrons includes *two* independent length scales, $L_h \propto T^{-1}$ and $L_\phi \propto T^{-1/2}$. They govern the temperature driven scaling outwards and towards the unstable fixed point [7], respectively (Fig. 1). Both are related to the corresponding temporal scales τ_h and τ_ϕ via the diffusion law ($z = 2$): $\tau_h \sim DL_h^2 \propto T^{-2}$ and $\tau_\phi \sim DL_\phi^2 \propto T^{-1}$. The Coulomb interaction therefore does *not* change the *true* dynamical exponent z from 2 to 1; instead, it leads to the emergence of the two different scales. It is only if one uses the usual representation of the length scales in the form $L_h \propto T^{-1/z_1}$ and $L_\phi \propto T^{-1/z_2}$ that there appears the dynamical exponent $z_1 = 1$, whereas z_2 remains equal to 2 [8]. The typical energy transfer is T and the phase-breaking rate τ_ϕ^{-1} is also of order T; however, the scattering rate τ_h^{-1} behaves as T^2. The corresponding cutoff L_h has nothing to do with the phase breaking: the temperature smearing of the transition is controlled by charging effects similar to those in the Coulomb blockade regime. The shape of the Coulomb gap in the one-particle DOS at the critical point has no direct relation to either of the dynamical exponents z_1 or z_2. Separately, we argue that $\rho_1(\omega)$ vanishes as $\exp[-\alpha \ln^2(T_c/|\omega|)]$, where $\alpha \sim 1$ and T_c is a characteristic width of the gap.

Our basic point in the description of the dynamical scaling is that the QH system at the critical point is metallic (in contrast to the critical system at a conventional Anderson transition in three dimensions) and it makes perfect sense to treat it as an ordinary dirty metal with $g \sim 1$. We therefore begin with the effect of electron-electron scattering on the quantum interference of diffusons [9] in a *weakly* disordered metal ($g \gg 1$) with completely broken time-reversal symmetry. To the best of our knowledge, this has not been spelled out clearly

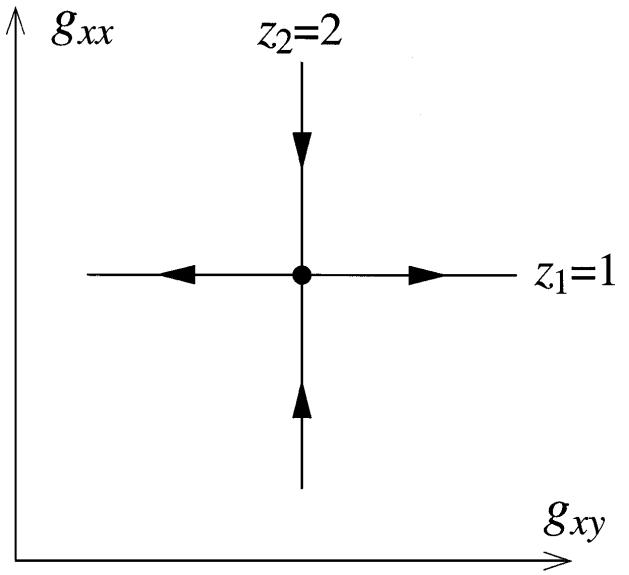

FIG. 1. Scaling with lowering T outwards and towards the unstable fixed point is governed by different length scales with temperature exponents $z_1 = 1$ and $z_2 = 2$, respectively.

in the literature. The diffusion propagator $\mathcal{D}_{\omega q}^{\omega_0}$ for interacting electrons is a function of two frequencies—only in the absence of interactions $\mathcal{D}_{\omega q}^{\omega_0} \propto \delta(\omega_0)$. It is convenient to choose the mixed representation $\mathrm{D}_{\omega q}^{t_0} = \int \frac{d\omega_0}{2\pi} \exp(-i\omega_0 t_0) \mathcal{D}_{\omega q}^{\omega_0}$ and regard the delay time t_0 as a parameter. The Dyson's equation assumes then the algebraic form $[\mathrm{D}_{\omega q}^{t_0}]^{-1} = [\mathrm{D}_{\omega q}^{(0)}]^{-1} - \Sigma_{\omega q}^{t_0}$, where the bare propagator $\mathrm{D}_{\omega q}^{(0)} = 1/(-i\omega + Dq^2)$. We define the *diffuson decay rate* $1/\tau_\phi^{\mathrm{D}}(t_0) = -\mathrm{Re}\Sigma_{\omega q}^{t_0}$ as a function of t_0 (assuming that the weak interaction does not renormalize $\mathrm{D}_{\omega q}^{t_0}$ on the microscopic scale). Particle number conservation dictates that $1/\tau_\phi^{\mathrm{D}}(0) = 0$, since the dynamical part of the density-density correlator $\langle nn \rangle_{\omega q}$ is expressed in terms of the integral $\int \frac{d\omega_0}{2\pi} \mathcal{D}_{\omega q}^{\omega_0}$. Thus, in contrast to the more familiar Cooperon, $\mathcal{D}_{\omega q}^{\omega_0}$ cannot be characterized by a *single* phase-breaking time (this should also be contrasted with the cutoff of the full diffusion propagator by a constant τ_ϕ^{D}, cf. [10]). To calculate $1/\tau_\phi^{\mathrm{D}}(t_0)$, we use the method [11], within the framework of which the electron-electron interaction is mediated by thermal fluctuations of a classical ($\omega \ll T$) electromagnetic field with the correlator $\langle VV \rangle_{\omega q} = 4\pi e^2 \nu_s T/\varepsilon(\omega^2 + \nu_s^2 q^2)$, where $\nu_s = (e^2/\varepsilon\hbar)g$ is the charge-spreading velocity, ε the bare dielectric constant (Nyquist noise). We transform to real space by writing the equation for the diffuson in the form

$$\left\{ \frac{\partial}{\partial t} - D\frac{\partial^2}{\partial \mathbf{r}^2} + \frac{i}{\hbar}\left[V\left(\mathbf{r}, t - \frac{t_0}{2}\right) - V\left(\mathbf{r}, t + \frac{t_0}{2}\right)\right]\right\} \mathrm{D}^{t_0}(\mathbf{r}, t) = \delta(\mathbf{r})\delta(t). \quad (1)$$

Notice the crucial difference between this equation and that for the Cooperon (cf. [11]): in the latter case the times t and t_0 are interchanged in the argument of the effective potential; as a result, t_0 becomes a "mute variable"—the averaged Cooperon does not depend on t_0 and this is why it is characterized by the single time τ_ϕ^{C}. Calculating the correlator of the potential in Eq. (1), we observe that $\tau_\phi^{\mathrm{D}}(t_0)$ can be obtained similarly to τ_ϕ^{C} by introducing the effective interaction $\langle VV \rangle_{\omega k}^{t_0} = \langle VV \rangle_{\omega k}(1 - \cos \omega t_0)$. It follows immediately that in the limit $t_0 \gg \tau_\phi^{\mathrm{D}}(t_0)$, where the oscillating term $\cos \omega t_0$ can be safely ignored, the particle-hole and particle-particle propagators decay in the same way: $\tau_\phi^{\mathrm{D}}(\infty) = \tau_\phi^{\mathrm{C}}$. The difference shows up at smaller t_0: one gets with logarithmic accuracy the equation for the decay rate of $\mathrm{D}_{\omega q}^{t_0}$: $1/\tau_\phi^{\mathrm{D}}(t_0) = 2\int \frac{d^2\mathbf{k}}{(2\pi)^2} \int \frac{d\omega}{2\pi} \langle VV \rangle_{\omega, \mathbf{k}+\mathbf{q}}^{t_0} \mathrm{Re}\mathrm{D}_{\omega k}^{t_0}$. Solving it, we obtain the compact expression

$$\frac{1}{\tau_\phi^{\mathrm{D}}(t_0)} = \frac{T}{g} \ln \frac{T}{D \max\{q^2, (Dt_0)^{-1}, [\nu_s \tau_\phi^{\mathrm{D}}(t_0)]^{-2}\}}. \quad (2)$$

This formula tells us that for $q \sim [D\tau_\phi^{\mathrm{D}}(t_0)]^{-1/2}$, which are relevant in the calculation of the conductivity, the decay rate starts to fall off as $\ln(Tt_0)$ at $t_0 \lesssim \tau_\phi^{\mathrm{D}}(\infty)$.

In the extreme of small $t_0 \ll T^{-1}$ the quasiclassical treatment is no longer accurate, but an estimate can be readily obtained by cutting off the frequency integration at $\omega \sim T$—it follows that the dephasing rate vanishes algebraically at zero t_0: $1/\tau_\phi^{\mathrm{D}}(t_0) \sim (T/g)(Tt_0)^2$.

Now let us look at the effect of the interaction on the quantum interference of diffusons. In the unitary limit, the leading weak-localization correction is given by the familiar expression $\delta g^{\mathrm{D}} \sim g^{-1}\ln(L/l)$ [9], where l is the mean free path (or the Larmor radius, when it is smaller), L an inelastic scattering length. However, the mechanism of the infrared cutoff in the high-B limit deserves comment, since the dephasing time $\tau_\phi^{\mathrm{D}}(t_0)$ tends to infinity as $t_0 \to 0$. The quasiclassical treatment of the Coulomb interaction allows one to calculate first the contribution to g from diffusons $\mathrm{D}^{t_0}(\mathbf{r}, t)$ moving in a given (as if externally applied) Nyquist potential. The Gaussian average over the thermal electromagnetic fluctuations ($\langle \ldots \rangle$ below) can then be safely performed. For the leading correction, this gives $\delta g^{\mathrm{D}} = g^{-1}\int_0^\infty dt \langle A(t)\rangle$, where $A = A_2 + A_3$ is a sum of two- and three-diffuson terms [9] (a proper cutoff on the ballistic scale is assumed). Consider the simplest two-diffuson contribution

$$A_2(t) = 2D^2 \int_0^t dt' \mathrm{D}^{t'-t}(0,t')\mathrm{D}^{t'}(0,t-t'), \qquad (3)$$

which already reveals the peculiarity of the dephasing in the unitary case. Though one could have expected that $\langle A_2(t)\rangle$ would decay exponentially at $t \gg \tau_\phi^{\mathrm{D}}(\infty)$, it can be readily seen from Eq. (3) that $\langle A_2(t)\rangle$ remains singular on the scale of $\tau_\phi^{\mathrm{D}}(\infty)$. The phase coherence is preserved because of the vanishing of the dephasing rate at $t' = 0$ and $t' = t$. A similar "breakdown" of the dephasing occurs in $\langle A_3(t)\rangle$. However, adding all the pieces, we find that the total contribution to δg^{D}, $\langle A(t)\rangle \propto \exp[-t/\tau_\phi^{\mathrm{D}}(\infty)]$, decays on the scale of the shortest dephasing time. This proves that the interaction-induced cutoff for δg^{D} is given by the phase-breaking length related to $\tau_\phi^{\mathrm{D}}(\infty)$ (which contrasts with the result of Ref. [12], where the inelastic cutoff of the weak localization in the unitary limit was identified with a much longer energy-relaxation length).

We turn now to the interaction-induced dephasing at the integer QH transition. We assume that the interaction is weak enough not to break down the integer QH effect, i.e., $e^2/\varepsilon\lambda \ll \Gamma$, where λ is the magnetic length, Γ the width of the disorder-broadened Landau level. It is then legitimate to repeat the above analysis of the phase breaking right at the QH metallic point by endowing the diffusion coefficient with a strong dispersion at $Dq^2/\omega \gtrsim 1$ [4]. The power-law dispersion at large q^2/ω signals that the QH metal starts to develop the critical eigenfunction correlations. However, as follows from the calculation with constant D, this does not change the dependence of L_ϕ on T, since the relevant Dq^2/ω are of order unity. Specifically, an estimate can be readily obtained by setting $g \sim 1$ in Eq. (2), which gives $T\tau_\phi^{\mathrm{D}}(\infty) \sim 1$ and $L_\phi \sim (D/T)^{1/2}$ ($z_2 = 2$). Notice that when the Fermi energy coincides with E_c, the localization effects can be neglected at all $\omega \ll \Gamma$, since $\xi \gg (D/\omega)^{1/2}$ within the energy band of width ω around E_c. In sum, the scale on which the dephasing occurs at the critical point is certainly $(D/T)^{1/2} \ll L_h$. We are led to conclude that while the phase breaking controls the temperature scaling of g_c right at the critical point, it does *not* control the observed metal-insulator crossover.

The reason for the strong increase of the cutoff L_h as compared to L_ϕ is that away from the critical point transport is governed by charging effects: the Coulomb blockade on the scale of ξ drastically narrows the crossover region. Indeed, one can identify two characteristic energies on the scale of ξ: the charging energy $U_c \sim e^2/\varepsilon\xi$ and the "on-site" energy spacing $\Delta \sim 1/(\partial n/\partial\mu)\xi^2$. Near the transition $U_c \gg \Delta$. The naive description of scaling in terms of L_ϕ/ξ amounts to the assumption that the QH system shows crossover at $T/\Delta \sim 1$. It is evident, however, that the system behaves as a metal only if T exceeds U_c—otherwise the scattering is blocked as in the usual Coulomb blockade regime. The QH system at given E_F can thus be modeled as a dense array of quantum dots of size ξ coupled via the tunneling integral $\sim\Delta$. The scaling form of g then reads

$$g = g_c F(U_c/T), \qquad (4)$$

or, equivalently, $g = g_c F(L_h/\xi)$ with $L_h \sim e^2/\varepsilon T$, so that $z_1 = 1$ (these arguments parallel those in [13], where $F(x)$ was argued to fall off at $x \to \infty$ as $\ln F \sim -x^{1/2}$). Hence, the scaling around the unstable fixed point indeed necessitates dealing with *two* scales, L_h and L_ϕ (Fig. 1). Also, while the typical energy transfer and the dephasing rate are both $\sim T$, the scattering rate $\tau_h^{-1} \sim DL_h^{-2}$ is much smaller:

$$1/\tau_h \sim T^2/T_c, \qquad T_c \sim e^4/\varepsilon^2 D. \qquad (5)$$

To test the two-scale picture with $z_1 \neq z_2$ experimentally, we suggest to measure the temperature dependent correction to the critical conductivity $\delta g_c(T)$. Specifically, according to numerical simulations [1,14], the finite-size correction to g_c scales as L^{-y} with $y \simeq 0.4$–0.5 (in fact, it can be shown analytically [15] that y is not an independent exponent; namely, there exists the nontrivial relation $y = \eta$, where $\eta \simeq 0.4$ is the usual critical exponent of eigenfunction correlations [4]). We predict that, while the smearing of the transition is controlled by L_h ($z_1 = 1$), the critical conductivity scales with L_ϕ ($z_2 = 2$), i.e., $\delta g_c \propto T^{y/2}$. Another possible test is based on the fact that $L_\phi \ll L_h$. Naively, one may well think that when L_h becomes larger, as $T \to 0$, than the system size L, there must appear strong mesoscopic fluctuations (say of the height of the conductivity peak). However, our approach suggests that this is not true, since in the range $L_\phi \ll L \ll L_h$ the width of the critical region is already T independent but the mesoscopic fluctuations are still suppressed (at $\tau_\phi^{-1} \sim T$, the only parameter that governs the amplitude of the fluctuations is L^2T/D).

The absence of the fluctuations at $L_h \gtrsim L$ would give a strong experimental support to the two-scale picture.

Finally, we discuss briefly the behavior of the one-particle DOS at the critical point $\rho_{1c}(\omega)$. It is a popular misconception that the reduction $z_1 \to 1$ signifies the linear vanishing of $\rho_{1c}(\omega) \propto |\omega|$ (see, e.g., [6]). In fact, several aspects require comment. First, as argued above, the true dynamical exponent is related to g_c and the *thermodynamic* DOS $\partial n/\partial\mu$, so that it is equal to 2 at the QH transition (merely reflecting the Einstein relation). Second, away from the critical point, the quasiparticle DOS ρ_h that appears in the hopping exponent [13] indeed behaves as $\rho_h \sim |\omega|\varepsilon^2/e^4$ at $|\omega| \lesssim U_c$; however, ρ_h does *not* coincide with ρ_1 unless the system is classical and electrons can be treated as point charges. The difference is due to the fact that in the classical treatment of the Coulomb gap [16] $\Delta/|\omega|$ is sent to ∞, whereas near the critical point Δ is the smallest energy scale. As a result, the rate of the charge spreading becomes a crucial factor in the suppression of ρ_{1c} in the metallic phase. The width of the interaction-induced gap in a metal grows with decreasing g as $\exp[-2(\pi g)^{1/2}]$ [17]. To calculate $\rho_{1c}(\omega)$, we use the elegant quasiclassical method suggested in [18], which works well in the conducting phase even if $g \sim 1$. Adjusting it to the high-B limit (in our case the screening length D/v_s is larger than the Larmor radius), we obtain at $g_c \sim 1$

$$\rho_{1c}(\omega) = (\partial n/\partial\mu)\exp[-S(\omega)], \qquad S \simeq \alpha \ln^2(T_c/|\omega|), \tag{6}$$

where the numerical coefficient $\alpha \sim 1$, and the width of the gap T_c is defined by Eq. (5). It is worth noticing that the localization-induced dispersion of the diffusion coefficient at large q^2/ω, which is the only peculiarity of the QH critical point as compared to the Drude metal, is of little importance here (in contrast to the conventional two-dimensional metal, where the localization effects get in the way of the method [18] at $\omega \to 0$). Note also the shape of the gap at the transition—ρ_{1c} vanishes faster than any power of ω. This should be contrasted with both the power-law behavior of ρ_1 at the Anderson transition in $2 + \epsilon$ dimensions [19] and the naive power counting at the QH transition [6]. This result also brings up the question of whether the Hartree-Fock method [20], within the framework of which a linear vanishing of ρ_{1c} was observed numerically, captures all the essential physics. Away from the critical point, the "log-normal" suppression of the DOS saturates with decreasing ω at $|\omega| \sim U_c$ (which means that the charge spreading stops on the scale of ξ). In the insulating phase, the linear [16] vanishing of ρ_1 should be expected at $\omega = 0$, but with a slope suppressed by the factor of $\exp[-S(U_c)]$.

To summarize, we have argued that the temperature driven scaling at the integer QH transition is governed by two independent length scales with the temperature exponents $z_1 = 1$ and $z_2 = 2$. The smearing of the transition is controlled by charging effects ($z_1 = 1$), whereas the interaction-induced phase breaking ($z_2 = 2$) is responsible for corrections to the critical conductivity. We suggested experimental tests of the two-scale picture.

We thank J. Hajdu, B. Halperin, D. Khmelnitskii, A. Mirlin, N. Read, and P. Wölfle for interesting discussions. This work was supported by the Deutsche Forschungsgemeinschaft, by the German-Israeli Foundation, and by the Russian Foundation for Basic Research under Grant No. 96-02-17894a.

*Also at A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

†Also at L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia.

[1] B. Huckestein, Rev. Mod. Phys. **67**, 357 (1995).

[2] H. P. Wei, D. C. Tsui, M. A. Paalanen, and A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1294 (1988).

[3] S. L. Sondhi, S. M. Girvin, J. P. Carini, and D. Shahar, Rev. Mod. Phys. **69**, 315 (1997).

[4] J. T. Chalker and G. J. Daniell, Phys. Rev. Lett. **61**, 593 (1988).

[5] The scaling form of the irreducible density-density correlator $\mathrm{Im}\langle nn\rangle_{\omega q} = q^{2-z} f(\omega/q^z)$ is inconsistent, unless $z = 2$, with the condition that both $\partial n/\partial\mu = \lim_{q\to 0}\langle nn\rangle_{0q}$ and $g_c = -ih \lim_{\omega\to 0}\omega \lim_{q\to 0} q^{-2}\langle nn\rangle_{\omega q}$ are finite.

[6] D.-H. Lee and Z. Wang, Phys. Rev. Lett. **76**, 4014 (1996).

[7] D. E. Khmel'nitskii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1983)]; A. M. M. Pruisken, in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer, Berlin, 1990).

[8] We compare the dynamical exponents for two different directions of the renormalization group flow around the unstable fixed point. The possibility of multiple divergent time scales for one direction of the flow was considered in detail in D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[9] S. Hikami, Phys. Rev. B **24**, 2671 (1981).

[10] C. Castellani, C. Di Castro, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **56**, 1179 (1986).

[11] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982).

[12] V. K. Dugaev and D. E. Khmel'nitskii, Zh. Eksp. Teor. Fiz. **90**, 1871 (1986) [Sov. Phys. JETP **63**, 1097 (1986)].

[13] D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **70**, 3796 (1993).

[14] Z. Wang, B. Jovanović, and D.-H. Lee, Phys. Rev. Lett. **77**, 4426 (1996).

[15] M. Janssen and D. G. Polyakov (to be published).

[16] A. L. Efros and B. I. Shklovskii, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[17] B. L. Altshuler and A. G. Aronov, in Ref. [16].

[18] L. S. Levitov and A. V. Shytov, Pis'ma Zh. Eksp. Teor. Fiz. **66**, 200 (1997) [JETP Lett. **66**, 214 (1997)].

[19] A. M. Finkel'stein, Zh. Eksp. Teor. Fiz. **86**, 367 (1984) [Sov. Phys. JETP **59**, 212 (1984)].

[20] S.-R. E. Yang and A. H. MacDonald, Phys. Rev. Lett. **70**, 4110 (1993).