Magnetic Edge States in a Magnetic Quantum Dot

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(Received 29 May 1997)

The formation of magnetic edge states along with corresponding classical trajectories is investigated for a magnetic quantum dot with inhomogeneous distributions of magnetic fields. The magnetic edge states are found to circulate either clockwise or counterclockwise along the boundary region of the quantum dot, depending on the number of missing flux quanta, and exhibit quite different properties, as compared to the conventional ones which are induced by electrostatic confinements in the quantum Hall system. [S0031-9007(98)05359-9]

PACS numbers: 73.20.Dx, 72.15.Gd, 73.40.–c

In past decades, advances in microfabrication techniques have made it possible to further confine a two-dimensional electron gas (2DEG) through built-in electrostatic potentials such as quantum wires, dots, or rings on a mesoscopic scale. Of special interest in these confined systems is the electron transport behavior in the quantum Hall regime, where the current-carrying edge states, formed near the boundary region, play an important role in describing the resonant tunneling, Aharonov-Bohm oscillation, nonlocal magnetoresistance, and Coulomb-blockade oscillation, etc. Recently, with the application of spatially inhomogeneous magnetic fields, a number of alternative magnetic structures were proposed on the 2DEG, such as magnetic quantum dots using a scanning tunneling microscope lithographic technique [1], magnetic superlattices by the patterning of ferromagnetic materials integrated by semiconductors [2], type-II superconducting materials deposited on conventional heterostructures [3], and nonplanar 2DEG systems grown by a molecular beam epitaxy [4].

For the 2DEG applied by inhomogeneous magnetic fields, which provide two different magnetic domains, as shown schematically in Fig. 1, the current-carrying states (hereafter referred to as the magnetic edge states in close analogy with electrostatically induced conventional ones) exist near the boundary between the two domains [5]. These magnetic edge states have quite different properties from the conventional ones; thus, a variety of new phenomena associated with the magnetic structures are expected in the electron transport. However, to our knowledge, very little attention has been paid to this problem [6].

In this paper, we investigate the nature of magnetic edge states in a magnetic quantum dot which is formed by inhomogeneous magnetic fields; electrons are apparently confined to a plane and, within that plane, the magnetic field is zero within a circular disk and constant B outside it [7]. We calculate exactly the single electron eigenstates and energies of a magnetic quantum dot as a function of magnetic field, using a single scaled parameter $s = \pi r_0^2 B/\phi_0$, which represents the number of missing magnetic flux quanta within the dot, where r_0 is the radius of the quantum dot and $\phi_0(= h/e)$ is the flux quantum. We find two types of edge states which circulate in opposite directions to each other along the boundary of the magnetic dot and exhibit quite different energy dependences on angular momentum. We find a close relation between the quantum mechanical eigenstates and the classical trajectories in the magnetic quantum dot; the quantum mechanical eigenstate corresponds to a certain ensemble average of the classical motions which consist of straight line paths in the dot region and cyclotron orbits with a quantized radius in the outside region. These radius and central positions of the cyclotron orbits depend critically on the value of s. For a small conductor with a magnetic quantum dot at the center, the calculated magnetoconductances show aperiodic oscillations instead of the Aharonov-Bohm-type periodic oscillations [8], and this behavior is attributed to the characteristics

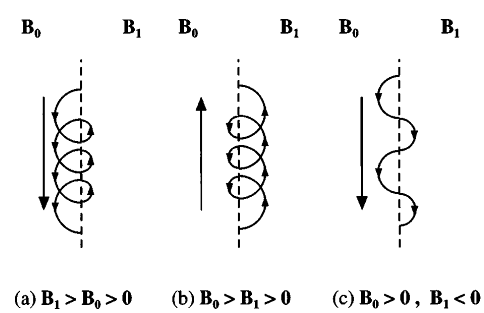

FIG. 1. Schematic diagram of classical trajectories of electrons for the magnetic edge states on the magnetic domain boundary.

of the magnetic edge states, which are absent in the conventional ones.

The single particle Schrödinger equation for a two-dimensional magnetic quantum dot is $(\vec{p} + e\vec{A})^2/(2m^*)\psi(\vec{r}) = E\psi(\vec{r})$, where m^* is the effective mass of electron and e is the absolute value of the electron charge. In polar coordinates (r, θ) on the plane, the vector potential $\vec{A}$ can be chosen as 0 for $r < r_0$ and $(r^2 - r_0^2)B/(2r)\hat{\theta}$ for $r > r_0$, so that $B = 0$ for $r < r_0$ and nonzero $B\hat{z}$ otherwise. The wave functions and the energies are easily determined by the continuity of the wave functions and their derivatives at the boundary of the dot. Since the wave functions are separables, i.e., $\psi_{nm}(\vec{r}) = R_{nm}(r)e^{im\theta}$, where m is the angular momentum quantum number and n $(= 0, 1, 2, \ldots)$ is the number of nodes in the radial wave function, the equation for the radial part is written as

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} + 2E\right)R_{nm}(r) = 0 \quad (r < r_0), \tag{1}$$

$$\left\{\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{(m - s)^2}{r^2} - r^2 + 2[E - (m - s)]\right\}R_{nm}(r) = 0 \quad (r \geq r_0). \tag{2}$$

Here $R_{nm}(r) = C_1 J_{|m|}(\sqrt{2E}\, r)$ for $r < r_0$ and $R_{nm}(r) = C_2 r^{|m-s|}e^{-r^2/2}U(a, b; r^2)$ for $r \geq r_0$. In this case, all quantities are expressed in dimensionless units by allowing $\hbar\omega_L[= \hbar eB/(2m^*)]$ and the inverse length $\beta = \sqrt{m^*\omega_L/\hbar}$ to be 1. Then, since $\hbar^2/m^* = \hbar\omega_L/\beta^2 \to 1$ and $r_0 \to \sqrt{s}$, $s = B\pi r_0^2 e/h$ is only the relevant parameter. The function J_m is the Bessel function of order m and U is the confluent hypergeometric function with $a = -(E - m_{\rm eff} - |m_{\rm eff}| - 1)/2, b = |m_{\rm eff}| + 1$, and $m_{\rm eff} = m - s$. It is noted that Eq. (2) has the same form as that of the uniform magnetic field case, except that the angular momentum m is replaced by the effective angular momentum $m_{\rm eff}$.

To see the significance of $m_{\rm eff}$, let us consider momentarily the 2DEG in a uniform magnetic field, in which the eigenstates are described by the degenerate Landau levels, $E_i = \hbar\omega_c(i + 1/2)$, where $\omega_c = eB/m^*$. When the symmetric gauge is chosen, n and m remain good quantum numbers, and the probability density of the eigenstate $(n = 0, m)$ has a maximum at $r = \sqrt{|m|}$ in dimensionless units. In this case, the quantum mechanical eigenstate (n, m) with the eigenvalue E_{nm} corresponds to the ensemble average of the classical cyclotron motions [9] with the radius r_i and its center located at r_j from the origin, which satisfies the following relations from the conservations of energy and angular momentum:

$$\begin{aligned} r_i &= \sqrt{\frac{E_{nm}}{2}} = \sqrt{\frac{2n + |m| + m + 1}{2}}, \\ r_j &= \sqrt{r_i^2 - m}. \end{aligned} \tag{3}$$

However, because of the uncertainty principle, the central position of the cyclotron orbit cannot be determined quantum mechanically.

In the magnetic quantum dot, the Landau level degeneracy is lifted for the states near the dot. From Eqs. (1) and (2), if the effective potential $V_{\rm eff}(r)$ is defined as

$$V_{\rm eff}(r) = \begin{cases} \dfrac{m^2}{2r^2} & (r < r_0) \\ \dfrac{m_{\rm eff}^2}{2r^2} + \dfrac{r^2}{2} + m_{\rm eff} & (r \geq r_0) \end{cases}, \tag{4}$$

the minimum of $V_{\rm eff}(r)$ always occurs at $r = r_0$ $(= \sqrt{s})$ for the states with $|m_{\rm eff}| < s$, i.e., $0 < m < 2s$, which correspond to the magnetic edge states circulating counterclockwise, as we will see below. The $m = 0$ state is widely distributed over the dot due to the lack of the centrifugal force, and the minimum of $V_{\rm eff}(r)$ for the states with $|m_{\rm eff}| > s$, i.e., $m < 0$ or $m > 2s$, is located at $r = \sqrt{|m_{\rm eff}|}$ outside the quantum dot, similar to the case of uniform magnetic fields. The states with $m < 0$, which exist near the dot, give rise to the magnetic edge states circulating clockwise. Figure 2 shows the energy levels of the magnetic quantum dot for different values of m at $s = 5$, the radius of which is about 500 Å for magnetic fields of teslas. The lowest energy state occurs at $m = 0$ and the degeneracy of the Landau levels are removed, as shown in Fig. 2. This result indicates that the inhomogeneity of magnetic fields perturbs mostly the states near the boundary of the quantum dot, and this perturbation is caused by the missing s flux quanta. The probability currents [10] $I_{nm} = \frac{1}{\hbar}\frac{\partial E_{nm}}{\partial m}$ for the perturbed states are found to be nonzero, resulting in the magnetic edge states; for $m > 0$, I_{nm} have positive values for counterclockwise circulations, whereas for $m < 0$, I_{nm} have negative values for clockwise circulations.

In Fig. 3, the energy levels are plotted as a function of magnetic field s for different values of (n, m), with the energy $\hbar\omega_L$ set to 1 at $s = 5$ and the radius r_0 fixed.

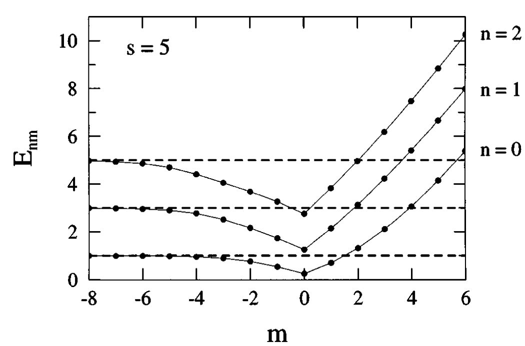

FIG. 2. Dependence of the energy eigenvalues E_{nm} on the angular momentum m for $s = 5$. Dashed lines represent the bulk Landau levels.

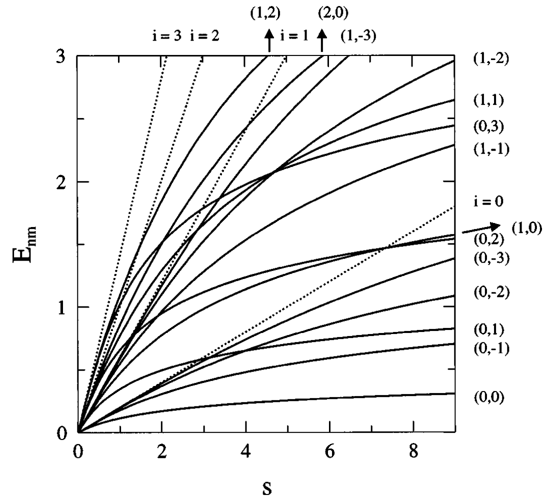

FIG. 3. Energy spectra as a function of s. The energy unit of $\hbar\omega_L = 1$ at $s = 5$ is used. Dotted lines represent the Landau levels.

As the magnetic field increases, the deviations of energies from the bulk Landau levels become significant, which leads to the magnetic edge states near the boundary of the quantum dot. In the limit of $B \to \infty$, we find that the energies approach the conventional circular dot which is electrostatically confined by hard walls without magnetic fields. From the analysis used for the uniform field case, we can also show that the (n, m) state corresponds exactly to the ensemble average of the classical motions which consist of the straight line paths in the dot region and the cyclotron orbits with the radius r_i and the center located at r_j outside the dot. These straight lines and cyclotron orbits intersect each other at the dot boundary. In this case, the relations [11] between the (n, m) states and the corresponding r_i and r_j values are determined from the conservations of energy and angular momentum for the magnetic quantum dot and are written as

$$r_i = \sqrt{\frac{E_{nm}}{2}}, \qquad r_j = \sqrt{r_i^2 - m_{\mathrm{eff}}}\,. \tag{5}$$

Equation (5) has the same form as Eq. (3), except that E_{nm} calculated from Eqs. (1) and (2) is lifted from the bulk Landau level in Eq. (3) and m is replaced by m_{eff} due to the inhomogeneity of magnetic fields. The classical trajectories for the $(0, 0)$, $(0, -1)$, and $(0, 1)$ states are drawn in Fig. 4, showing a clear correspondence between the quantum eigenstates and the classical motions; the probability densities $|R_{nm}(r)|^2$ and the directions of the probability currents I_{nm} correspond to the classical motions. The classical trajectory corresponding to the $(0, 0)$ state carries no current because it always passes through the origin, and the classical motions of the $(0, -1)$ and $(0, 1)$ states correspond to the probability currents of the states in the clockwise and counterclockwise directions, respectively. We find that our correspondence analysis may answer the important question as to whether the classical motions corresponding to the quantum eigenstates are periodic or not. In the magnetic quantum dot, periodic motions occur if the angle α made by two lines connected from the origin to the centers of two successive orbits [see Fig. 4(c)] is $2\pi p/q$, where p and q are integers. From a simple geometrical argument, α is found to satisfy the relation $\cos(\alpha/2) = (r_i^2 + r_j^2 - r_0^2)/2r_i r_j$. However, at this moment, it is difficult to give a definite answer because of the numerical errors for evaluating E_{nm}.

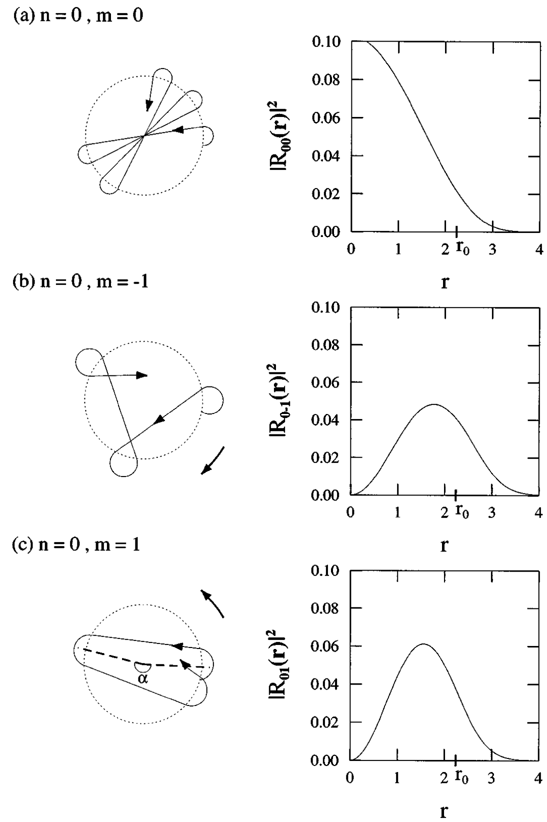

FIG. 4. Classical trajectories of electrons and corresponding probability densities for the eigenstates (a) $(0, 0)$, (b) $(0, -1)$, and (c) $(0, 1)$.

Finally, we discuss the difference between the magnetic edge states and conventional ones, by investigating the quantum interference effect in the magnetic quantum dot. We consider a small two-dimensional conductor with a magnetic quantum dot at the center. For magnetic fields which give the quantum Hall plateaus, the transport along the boundary of the sample, which is usually promoted by conventional edge states, can be backscattered by the resonant tunneling into the magnetic edge states along the boundary of the dot, because of the impurity effect in the narrow region between two boundaries. In usual quantum dots or ring structures, the resonant tunneling effect in magnetoresistance measurements gives rise to the Aharonov-Bohm oscillations [8,12], which are periodic with magnetic field. In the magnetic quantum dot considered here, we do not see such periodic oscillations. We calculate the two-terminal conductance, which is the inverse of the sum of magnetoresistance and Hall resistance, taking into account the

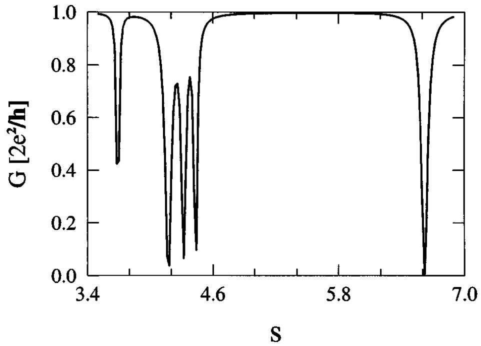

FIG. 5. Magnetoconductance as a function of s.

resonant backscattering via the magnetic edge channels as follows:

$$G(B) = \frac{2e^2}{h}\left[1 - \sum_{n,m} \frac{\Gamma^2}{[E_F - E_{nm}(B)]^2 + \Gamma^2}\right], \quad (6)$$

where Γ is the elastic resonance width and a constant value of $\Gamma = 0.005$ is used for simplicity. The calculated conductance is plotted as a function of magnetic field in Fig. 5, with the Fermi energy of $E_F = 2$ in units of Fig. 3. In this case, the magnetic fields represented by s are in the $\nu = 2$ quantum Hall plateau region, where ν is the Landau level filling factor. We find that the oscillations are not periodic, in contrast to the Aharonov-Bohm-type oscillations. The first dip in the conductance at about $s = 3.7$ is due to the resonant backscattering via the $(1, -3)$ magnetic edge state. The other dips are found to be associated with the $(0, 3)$, $(1, 1)$, $(1, -2)$, and $(1, -1)$ states in the increasing order of s. In the narrow ring structure of Jain [8], the intervals between the dips were shown to be periodic, which indicates the subsequent change of one flux quantum passing through the inner boundary. In our magnetic dot structure, the resonances occur via the two different magnetic edge states circulating in different directions, depending on the sign of m. Since there is no magnetic field inside the magnetic dot, the magnetic edge states may not enclose the magnetic flux, resulting in the missing of flux quanta, which is absent in the edge states formed by electrostatic confinements.

In conclusion, we have investigated the electronic structure of a magnetic quantum dot, and the formation of the edge states corresponded to the classical trajectories. The magnetic edge states depend critically on the number of missing flux quanta and show quite distinctive aperiodic oscillations in magnetoconductance. The significance of this paper can be extended to more complex systems [11] and applied to the edge states of composite fermions [13] in the fractional quantum Hall system with a spatially varying electrostatic potential, for example, in an antidot, because the effective magnetic field in the context of composite fermions varies with the density of composite fermions.

This work was supported in part by the SPRC of Jeonbuk National University, by the CTPC and CMS at KAIST, and by the Korea Ministry of Education, Grant No. BSRI 96-2435.

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