Simultaneous Observation of Multiple Nonlocal Eigenmodes of an Inhomogeneity-Driven Plasma Instability

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(Received 19 June 1997)

Multiple eigenmodes of the inhomegeneous energy-density-driven instability are observed simultaneously with large and comparable amplitude. This is in sharp contrast to the usual case in plasmas in which one eigenmode dominates or the laboratory device's geometry is responsible for multiple local eigenmodes. Each feature in the experimentally observed spectrum is unambiguously identified with a specific eigenstate of the theoretical model. [S0031-9007(98)05297-1]

PACS numbers: 52.35.Qz, 52.35.Fp

It is well known that inhomogeniety in plasma parameters can result in unstable waves [1]. The best known of these are the low-frequency waves associated with the drift [2] and Kelvin-Helmholtz [3] instabilities which are driven by a gradient in the density and in the transverse-velocity shear, respectively. The character of these waves can be coherent or turbulent and the influence of these waves on the plasma equilibrium in general, and on particle transport in particular, has been the subject of many years of serious study in fusion and space applications.

In a laboratory experiment, waves are usually observed as one or more normal modes of the plasma geometry. For example, radial [4], azimuthal [5], axial [6], and toroidal [7] normal modes have been identified in cylindrical and toroidal geometries and appear as highly resolved features in the fluctuation spectra. Multispiked spectra, caused under these circumstances by the existence of plasma edges, would not be expected in the expansive plasmas in space, except in the presence of density structuring; for example, at a boundary layer or in the case of filamentation. In this Letter, observations are reported of eigenmodes that originate from a transverse-flow structure having a scale size L much smaller than the plasma dimensions but larger than an ion gyroradius ρ_i.

In previous investigations of the inhomogeneous energy-density driven (IEDD) instability, in which potassium [8] and argon [9] plasmas were used, a single spectral feature was reported, with a width ranging from narrow ($\Delta f/f \approx 2\%$) to broad ($\Delta f/f \approx 30\%$). Both the interpretation of the spikiness in the spectrum and the ramifications on the spectrum of the existence of multiple eigenmodes could only be speculated [8,10]. In the plasma experiments reported here, where sodium is used [11] for achieving small values of $\epsilon \equiv \rho_i/L$, the number of observed eigenmodes is seen to increase with decreasing ϵ, as expected, and the identity of individual eigenmodes is readily confirmed by their unique amplitude dependence on the magnetic field.

Experiments are performed in a single-ended Q-machine [12] plasma column (3-cm radius, 3-m length) with the following parameters: density $n \approx 10^9$ cm^{-3}, ion and electron temperatures $T_i \approx T_e \approx 0.2$ eV, undisturbed plasma potential $V_{p0} \approx -2$ V, ion-electron mass ratio $m_i/m_e = 4.2 \times 10^4$ (sodium), magnetic field $B = 1.5$ kG, and neutral pressure (during the experiment) $P_n < 10^{-6}$ torr. Density and plasma potential are measured using Langmuir probes [13] and emissive probes, respectively. A segmented disk electrode [14] is used to produce and control the transverse plasma flow and the parallel electron drift velocity [15]. The disk electrode is made of five coplanar, concentric circular segments heated to prevent surface contamination. The applied voltages, V_0, V_1, V_2, V_3, and V_4, on the inner button segment ($r \leq 0.80$ cm) and on the adjacent annular segments ($0.90 \leq r \leq 1.35$ cm, $1.40 \leq r \leq 1.85$ cm, $1.90 \leq r \leq 2.35$ cm, and $2.40 \leq r \leq 2.85$ cm, respectively) are set with independent power supplies.

The electron drift v_d, relative to the ions, is produced by biasing the segments positively with respect to V_{p0}, yielding a magnetic-field-aligned current. Typical values of $v_d/v_{te} = 0.1$ ($v_d/v_{ti} = 20$) are obtainable at disk-electrode voltages of approximately 20 V. When the segments are biased simultaneously but unequally, a localized radial electric field is produced that results in a localized $\mathbf{E} \times \mathbf{B}$ flow, with speed v_E. Values of $v_E/v_{te} = 0.01$ ($v_E/v_{te} = 2$), where v_{te} (v_{ti}) is the electron (ion) thermal speed, are typically obtainable. The inhomogeneous transverse-velocity profile has a radial dimension that can be adjusted within the range $0.3 < 2L < 2.5$ cm. The IEDD wave experiments reported here are the first to use an *outward* electric field to destabilize the instability; all previous experiments used an inward electric field. This enables values of v_d to be large enough across the entire transverse-flow profile that the entire profile contributes to the instability.

The measurement of k_θ and k_z involves the cross correlation of simultaneously acquired ion-saturation current fluctuations from two separated Langmuir probes mounted together and inserted radially into the plasma column to a point where the amplitude of each mode is large. At the radial position $r = 0.4$ cm, values of k_θ and k_z are 2.8 and (0.11 ± 0.01) cm^{-1}, respectively. The eigenmodes propagate primarily azimuthally, in the direction of $\mathbf{E} \times \mathbf{B}$. The axial component of the propagation is in the direction of the electron drift, with $\omega_\mathrm{I}/(k_z v_d) \approx 0.7$, where ω_I is the frequency of an eigenmode in the frame of the $\mathbf{E} \times \mathbf{B}$ flow. The azimuthal mode structure is that of an azimuthal normal mode (i.e., $k_\theta = m/r$) with mode number $m = 1$. Although the wave exists along the entire length of the plasma column, the axial mode structure does not appear to be that of a normal mode since the axial wavelength is short ($\lambda_z = 0.57$ m $\pm$ 10%) compared to the plasma-column length ($L_p = 3.0$ m) and L_p/λ_z is not an integer multiple of one-half.

In Fig. 1 are three pairs of radial profiles consisting of the plasma potential $V_p(r)$ (dashed line) and the radial electric field (solid line) determined by $E_r = -dV_p/dr$. Figures 1(a)–1(c) represent the small-L, medium-L, and large-L cases, respectively. One electric-field structure, with a FWHM of $2L = 0.5$ cm, stands out in the small-L case. In the medium-L case, two structures, each with $2L = 0.4$ cm, are apparent; however, they act together as a single structure, as in quantum mechanics, with an effective FWHM of 1.0 cm. In the large-L case, the effective full width at half maximum (FWHM) is 2.5 cm, although up to four electric-field structures are resolved by the emissive probe measurement. The value of ϵ associated with Figs. 1(a)–1(c) is 0.50, 0.34, and 0.10, respectively; however, this value can be doubled or halved over the operating range of the magnetic field due to the sensitivity of ρ_i and the insensitivity of L to B [11].

Previous IEDD wave studies [8,9], in which $0.7 \leq \epsilon \leq 1$, encountered a single dominant feature in the fluctuation spectrum, as expected [8] for large ϵ. Figure 2 demonstrates that two dominant spectral features are encountered in the medium-L case and that the number of eigenmodes increases with magnetic field (i.e., as ϵ decreases). The model [16] predicts two and four eigenmodes, closely spaced in frequency, for conditions matching Figs. 2(a) and 2(b), in excellent agreement with the experiment. Note that $\omega < \omega_{ci}$ for some spectral features, which is impossible for the homogeneous case of the current-driven electrostatic ion-cyclotron (CDEIC) instability [17].

The eigenfunction associated with each spectra feature extends across the entire profile of electric-field structures. This is evidenced in Fig. 2 by the close resemblance of spectra recorded simultaneously on all of the segments of the disk electrode. Moreover, the radial profile of mode amplitude, shown as solid circles in Fig. 1(b), extends across the electric-field structures and peaks off axis, as is typical for IEDD waves [18], with the maximum *between* the electric-field structures.

Figure 3 shows the magnetic-field dependence of the eigenmode amplitudes for the medium-L case using the parameter $b \equiv (k_\theta \rho i)^2$ for the horizontal axis. For this medium-L case, one eigenmode ($j = 1$) is present at low B (i.e., large b), two eigenmodes ($j = 1, 2$) are present at higher B, and three eigenmodes ($j = 1, 2, 3$) are present at even higher B (i.e., low b). The solid circles (connected by dashed lines) are the maximum fast Fourier transform (FFT) amplitudes of the experimentally observed spectral features. The solid line, for which b

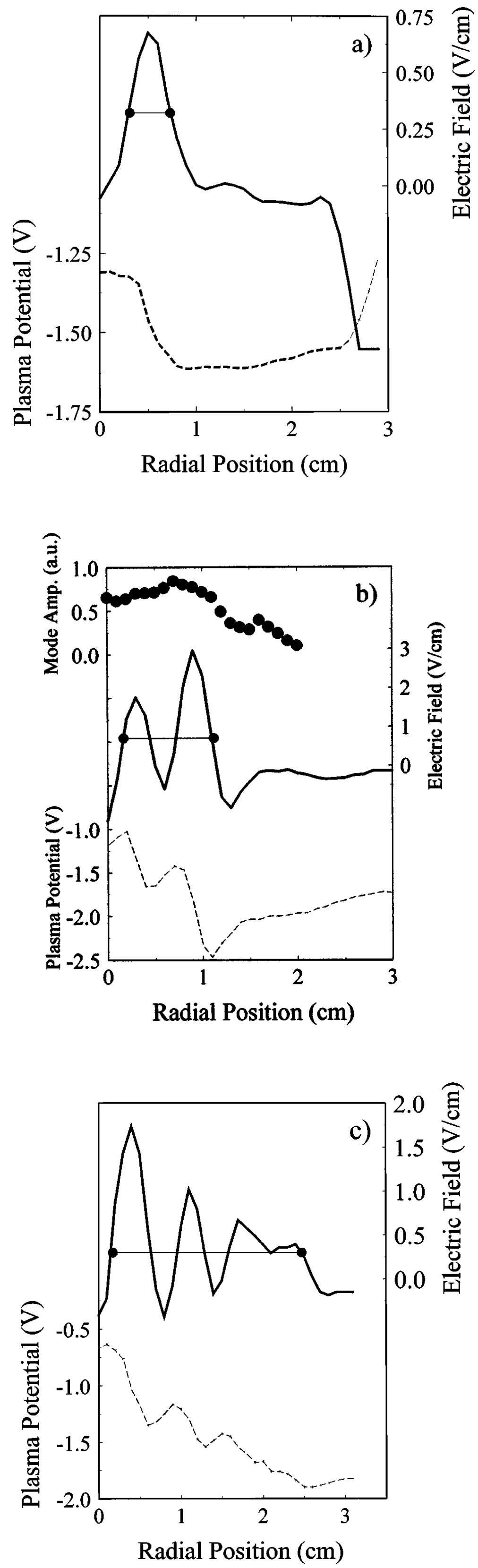

FIG. 1. Plasma potential (dashed line) and electric field (solid line). Segments not floating are (a) $V_0 = 50$ V, (b) $V_0 = 50$ V, $V_1 = 50$ V, and (c) $V_0 = 30$ V, $V_1 = 5$ V, $V_2 = 2$ V, $V_3 = 1$ V, $V_4 = -5$ V. The horizontal line indicates the dimension $2L$ of the electric-field structure.

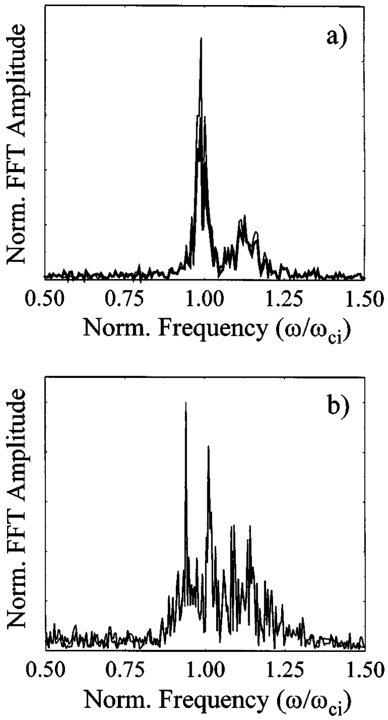

FIG. 2. Spectrum of oscillations in the current collected by the annulus (thin line) and the button (thick line) segments for the case of Fig. 1(b) in which (a) $\epsilon = 0.34$ ($B = 1.28$ kG) and (b) $\epsilon = 0.24$ ($B = 1.75$ kG).

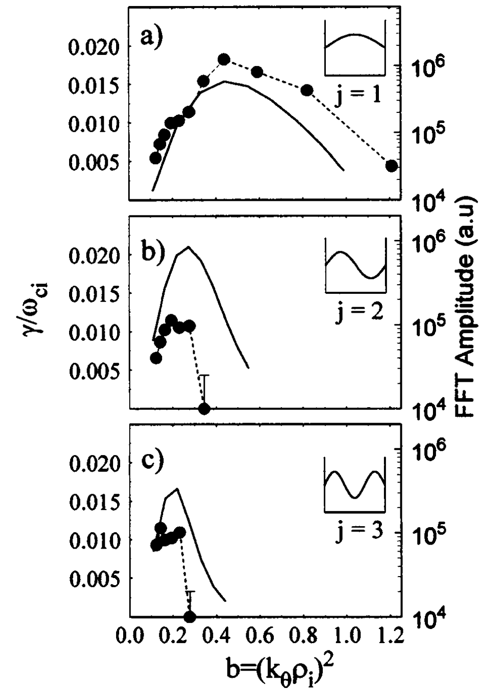

FIG. 3. FFT amplitude (experiment) and growth rate (theory) for the jth eigenmode, where $j = 1$(a), 2(b), and 3(c). The measured value $k_\theta = 2.8$ cm^{-1} is used to convert values of B to values of $b \equiv (k_\theta \rho_i)^2$.

is defined as $(k_{y\rho i})^2$, is the predicted growth rate for the ground state ($j = 1$), the second eigenstate ($j = 2$), and the third eigenstate ($j = 3$). Although measurements of growth rate and saturated amplitude [11,15] demonstrate their proportionality, the emphasis should be on comparing the cutoff values of b for the experiment and theory, since these observed and predicted values are not affected by the mechanism responsible for the saturation of the mode amplitude in the experiment. Notice that, as j increases, the separation between the upper and lower cutoff values of b for a given eigenmode shrinks and the range of b over which the mode is unstable decreases. This method unambiguously identifies the observed eigenmodes with the $j = 1, 2$, and 3 eigenstates.

The effects of inhomogeneous transverse flow with and without parallel current have been described using a full kinetic analysis by Ganguli *et al.* [16]. The steady-state equilibrium used in the model consists of a uniform magnetic field along the z direction, a localized transverse flow profile $\nu_E(x)$ characterized by a maximum value ν_E^0 (in the $-y$ direction) and a FWHM of $2L$, a parallel electron drift speed ν_d relative to the ions, and a perpendicular density gradient. The general eigenvalue condition in this model can be solved numerically for a smooth $\nu_E(x)$ profile; however, a sharp-boundary version [16] conveniently shows the essential shear effects [19,20]. The sharp-boundary model consists of an isolated region where ν_E has a nonzero constant value, separated by an infinitesimally thin boundary from the surrounding region, where ν_E is zero. The width of the first region is $2L$. This instability is driven by transverse flow inhomogeneity which results in an inhomogeneous wave energy density. Hence, it is referred to as the IEDD instability [10]. In the cylindrical geometry of the experiment, it is the radial profile of azimuthal velocity that is inhomogeneous, with $\nu_\theta \approx 0$ for $|r - r_0| > L$ and $\nu_\theta \approx \nu_E$ for $|r - r_0| < L$, where r_0 is the center of the inhomogeneity.

The inhomogeneous profile of the plasma flow transverse to the magnetic field yields multiple solutions of the dispersion relation which is in the form of an eigenvalue condition [16]. Each predicted eigenstate, or eigenmode, corresponds to a complex root with properties that vary with changes in the flow profile. The implications of the multiple eigenmodes, denoted by the mode number j, were recently studied in detail [19,20]. The number of roots in the system for a given set of parameters is determined by ν_E^0 and $2L$, has a minumum of one at small ν_E^0 and small L, and increases with increasing ν_E^0 or decreasing ϵ. These complex roots can have comparable growth rates, resulting in fluctuations with broadband spiky spectra.

The comparison in Fig. 3 between the experiment and the smooth-boundary theory confirms not only the presence of a set of inhomogeneity-related eigenmodes associated with the IEDD instability, but also the specific identity of each eigenmode. The qualitative comparison of the b dependence of each eigenmode's experimentally observed saturated amplitude and theoretically predicted linear-theory growth rate is convincing. However, what reinforces this interpretation is that experiment and theory show the same ordering of the lower, center, and upper values of b of one eigenmode relative to the others in the family of eigenmodes.

Sounding rocket data from SCIFER and AMICIST provide the best evidence yet of the IEDD waves in the auroral acceleration zone, where gyroresonant wave-particle interactions are believed to be responsible for the unexpectedly large function of heavy ions of ionospheric origin in the magnetosphere [21]. The SCIFER experiment [22] documented the microphysical signatures of auroral overflow in the cleft-ion fountain, believed to be the principal source of mass for the magnetosphere, especially O^+, as the payload passed through several discrete aurora. SCIFER provided continuous spatial/temporal resolution 2 orders of magnitude better than that achieved by previous orbiting spacecraft and correlated one-to-one the structured transverse ion acceleration events with broadband, low-frequency electric fields and plasma density depletions. The AMICIST experiment [23] did the same thing with the nightside auroral oval where the outflow is smaller but the physics appears to be the same. The exact correlation between transverse ion energization, magnetic-field-aligned electron current, and broadband low-frequency waves was concluded to be the unmistakable signature of current-driven electrostatic waves. The lack of spectral features ordered by the ion-cyclotron frequencies in the observed spectrum and the synchronous variations in wave intensity and the level of velocity shear in the data [24] suggest that IEDD waves are better suited than CDEIC or ion-acoustic waves for interpreting these space measurements. Bonnell *et al.* [23] provide specific values for many important wave parameters that match very well with the IEDD waves parameters measured in the WVU Q Machine.

In summary, the IEDD instability has been observed in a sodium plasma for the first time, allowing the identification of multiple radial eigenmodes. A well resolved, but closely spaced, group of IEDD wave spectral features are observed with large and comparable amplitude, a first for any ion-cyclotron wave. No tendency for these features to coalesce or otherwise lose their individual identities is detected. This is in sharp contrast to the conventional wisdom, established over three decades of basic plasma experiments, according to which multiple simultaneous eigenmodes with similar three-dimensional mode structure are unexpected. This multieigenmode signature is related not to the geometry of the device boundaries but to the localized flow profile, and, therefore, could be expected in signatures of IEDD waves in space where the flow is structured, even if the density is not structured.

This work is supported by the Office of Naval Research and the National Science Foundation. Useful discussions with G. Ganguli are gratefully acknowledged.

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