Optical Solitons in N-Coupled Higher Order Nonlinear Schrödinger Equations

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We consider the coupled higher order nonlinear Schrödinger (CHNLS) equations which govern the propagation of the fields in a birefringent fiber with all higher order effects like the third order dispersion, Kerr dispersion, and stimulated Raman scattering. We generalize the 2×2 Ablowitz-Kaup-Newell-Segur method to the 5×5 eigenvalue problem and construct the Lax pair. The exact soliton solutions are explicitly obtained using the Darboux-Bäcklund transformation. A similar case of study is extended to three coupled HNLS equations and hence generalized to N-coupled equations. [S0031-9007(97)04790-X]

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The next generation of optical communication is definitely going to be revolutionized by the all soliton communication link. In 1973, the results of Hasegawa and Tappert [1] proved that the major constraint in the optical fiber, namely, the group velocity dispersion (GVD) could be exactly counterbalanced by the self-phase modulation (SPM). SPM is the dominant nonlinear effect in silica fibers due to the Kerr effect. The theoretical results of Hasegawa and Tappert [1] were greatly supported by the experimental demonstration of optical solitons by Mollenauer *et al.* [2] in 1980.

For handling more channels, it is necessary to transmit ultrashort soliton pulses at a high bit rate. In 1986, Mitschke and Mollenauer [3] reported that the ultrashort soliton pulses (USP) suffer from self-frequency shift due to Raman effect. The USP not only suffer from Raman effect but also from third order dispersion (TOD) and Kerr dispersion (otherwise called the self-steepening) [4–7]. Normally, the temporal broadening due to TOD will be very negligible when compared to GVD. But, a considerable amount of asymmetrical broadening in the time domain will be produced by TOD for USP. The Kerr dispersion is due to the intensity dependence of group velocity. This forces the peak of the pulse to travel slower than the wings, which causes asymmetrical spectral broadening of the pulse. Raman effect gives self-frequency shift to the pulse. The self-frequency shift is a self-induced redshift in the pulse spectrum arising from stimulated Raman effect: the long wavelength components of the pulse experience Raman gain at the expense of the short wavelength components, resulting in an increasing redshift as the pulse propagates. It has been recognized that the self-frequency shift is potentially a detrimental effect in soliton communication systems due to the fact that the power fluctuations at the source translate into frequency fluctuations in the fiber through the power dependence of the soliton self-frequency shift and hence into timing jitter at the receiver [8]. With all these effects, the wave propagation is governed by the higher order nonlinear Schrödinger (HNLS) equation [4–7]. Recently, it has been shown that the HNLS equation allows soliton-type propagation for some particular choices of parameters and also obtained the exact N-soliton solutions [9]. One of the integrable cases was already considered by Sasa and Satsuma in 1991 [10].

For handling more channels, it is necessary to achieve wavelength division multiplexing (WDM) [11] using solitons. In this case, at least two optical fields are to be transmitted. In 1974, Manakov proposed the coupled NLS equation [12]. In that, he derived the coupled NLS equation from the NLS equation by considering the total field comprising of two fields with left and right polarizations. In a similar way, we have proposed the coupled HNLS (CHNLS) equations and have shown that the system is integrable for a particular form using the Painlevé analysis [13]. The integrable form of CHNLS equations is

$$\begin{aligned} &iq_{1Z} + \tfrac{1}{2}q_{1TT} + (|q_1|^2 + |q_2|^2)q_1 + i\varepsilon[q_{1TTT} + \\ &\qquad 6(|q_1|^2 + |q_2|^2)q_{1T} + 3q_1(|q_1|^2 + |q_2|^2)_T] = 0\,, \\ &iq_{2Z} + \tfrac{1}{2}q_{2TT} + (|q_1|^2 + |q_2|^2)q_2 + i\varepsilon[q_{2TTT} + \\ &\qquad 6(|q_1|^2 + |q_2|^2)q_{2T} + 3q_2(|q_1|^2 + |q_2|^2)_T] = 0\,. \end{aligned} \tag{1}$$

Equation (1) is the coupled form of the HNLS equation considered by Sasa and Satsuma [10]. If we put the condition $q_2 = 0$, Eq. (1) reduces to the completely integrable HNLS equation. The coupled HNLS equation described by Tasgal and Potasek [14] is the coupled form of the Hirota equation [15]. The equation considered in [14] includes higher order effects like the TOD and Kerr dispersion, which is found to be the next hierarchy of the integrable coupled NLS equation [16]. The CHNLS equation includes all the higher order effects like TOD, Kerr dispersion, and the stimulated Raman effect. If we take the limit of the mixed derivative (last two) terms tending to zero, Eq. (1) reduces to the coupled HNLS equation described by Tasgal and Potasek [14].

In the concluding remarks of our paper [13], we have mentioned that it is very difficult to construct the linear eigenvalue problem for Eq. (1). The main aim of this paper is to establish the complete integrability properties of (1).

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In order to analyze Eq. (1), it is rather convenient to introduce variable transformations,

$$E_1(t,z) = q_1(T,Z)\exp\left\{\frac{-i}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\},$$
$$E_2(t,z) = q_2(T,Z)\exp\left\{\frac{-i}{6\varepsilon}\left(T - \frac{Z}{18\varepsilon}\right)\right\}, \qquad (2)$$
$$z = Z, \qquad t = T - \frac{Z}{12\varepsilon}.$$

Then, Eq. (1) is reduced to a coupled complex modified Korteweg–de Vries (KdV)-type equation,

$$\begin{aligned} E_{1z} + \varepsilon[E_{1ttt} + 6(|E_1|^2 + |E_2|^2)E_{1t} + \\ 3E_1(|E_1|^2 + |E_2|^2)_t] = 0, \\ E_{2z} + \varepsilon[E_{2ttt} + 6(|E_1|^2 + |E_2|^2)E_{2t} + \\ 3E_2(|E_1|^2 + |E_2|^2)_t] = 0. \end{aligned} \qquad (3)$$

To construct the explicit Lax pair, we generalize the 2×2 Ablowitz-Kaup-Newell-Segur (AKNS) method [17] to a 5×5 eigenvalue problem and obtain the Lax pair for Eq. (3). It should be noted that the HNLS equation described by Sasa and Satsuma [10] admits a 3×3 eigenvalue problem.

We generalize the 2×2 AKNS method to the 5×5 eigenvalue problem and we derive the Lax pair for the coupled complex modified KdV equations (3) in the form

$$\begin{matrix}\Psi_t = U\Psi \\ \Psi_z = V\Psi\end{matrix} \qquad \Psi = (\Psi_1\ \Psi_2\ \Psi_3\ \Psi_4\ \Psi_5)^T \qquad (4)$$

$$U = \begin{pmatrix} -i\lambda & E_1 & E_1^* & E_2 & E_2^* \\ -E_1^* & i\lambda & 0 & 0 & 0 \\ -E_1 & 0 & i\lambda & 0 & 0 \\ -E_2^* & 0 & 0 & i\lambda & 0 \\ -E_2 & 0 & 0 & 0 & i\lambda \end{pmatrix},$$

$$\begin{aligned} V = {} & \frac{8i\varepsilon\lambda^3}{5}\begin{pmatrix} -4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} + 4\varepsilon\lambda^2 \begin{pmatrix} 0 & E_1 & E_1^* & E_2 & E_2^* \\ -E_1^* & 0 & 0 & 0 & 0 \\ -E_1 & 0 & 0 & 0 & 0 \\ -E_2^* & 0 & 0 & 0 & 0 \\ -E_2 & 0 & 0 & 0 & 0 \end{pmatrix} \\ & - 2i\varepsilon\lambda \begin{pmatrix} -2A & -E_{1t} & -E_{1t}^* & -E_{2t} & -E_{2t}^* \\ -E_{1t}^* & |E_1|^2 & (E_1^*)^2 & E_1^*E_2 & E_1^*E_2^* \\ -E_{1t} & E_1^2 & |E_1|^2 & E_1E_2 & E_1E_2^* \\ -E_{2t}^* & E_1E_2^* & E_1^*E_2^* & |E_2|^2 & (E_2^*)^2 \\ -E_{2t} & E_1E_2 & E_1^*E_2 & E_2^2 & |E_2|^2 \end{pmatrix} \\ & - \varepsilon \begin{pmatrix} 0 & 4AE_1 + E_{1tt} & 4AE_1^* + E_{1tt}^* & 4AE_2 + E_{2tt} & 4AE_2^* + E_{2tt}^* \\ -4AE_1^* - E_{1tt}^* & E_1E_{1t}^* - E_1^*E_{1t} & 0 & E_2E_{1t}^* - E_1^*E_{2t} & E_2^*E_{1t}^* - E_1^*E_{2t}^* \\ -4AE_1 - E_{1tt} & 0 & E_1^*E_{1t} - E_1E_{1t}^* & E_2E_{1t} - E_1E_{2t} & E_2^*E_{1t} - E_1E_{2t}^* \\ -4AE_2^* - E_{2tt}^* & E_1E_{2t}^* - E_2^*E_{1t} & E_1^*E_{2t}^* - E_2^*E_{1t}^* & E_2E_{2t}^* - E_2^*E_{2t} & 0 \\ -4AE_2 - E_{2tt} & E_1E_{2t} - E_2E_{1t} & E_1^*E_{2t} - E_2E_{1t}^* & 0 & E_2^*E_{2t} - E_2E_{2t}^* \end{pmatrix}, \end{aligned}$$

where $A = (|E_1|^2 + |E_2|^2)$.

It should be noted that the above V structure is different from the usual AKNS eigenvalue problem for the coupled NLS equations where we have the 3×3 matrix form for V. This change is mainly due to the last term in Eq. (3). Using a different method, a similar type of eigenvalue problem has been investigated by Newell [18].

Hence, the Lax pair confirms the complete integrability of Eq. (3) and thereby Eq. (1). From the knowledge of the Lax pair, one can construct the soliton solutions using various methods. Here, we use the Darboux-Bäcklund transformation and obtain the explicit soliton solution.

To derive the Bäcklund transformation (BT) of Eq. (3), let us write down Eq. (4) in the coupled Riccati form. Introducing new variables (or pseudopotentials [19])

$$\Gamma_1 = \frac{\Psi_1}{\Psi_5}; \qquad \Gamma_2 = \frac{\Psi_2}{\Psi_5}; \qquad \Gamma_3 = \frac{\Psi_3}{\Psi_5}; \qquad \Gamma_4 = \frac{\Psi_4}{\Psi_5}, \qquad (5)$$

Eq. (4) yields

$$\Gamma_{1t} = -2i\lambda\Gamma_1 + E_1\Gamma_2 + E_1^*\Gamma_3 + E_2(\Gamma_4 + \Gamma_1^2) + E_2^*, \qquad (6a)$$

$$\Gamma_{2t} = -\Gamma_1 E_1^* + \Gamma_1\Gamma_2 E_2, \qquad (6b)$$

$$\Gamma_{3t} = -\Gamma_1 E_1 + \Gamma_1\Gamma_3 E_2, \qquad (6c)$$

$$\Gamma_{4t} = -\Gamma_1 E_2^* + \Gamma_1\Gamma_4 E_2. \qquad (6d)$$

Now let us seek a transformation of variables $\Gamma_1 \to \Gamma_1'$, $\Gamma_2 \to \Gamma_2'$, $\Gamma_3 \to \Gamma_3'$, $\Gamma_4 \to \Gamma_4'$, $\lambda \to \lambda'$, $E_1 \to E_1'$, and $E_2 \to E_2'$ which keeps the form of Eqs. (6) invariant. The simplest transformation can be tried by setting $\Gamma_1' = \Gamma_1$, $\Gamma_2' = \Gamma_2$, $\Gamma_3' = \Gamma_3$, $\Gamma_4' = \Gamma_4$, $\lambda' = \lambda^*$, looking for E_1' and E_2' in the form

$$E_1 - E_1' = \frac{2i(\lambda - \lambda^*)\Gamma_1^*\Gamma_3}{1 + |\Gamma_1|^2 + |\Gamma_2|^2 + |\Gamma_3|^2}, \qquad (7a)$$

$$E_2 - E_2' = \frac{2i(\lambda - \lambda^*)\Gamma_1^*}{1 + |\Gamma_1|^2 + |\Gamma_2|^2 + |\Gamma_3|^2}. \qquad (7b)$$

Equations (7) define the Bäcklund transformation for Eq. (3). In that, the primed quantities correspond to N-soliton solutions and the unprimed quantities correspond to the (N-1) soliton solutions. This means that, on the basis of a known solution (seed solution) to Eq. (3), we are able to find pseudopotentials (6), and making use of (7) we may then find the desired potentials E_1 and E_2, i.e., new solutions of Eq. (3).

For instance, the trivial solution of Eq. (3) $E_1 = E_2 = 0$ corresponds to the following pseudopotentials (with $\lambda = i\beta$):

$$\Gamma_1(0) = c_1 \exp(2\beta t - 8\varepsilon\beta^3 z)\,; \tag{8a}$$

$$\Gamma_2(0) = c_2\,; \tag{8b}$$

$$\Gamma_3(0) = c_3\,; \tag{8c}$$

$$\Gamma_4(0) = c_4\,, \tag{8d}$$

where c_1, c_2, c_3, and c_4 are arbitrary integration constants. So, we can find new solutions of Eq. (3) from (7) which is generated by the trivial one

$$E_1(1) = 2\beta\,\frac{c_3}{c_1^*}\,\mathrm{sech}(2\beta t - 8\varepsilon\beta^3 z)\,, \tag{9a}$$

$$E_2(1) = 2\beta\,\frac{1}{c_1^*}\,\mathrm{sech}(2\beta t - 8\varepsilon\beta^3 z)\,. \tag{9b}$$

Expressions (9) give the single soliton solutions of Eq. (3). Similarly using $\Gamma_1(0)$, $\Gamma_2(0)$, $\Gamma_3(0)$, $\Gamma_4(0)$, $E_1(1)$, and $E_2(1)$, one can generate the N-soliton solutions of Eq. (3) in a recursive manner.

For the simultaneous transmission of three fields in a fiber, one has to consider three coupled HNLS equations in the form

$$\begin{gathered}
iq_{1Z} + \tfrac{1}{2}q_{1TT} + q_1\sum_{n=1}^{3}|q_n|^2 + \\
i\varepsilon\left[q_{1TTT} + 6q_{1T}\sum_{n=1}^{3}|q_n|^2 + 3q_1\left(\sum_{n=1}^{3}|q_n|^2\right)_T\right] = 0\,, \\
iq_{2Z} + \tfrac{1}{2}q_{2TT} + q_2\sum_{n=1}^{3}|q_n|^2 + \\
i\varepsilon\left[q_{2TTT} + 6q_{2T}\sum_{n=1}^{3}|q_n|^2 + 3q_2\left(\sum_{n=1}^{3}|q_n|^2\right)_T\right] = 0\,, \\
iq_{3Z} + \tfrac{1}{2}q_{3TT} + q_3\sum_{n=1}^{3}|q_n|^2 + \\
i\varepsilon\left[q_{3TTT} + 6q_{3T}\sum_{n=1}^{3}|q_n|^2 + 3q_3\left(\sum_{n=1}^{3}|q_n|^2\right)_T\right] = 0\,.
\end{gathered} \tag{10}$$

Using a similar type of transformation as that of Eq. (2), one can reduce Eq. (10) to three coupled complex modified KdV-type equations. The eigenvalue problem for the three coupled complex modified KdV equations can be constructed as

$$\begin{aligned}\Psi_t &= U\Psi \\ \Psi_z &= V\Psi\end{aligned} \qquad \Psi = (\Psi_1\ \Psi_2\ \Psi_3\ \Psi_4\ \Psi_5\ \Psi_6\ \Psi_7)^T, \tag{11}$$

where

$$U = \begin{pmatrix} -i\lambda & E_1 & E_1^* & E_2 & E_2^* & E_3 & E_3^* \\ -E_1^* & i\lambda & 0 & 0 & 0 & 0 & 0 \\ -E_1 & 0 & i\lambda & 0 & 0 & 0 & 0 \\ -E_2^* & 0 & 0 & i\lambda & 0 & 0 & 0 \\ -E_2 & 0 & 0 & 0 & i\lambda & 0 & 0 \\ -E_3^* & 0 & 0 & 0 & 0 & i\lambda & 0 \\ -E_3 & 0 & 0 & 0 & 0 & 0 & i\lambda \end{pmatrix},$$

$$\begin{aligned}
V = {}& \frac{8i\varepsilon\lambda^3}{7}\begin{pmatrix} -6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + 4\varepsilon\lambda^2\begin{pmatrix} 0 & E_1 & E_1^* & E_2 & E_2^* & E_3 & E_3^* \\ -E_1^* & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_2^* & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_3^* & 0 & 0 & 0 & 0 & 0 & 0 \\ -E_3 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
& + 2i\varepsilon\lambda\begin{pmatrix} -2B & -E_{1t} & -E_{1t}^* & -E_{2t} & -E_{2t}^* & -E_{3t} & -E_{3t}^* \\ -E_{1t}^* & |E_1|^2 & (E_1^*)^2 & E_1^*E_2 & E_1^*E_2^* & E_1^*E_3 & E_1^*E_3^* \\ -E_{1t} & E_1^2 & |E_1|^2 & E_1E_2 & E_1E_2^* & E_1E_3 & E_1E_3^* \\ -E_{2t}^* & E_1E_2^* & E_1^*E_2^* & |E_2|^2 & (E_2^*)^2 & E_2^*E_3 & E_2^*E_3^* \\ -E_{2t} & E_1E_2 & E_1^*E_2 & E_2^2 & |E_2|^2 & E_2E_3 & E_2E_3^* \\ -E_{3t}^* & E_1E_3^* & E_1^*E_3^* & E_2E_3^* & E_2^*E_3^* & |E_3|^2 & (E_3^*)^2 \\ -E_{3t} & E_1E_3 & E_1^*E_3 & E_2E_3 & E_2^*E_3 & E_3^2 & |E_3|^2 \end{pmatrix} \\
& + \varepsilon\begin{pmatrix} 0 & 4BE_1 + E_{1tt} & 4BE_1^* + E_{1tt}^* & 4BE_2 + E_{2tt} & 4BE_2^* + E_{2tt}^* & 4BE_3 + E_{3tt} & 4BE_3^* + E_{3tt}^* \\ -4BE_1^* - E_{1tt}^* & E_1E_{1t}^* - E_1^*E_{1t} & 0 & E_2E_{1t}^* - E_1^*E_{2t} & E_2^*E_{1t}^* - E_1^*E_{2t}^* & E_3E_{1t}^* - E_1^*E_{3t} & E_3^*E_{1t}^* - E_1^*E_{3t}^* \\ -4BE_1 - E_{1tt} & 0 & E_1^*E_{1t} - E_1E_{1t}^* & E_2E_{1t} - E_1E_{2t} & E_2^*E_{1t} - E_1E_{2t}^* & E_3E_{1t} - E_1E_{3t} & E_3^*E_{1t} - E_1E_{3t}^* \\ -4BE_2^* - E_{2tt}^* & E_1E_{2t}^* - E_2^*E_{1t} & E_1^*E_{2t}^* - E_2^*E_{1t}^* & E_2E_{2t}^* - E_2^*E_{2t} & 0 & E_3E_{2t}^* - E_2^*E_{3t} & E_3^*E_{2t}^* - E_2^*E_{3t}^* \\ -4BE_2 - E_{2tt} & E_1E_{2t} - E_2E_{1t} & E_1^*E_{2t} - E_2E_{1t}^* & 0 & E_2^*E_{2t} - E_2E_{2t}^* & E_3E_{2t} - E_2E_{3t} & E_3^*E_{2t} - E_2E_{3t}^* \\ -4BE_3^* - E_{3tt}^* & E_1E_{3t}^* - E_3^*E_{1t} & E_1^*E_{3t}^* - E_3^*E_{1t}^* & E_2E_{3t}^* - E_3^*E_{2t} & E_2^*E_{3t}^* - E_3^*E_{2t}^* & E_3E_{3t}^* - E_3^*E_{3t} & 0 \\ -4BE_3 - E_{3tt} & E_1E_{3t} - E_3E_{1t} & E_1^*E_{3t} - E_3E_{1t}^* & E_2E_{3t} - E_3E_{2t} & E_2^*E_{3t} - E_3E_{2t}^* & 0 & E_3^*E_{3t} - E_3E_{3t}^* \end{pmatrix},
\end{aligned}$$

where $B = \sum_{n=1}^{3} |E_n|^2$.

From the above, it is clear that if the number of propagating fields is increasing, there will be a corresponding increase in the order of the U and V matrices. By choosing the proper U matrix (for $N = 4, 5, \ldots, N$), one can construct the corresponding V matrix using the general AKNS method. But, in a simpler manner, it is possible to write down the V matrix (for $N = 4, 5, \ldots, N$) by looking at the symmetries seen in the V matrices for $N = 2, 3$.

To obtain the Bäcklund transformation, we introduce the variables

$$\Gamma_n = \frac{\Psi_n}{\Psi_7}, \qquad n = 1, 2, \ldots, 6 \tag{12}$$

and obtain the BT as

$$E_1 - E_1' = \frac{2i(\lambda - \lambda^*)\Gamma_1^*\Gamma_3}{1 + \sum_{n=1}^{6} |\Gamma_n|^2},$$

$$E_2 - E_2' = \frac{2i(\lambda - \lambda^*)\Gamma_1^*\Gamma_5}{1 + \sum_{n=1}^{6} |\Gamma_n|^2},$$

$$E_3 - E_3' = \frac{2i(\lambda - \lambda^*)\Gamma_1^*}{1 + \sum_{n=1}^{6} |\Gamma_n|^2}. \tag{13}$$

Similarly, the one soliton solution for the three coupled complex modified KdV equations can be generated as

$$E_1(1) = 2\beta \frac{c_3}{c_1^*} \operatorname{sech}(2\beta t - 8\varepsilon\beta^3 z), \tag{14a}$$

$$E_2(1) = 2\beta \frac{c_5}{c_1^*} \operatorname{sech}(2\beta t - 8\varepsilon\beta^3 z), \tag{14b}$$

$$E_3(1) = 2\beta \frac{1}{c_1^*} \operatorname{sech}(2\beta t - 8\varepsilon\beta^3 z). \tag{14c}$$

In a similar way, for four coupled HNLS equations, one can construct a 9×9 eigenvalue problem, the corresponding BT, and a soliton solution. So, in general, for N-coupled HNLS equations the Lax pair can be constructed using the $(2N + 1) \times (2N + 1)$ eigenvalue problem.

Thus, in this Letter, we have generalized the 2×2 AKNS method to the 5×5 eigenvalue problem to the CHNLS equations. We have constructed the Lax pair and the exact soliton solution using Darboux-Bäcklund transformations. A similar procedure is extended for the three coupled HNLS equations. Finally, the method is generalized for N-coupled HNLS equations. Hence, with these results, we have proved that the CHNLS equations that describe the wave propagation of two and higher number of fields in a fiber system with all the higher order effects like TOD, Kerr dispersion, and stimulated Raman effect will allow soliton-type pulse propagation. This will help in achieving WDM using USP.

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[1] A. Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973).
[2] L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[3] F. M. Mitschke and L. F. Mollenauer, Opt. Lett. **11**, 657 (1986).
[4] Y. Kodama, J. Stat. Phys. **39**, 597 (1985).
[5] Y. Kodama and A. Hasegawa, IEEE J. Quantum Electron. **23**, 510 (1987).
[6] K. Porsezian and K. Nakkeeran, Phys. Rev. Lett. **74**, 2941 (1995); **76**, 3955 (1996).
[7] A. Hasegawa and Y. Kodama, *Solitons in Optical Communication* (Oxford University Press, New York, 1995).
[8] D. Wood, IEEE J. Lightwave Tech. **8**, 1097 (1990).
[9] M. Gedalin, T. C. Scott, and Y. B. Band, Phys. Rev. Lett. **78**, 448 (1997).
[10] N. Sasa and J. Satsuma, J. Phys. Soc. Jpn. **60**, 409 (1991).
[11] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, 1989).
[12] S. V. Manakov, Sov. Phys. JETP **38**, 248 (1974).
[13] K. Porsezian, P. Shanmugha Sundaram, and A. Mahalingam, Phys. Rev. E **50**, 1543 (1994).
[14] R. S. Tasgal and M. J. Potasek, J. Math. Phys. (N.Y.) **33**, 1208 (1992).
[15] R. Hirota, J. Math. Phys. (N.Y.) **14**, 805 (1973).
[16] W. Oevel and K. Porsezian (to be published).
[17] M. J. Ablowitz, D. J. Kaup, A. C. Newell, and H. Segur, Stud. Appl. Math. **53**, 249 (1974).
[18] A. C. Newell, Proc. R. Soc. London A **365**, 283 (1979).
[19] J. Corones, J. Math. Phys. (N.Y.) **17**, 756 (1976); **18**, 163 (1977).