Big Incoherent Solitons

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(Received 29 October 1997)

We give a classical geometric optics description of incoherent solitons—those launched by a diffuse source. This method is intuitive, advances predictions such as the existence of solitons of arbitrary cross section, and importantly, it provides a simple (universal) analytical description for the incoherent solitons of any nonlinear medium. Previously, analytical results were known for the $\ln I$ nonlinearity only. [S0031-9007(98)05343-5]

PACS numbers: 42.65.Tg, 03.40.Kf, 42.65.Jx

Solitons are waves that remain localized as they travel in a uniform nonlinear medium, whereas they would diffuse in a linear medium. In optics [1–3], they are candidates as building blocks for a potential all-optical technology where light guides and manipulates light. Significantly, Mitchell and Segev [4] have recently shown that solitons can be launched by an incandescent light bulb rather than the usual high power laser. We now give a classical geometric optics description of such solitons which leads to analytical results for any nonlinear medium.

The refractive index $n(\mathbf{x})$ of a nonlinear medium depends on the intensity $I(\mathbf{x})$ of the illuminating beam through the relation $n = n(I)$, where $\mathbf{x}$ is the spatial position. Accordingly, it is possible for a beam to create its own waveguide $n(\mathbf{x})$ [1]. The self-consistency relation, $n(\mathbf{x}) = n\{I(\mathbf{x})\}$, provides a mathematical procedure for obtaining analytical solutions [5].

Traditionally spatial solitons have been considered as one mode of their induced waveguide [1] but, crucial to our argument, it was recently shown that they could be two or more modes [6,7]. This core idea, taken with self-consistency [5,7], can be nicely generalized [8] to describe incoherent solitons. However, the modal method, while exact and general, is rather mathematical when many modes are involved and to date analytical results are limited to incoherent solitons of the $\ln I$ nonlinearity [9,10]. But, in analogy to coherent solitons [11], the apparent mathematical complexity is not fundamental to incoherent solitons themselves, but rather it is a consequence of the particular limits and models chosen. Accordingly, we now concentrate on highly incoherent (multimoded) solitons with a view towards simplicity. Such solitons have actually been observed in recent seminal experiments [4].

Light spreads from an incoherent source because of its diffuse irradiation and not because of diffraction. Now, it is well documented [12] that geometric optics provides an elegant and accurate description of diffusely illuminated multimoded waveguides. Because highly incoherent solitons induce such waveguides, they must also be describable by geometric optics. We next demonstrate this, first with an idealized example to convey the physics.

Consider an incoherent source, of radius ρ, lying in the x, y plane, whose intensity $I(r)$ is uniform along its surface at every radial position r. Light is emitted equally from each point of the source in a cone of angles θ up to $\theta_{\max}$, where θ is the inclination to the z direction. This source induces a step profile (linear) waveguide characterized by $n = n(I)$ for radial position $r < \rho$ and by $n = n(0) = n_0$ for $r > \rho$. Such a waveguide is well known [12] to trap all rays emitted from the source provided the rays undergo total internal reflection from the internal interface. This occurs when the maximum angle $\theta_{\max}$ of irradiance equals $\theta_c(I) = \cos^{-1}\{n_0/n(I)\}$, the complement of the usual critical angle for the induced waveguide, where $\theta_c^2(I) \cong \delta n^2/n_0^2$, $\delta n^2 = n^2(I) - n_0^2$, assuming $n(I) \cong n_0$ as in practice. Accordingly, a circularly symmetric beam of uniform intensity that emits radiation in a cone of angles up to $\theta_c(I)$ is a soliton because it self-consistently induces a (linear) waveguide which then propagates the light beam uniformly through space. The identical argument applies for solitons of arbitrary symmetry in cross section.

The above idealized source conveys much of the essential physics in the geometric optics approach, and it suggests that incoherent solitons of arbitrary cross section exist in any nonlinear medium. Now suppose that the above source intensity $I(\mathbf{r})$ is any (arbitrarily) smoothed out step function with $I(\mathbf{r}) \to 0$ as $\mathbf{r} \to \infty$. The nonlinear induced waveguide now has a graded refractive index profile $n(\mathbf{r}) = n\{I(\mathbf{r})\}$. In analogy with a step profile waveguide, all rays emitted from each radial position $\mathbf{r}$ of the source are trapped provided again that $\theta_{\max} = \theta_c$, but $\theta_c\{I(\mathbf{r})\}$ now depends on the radial position $\mathbf{r}$ and is obtained by replacing $n(I)$ in the above example by $n\{I(\mathbf{r})\}$. In other words, the incoherence or equivalently the cone of radiation from each point of the source contracts from its maximum at positions where I is maximum to zero where $I = 0$.

For any beam to be a stationary soliton, it must obey the self-consistency relation that it is guided *uniformly* along the waveguide it induces. Clearly, this places a constraint on the incoherence, i.e., on the allowed distribution of rays radiated diffusely from each position $\mathbf{r}$ in the beam's cross section. We characterize this distribution by the ray density function $D\{r, \theta\}$. Now it is a well known consequence of Liouville's theorem [13] that the density D of rays is uniform along any ray path. It then follows that D can be any function of the ray invariant if the beam is to be uniform along its induced waveguide. The invariant for trapped rays traveling along an axially uniform waveguide, characterized by $n(\mathbf{r})$, is given by the generalized Snell's law [12] at any axial distance z:

$$\beta = n(\mathbf{r}) \cos \theta . \tag{1}$$

Accordingly, the necessary incoherence to produce a stationary beam is obtained when D is any function of β, i.e., $D = D(\beta)$. We next link the beam intensity to $D(\beta)$ by recognizing that the sum of all rays emitted at position $\mathbf{r}$ in the beam's cross section must equal the intensity at $I(\mathbf{r})$. Thus,

$$I(\mathbf{r}) = 2\pi \int_0^{\theta_c(\mathbf{r})} D(\beta) \sin \theta d\theta = \frac{2\pi}{n(\mathbf{r})} \int_{n_0}^{n(\mathbf{r})} D(\beta) d\beta , \tag{2}$$

where $n(\mathbf{r})$ depends on I because the induced waveguide profile is $n = n\{I(\mathbf{r})\}$. Now, by differentiating Eq. (2), we find that the incoherence necessary to be a stationary soliton in any specified nonlinear medium $n(I)$ is $D(\beta) = (1/2\pi) \{\partial(In)/\partial n\}$ with n evaluated at β.

Here $n_0 < \beta < n\{I(\mathbf{r})\}$ or equivalently $0 < \theta < \theta_c\{I(\mathbf{r})\}$. The incoherence is zero ($D = 0$) for $\theta > \theta_c\{I(\mathbf{r})\}$. But, in practice $n(\mathbf{r}) \cong n_0$, so that θ_c is small. Accordingly the incoherence D required for a beam to be a soliton, i.e., the necessary distribution of rays radiated diffusely at each point in the beam cross section is given by the simple universal expression

$$D(\beta) \cong (n_0/2\pi) (\partial I/\partial n)|_{n=\beta} , \tag{3}$$

taken together with $\beta^2 - n_0^2 = n^2(\cos^2\theta - \cos^2\theta_c) \cong n_0^2[\theta_c^2\{I(\mathbf{r})\} - \theta^2]$. Here $\theta_c^2(I) \cong \{n^2(I) - n_0^2\}/n_0^2$. This result holds for arbitrary beam shape and cross sectional symmetry. Given any nonlinearity $n(I)$, Eq. (3) together with the expression for $\beta^2 - n_0^2$ provides the necessary incoherence to be a soliton. The incoherence $D\{\theta, I(\mathbf{r})\}$ depends on radial position $\mathbf{r}$ through intensity $I(\mathbf{r})$. It is unnecessary to preserve this implicit dependence on $\mathbf{r}$, so we take $D(\theta, I)$, with θ the direction of radiation.

Before providing some examples it is noteworthy to recall that the intensity profile $I(x, y)$ of stationary coherent solitons is dictated by the nonlinearity $n(I)$, and it must be circularly symmetric [7]. Whereas we have found that stationary incoherent solitons can have any intensity profile $I(x, y)$ with the nonlinearity instead setting the necessary diffuse irradiation at each position in the soliton cross section.

As a first example we consider the familiar cubic (Kerr) nonlinearity for which $n^2 = n_0^2 + \alpha I$, with α a material constant. From Eq. (3) we find that the soliton must have a uniform cone of irradiance $0 < \theta < \theta_c(I)$ as specified by $D(\theta, I) = n_0^2/\pi\alpha$ at each intensity in the soliton cross section. Recall from our introductory comments that the cone of irradiance $\theta_c(I)$ is largest at positions corresponding to the maximum value of I and zero at positions where I is zero. While this incoherence may seem contrived, it is, in fact, what is emitted from a long optical fiber of profile shape $n\{I(r)\}$ when it is illuminated by a conventional diffuse source [14].

Next we consider the incoherence necessary for beams to be solitons of a saturated Kerr nonlinearity characterized by $n^2 = n_0^2 + \alpha I/(1 + I/I_t)$, where I_t is given saturation intensity. This leads from Eq. (3) to $D(\theta, I) = (\alpha I_t^2/\pi n_0^2)/[\theta_M^2 - \theta_c^2(I) + \theta^2]^2$ with $\theta_M^2 = \alpha I_t/n_0^2$ the maximum θ_c^2 possible. Here is an example where the required irradiance at each intensity in the cross section is nonuniform in θ.

Finally we consider a logarithmic saturating medium of the form $n^2 = n_0^2 + \Delta \ln(1 + I/I_t)$, where $\Delta = \{n^2(I_t) - n_0^2\}/\ln 2$ characterizes the strength of the nonlinearity. If beams are to be solitons in such a medium, the irradiance at each intensity I is found from Eq. (3) to be

$$D(\theta, I) = (I_t/\pi\theta_t^2) e^{\{\theta_c^2(I) - \theta^2\}/\theta_t^2}, \tag{4}$$

where $\theta_t^2 = \Delta/n_0^2$. This incoherence $D(\theta, I)$ is illustrated in Fig. 1 and is also qualitatively like that for the saturating Kerr nonlinearity discussed above. For low intensities ($I \ll I_t$), we observe that $n^2 \cong n_0^2 + \alpha I$, $\theta_c \ll \theta_t$, and from Eq. (4) the irradiance D is again uniform as it is for a Kerr medium. For large intensities ($I \gg I_t$), $n^2 \cong n_0^2 + \Delta \ln(I/I_t)$, and from Eq. (4)

$$D(\theta, I) = \{I/\pi\theta_t^2\} e^{-\theta^2/\theta_t^2}, \tag{5}$$

using $\theta_c^2 = (\delta n^2/n_0^2) \cong \theta_t^2 \ln(I/I_t)$, where $\delta n^2 = n^2(I) - n_0^2$.

We anticipate that geometric optics is accurate when the beam spread due to diffraction $\theta_d \sim \lambda/\rho$ is insignificant compared to the maximum angle of diffuse irradiation θ_c. In other words, $I(\mathbf{r})$ must be a "smooth"

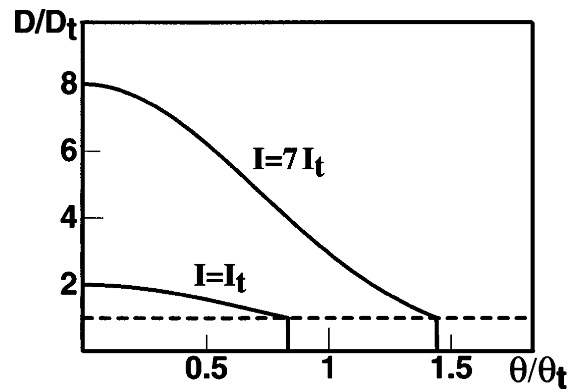

FIG. 1. The diffuse irradiation $D(\theta, I)$ necessary for a beam to be a soliton at two different intensities in the beam's cross section, where θ is the direction of radiation. Here $D_t = I_t/\pi\theta_t^2$ and $\theta_t = \sqrt{\Delta}/n_0$ are given properties of the logarithmic saturating nonlinearity $n^2 = n_0^2 + \Delta \ln(1 + I/I_t)$.

function with a "big" characteristic radius ρ obeying $\rho \gg \lambda n_0/(\delta n^2)^{1/2}$. This is also the condition for a soliton to be significantly incoherent or equivalently to be multimoded. We now compare our method with the only known exact result for solitons of arbitrary incoherence in the literature—that by Christodoulides *et al.* [9,10] for a circularly symmetric soliton in the $n^2 = n_0^2 + \Delta \ln(I/I_t)$ nonlinearity. The exact result is given by replacing θ_t^2 with $(1 - \rho_s^2/\rho^2)\theta_t^2$ in our Eq. (5), where ρ_s is the soliton radius for a coherent soliton, $\rho_s = \lambda/2\pi\sqrt{\Delta}$. The error in geometric optics is less than 6% when $\rho > 3\rho_s$.

To conclude, we have provided the first classical geometric optics description of solitons. Incoherent (multimoded) solitons differ from the usual coherent (single mode) solitons in several respects: (1) They spread in a linear medium because of diffuse irradiation and not by diffraction; (2) they have an arbitrary smoothed out intensity distribution and cross sectional symmetry, e.g., two circularly symmetric beams can travel in parallel; (3) they can be described by classical geometric optics, leading to a simple universal analytical expression for the beam incoherence required given any nonlinear medium $n(I)$, i.e., the necessary diffuse irradiation $D(\theta, I)$ required for each intensity I in the soliton cross section. We have not addressed the consequence of having the inappropriate incoherence. But in analogy to earlier studies [15], it is possible to show that Gaussian beams of a $\ln I$ nonlinearity would then have a periodic behavior. Our findings open the door to a purely geometric optics treatment of soliton dynamics using only Liouville's theorem for the invariance of D and the eikonal equation for the trajectories of rays [16].

We appreciate suggestions by Colin Pask and thank Moti Segev for sending us Refs. [8,10] prior to their publication. We are part of the Australian Photonics Cooperative Research Centre.

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[16] According to Liouville's theorem [13], the density $D(\mathbf{r}, \theta, \phi)$ of rays is uniform along any ray trajectory, that is $\frac{dD}{dz} = \frac{\partial D}{\partial z} + \nabla_\perp D \cdot \frac{d\mathbf{r}}{dz} + \frac{\partial D}{\partial z}\frac{d\theta}{dz} + \frac{\partial D}{\partial \phi}\frac{d\phi}{dz} = 0$ for paraxial rays. The eikonal equation [12,13] is $\frac{d}{ds}[n(\mathbf{R})\frac{d\mathbf{R}}{ds}] = \nabla n(\mathbf{R})$ which becomes $\frac{d^2\mathbf{r}}{dz^2} = \frac{\nabla_\perp n}{n_0}$ when the maximum and minimum values of n are nearly equal and the rays are approximately directed along the z axis. The refractive index n depends implicitly on the spatial coordinates $\mathbf{r}, z$ through the intensity I so that $n = n\{I(\mathbf{r}, z)\}$. Here $\mathbf{R} = (\mathbf{r}, z) = (x, y, z)$ is the position vector and s is the arc length. The angles define the direction of the ray and are given by $\cos\theta = dz/ds$, $\sin\theta\cos\phi = dx/ds$ and $\sin\theta\sin\phi = dy/ds$ so that $dx/dz = \tan\theta\cos\phi \cong \theta\cos\phi$ and $dy/dz = \tan\theta\sin\phi \cong \theta\sin\phi$.