Ultrasonic Band Gap in a Periodic Two-Dimensional Composite

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The first experimentally observed ultrasonic full band gap in periodic bidimensional composites for the longitudinal wave mode is described in this Letter. The structure consists of an aluminum alloy plate with a square periodic arrangement of cylindrical holes filled with mercury. No propagation wave exists at the frequency range between 1000–1120 kHz irrespective of the measurement direction. The experiment was performed by means of an ultrasonic transmission technique, and a measurement of the position dependence of the acoustic amplitude was also performed. [S0031-9007(97)05225-3]

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As it is well known, x-ray diffraction by crystals and electrons in a periodic potential [1,2], and photons [3–6] and acoustic waves [7,8] in periodic structures display frequency band gaps, or stop bands, where the wave propagation is forbidden with band gaps appearing around Bragg resonances. In these regions, waves decay exponentially along all propagation directions, since they are strongly reflected by the structure. The existence of a band gap has been studied theoretically using a plane-wave method [1,2,4–12]. Because of the periodicity of the structure, solutions of the wave equation, such as pressures or displacements in the case of acoustics, satisfy the Floquet-Bloch theorem, i.e., they are plane waves multiplied by a spatial function exhibiting the same periodicity as the lattice. Hence, the plane wave expansion of the periodic modulation function contains only plane waves with wave vectors that are vectors of the reciprocal lattice. Finally, the other physical parameters involved in the propagation problem, such as densities and Lamé coefficients in the case of acoustics, can also be expanded as Fourier series extended to reciprocal lattice vectors. The wave propagation problem is then transformed to an eigenvalue problem, and the dispersion relations along the main symmetry directions in the first Brillouin zone can be obtained. A full band gap appears when the bands are separated and no wave exists in a given frequency range irrespective of the propagation direction [1–12].

A very active search for acoustic band gaps in periodic structures is in progress. Theoretical works concerning acoustic stop bands in one-dimensional active piezocomposites [13,14] and bidimensional passive structures [7–12] have been recently published. The technological interest of band gaps in bidimensional periodic structures is related to the possibility of designing acoustic band-pass mirrors and, alternately, ultrasonic "silent blocks" [11]. The only experimental attempt to find such a band gap [15] was the measurement of sound attenuation by an artistic sculpture (obviously not specially designed for this purpose) which failed to find a full band gap [12].

As far as we know, no measurements of ultrasonic propagation in bidimensional composites have been performed searching for band gaps. The strategy of the present work is to scale the experimental structures (composites) to the physical range of the ultrasound, where standard propagation techniques can be used under conditions of plane wave excitation, as in the photonic case [3]. The structure chosen here consists of an aluminum alloy plate with a square periodic arrangement of cylindrical holes filled with mercury. We have used a favorable combination of materials where the cylindrical scatterers have a lower propagation velocity than the propagation medium, but the density of the scatterers is higher than that of the surrounding medium [7,9]. The acoustic propagation in a solid-liquid composite searching for band gaps has not yet been theoretically studied. Nevertheless, taking into account that the high velocity ratio between the host (aluminum) and the liquid scatterers (mercury) is about 4.4, and the high density contrast between the scatterers and the host is about 4.8, theoretical calculations for both scalar and elastic waves [7,9] predict the appearance of band gaps for certain ranges of perforating ratios of the samples. On the other hand, the acoustic impedances of both the host and the scatterers are similar (aluminum 18.2 $\mathrm{kg\,m^{-2}\,s^{-1}}$, mercury 19.7 $\mathrm{kg\,m^{-2}\,s^{-1}}$). In this experimental frame, the destructive interference mechanism is mainly due to the temporal delay introduced in the propagation through the scatterers. The good acoustic matching between aluminum and mercury largely decreases the longitudinal to transversal mode conversion at the aluminum-mercury interface. So, the present problem can be interpreted in the light of the scalar (or acoustic) wave propagation theory [7,9,12].

Our experimental setup is based on the well known ultrasonic transmission technique. This technique consists of the use of a couple of broadband ultrasonic transducers able to launch a plane wave signal from the emitter, through the inspected plate, which is then processed at the receiver. Ultrasonic propagation parameters of several sample plates made of aluminum alloy (ALPLAN

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MEC 7079 T 651) were measured. In the first measurement set, all plates had the same dimensions: 5 cm in length, 4 cm in width, and 1.5 cm thick. The plates were perforated with holes of 2 mm diameter forming square networks with different lattice parameters. In order to fill the holes with mercury, a thin plastic sheet (0.1 mm) made of polyethylene was gently bonded to the bottom of the plates by means of a narrow epoxy strip placed at the bottom boundary. Finally, mercury was injected into the holes with a dispenser. The ultrasonic transducers were a commercial variety of immersion type (Panametrics series Videoscan of diameter 20 mm). The emitter was excited with a broad band signal from a Panametrics 5052 Ultrasonic Analyzer. The received signal was captured by a digital oscilloscope (Tektronix TS 520, with a 8-bit digitizer) with real time fast Fourier transform (FFT) analysis capabilities. The oscilloscope displays the amplitude and the phase of the FFT for high energy excitation impulse signals. The phase shift is proportional to the wave number k. With these data, we can directly study the attenuation versus angular frequency ω, and the dispersion relation $\omega = \omega(k)$, along different symmetry directions of the lattice. The measurement frequency range is 0.5–2 MHz. The maximum signal-to-noise ratio (SNR) of the digital oscilloscope is about 50 dB. The diameter of the transducers and the SNR of the digitizer restrict the sample dimensions in this first experimental setup. In order to study the dispersion relations with a frequency sweep avoiding the FFT process, we have additionally used a Gain-Phase Analyzer HP 4194 A (10 Hz–100 MHz).

The perforating ratio is a crucial parameter for the appearance and width of band gaps because it gives the geometrical structure factor of the sample [1,2,7–12]. Plates with perforating ratios from 0% to 47% were made and studied. Only samples with perforating ratios from 30% to 47% exhibiting enough attenuation strength to be considered as potentially containing band gaps. The optimum sample had a perforating ratio of 40%, which is in reasonable agreement with theoretical predictions for the scalar wave case [7,9]. The longitudinal mode of this sample was analyzed in detail along [100] and [110] directions (see Figs. 1 and 2). In this case, the lattice parameter a is 2.8 mm.

Both emitter and receiver transducers are coupled to transversal walls of the plate. The reference spectrum is that recorded from a plate without holes and it shows a broad plateau around 1 MHz, as can be seen in Fig. 1(a). In all samples with longitudinal walls parallel to unit cell sides it is possible to correlate the first and second attenuation zones to the first and second Bragg resonances in the [100] direction [Fig. 1(a)]. In the case of the sample with a perforating ratio of 40%, a wide band gap appears around the first Bragg resonance in the [100] direction (750 kHz). As can be seen in Fig. 1(a), the stop-band edges are sharp and deep. The edges

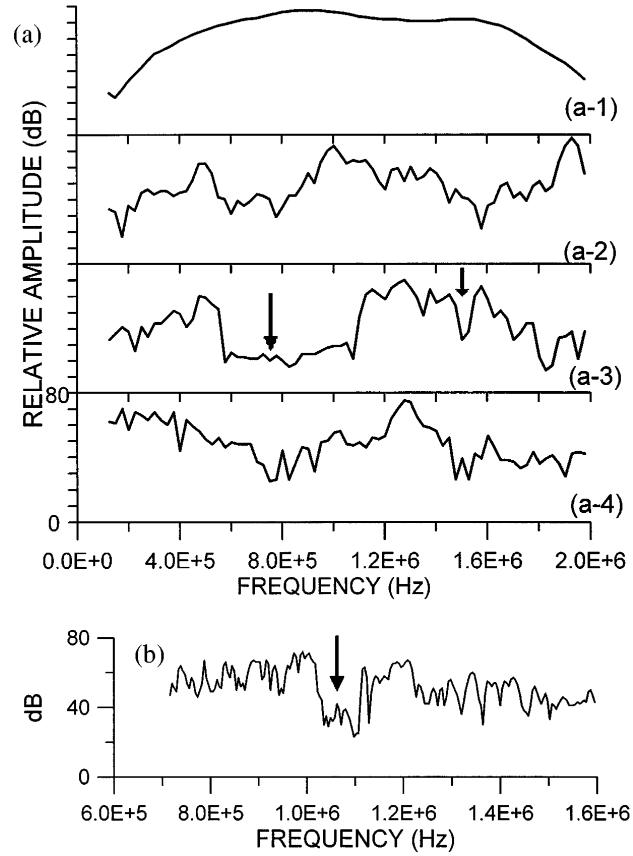

FIG. 1. Measurements of attenuation in the [100] direction (a). Sample without holes (a-1), with a perforating ratio of 30% (a-2), 40% (a-3), and 47% (a-4). The same measurement in the [110] direction for the sample with a perforating ratio of 40% (b). Arrows indicate the Bragg resonances.

decrease steeply by more than 45 dB, near the SNR limit, and are separated by 600 kHz. Thus, the gap/midgap ratio is 0.8, which is in accordance with the theoretical prediction for the above mentioned density contrast of 4.8, in the acoustic wave propagation case [7]. A plate with longitudinal walls parallel to the [110] direction, i.e., by rotating the unit cell by 45°, was made with a perforating ratio of 40%. This sample also shows strong attenuation, close to the SNR, which matches with the corresponding Bragg resonance in the [110] direction, 1060 kHz [Fig. 1(b)]. In this case with $a = 2.8$ mm, the dispersion relations along both symmetry directions of the first Brillouin zone, [100] ($0 < k < \pi/a$) and [110] ($0 < k < 2^{1/2}\pi/a$), in a reduced-zone scheme [1,2], exhibit a *full* band gap between the first and the second bands, where the wave propagation is forbidden along any direction in the frequency range between 1000 and 1120 kHz (Fig. 2). The above mentioned result shows that the experiment is truly two dimensional and sets the problem in the context of the band theory analogy [1–12].

In order to study the expected exponential decay of the propagation vibration amplitude at a single frequency at

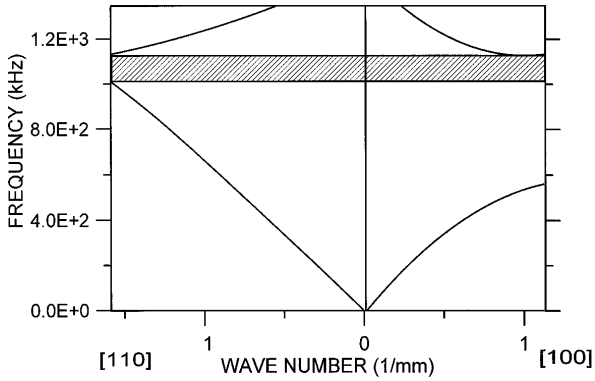

FIG. 2. Dispersion relations along both symmetry directions of the first Brillouin zone [100] and [110], for the sample with a perforating ratio of 40%. The full gap is depicted between the first and second bands.

the middle of the gap (1060 kHz), we made larger sample plates (10 cm in length and width, $a = 2.8$ mm, and with side walls parallel to [100] or [110] directions) excited by an array of specially designed transducers. We have used a polyvinylidene fluoride needle microhydrophone mechanically actuated by means of an automatic system. The hydrophone is acoustically coupled to the sample using silicone oil [16]. The surface vibration amplitudes at the center of the unit cells were recorded starting from the nearest position to the excitation plane. Such vibration, normal to the surface, is generated by the longitudinal wave mode due to the elastic Poisson coupling, and its amplitude is proportional to the longitudinal wave amplitude. The measurements along both [100] and [110] directions can be seen in Fig. 3, showing the expected exponential decay [1,5], in agreement with the experimental data in Ref. [3]. This behavior shows that the infinite attenuation inside the gap is independent of the propagation direction and it can be used as another signature on larger samples for the existence of a full band gap.

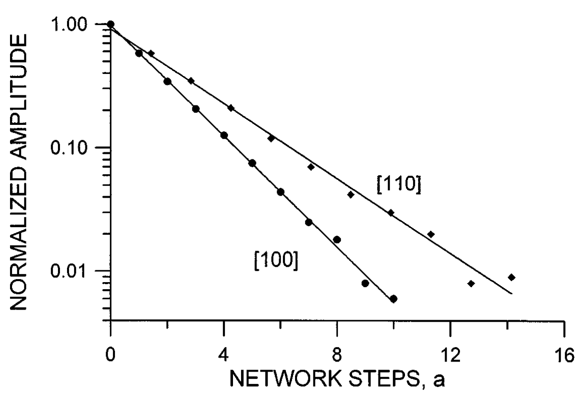

FIG. 3. Exponential decay measurements inside the band gap along both [100] and [110] directions for the sample with a perforating ratio of 40%. The slopes are: −0.51 and −0.34, respectively. Note a $1/e$ attenuation length of two unit cells.

By means of a technique described elsewhere [16], we have scanned the surface of a specially designed sample plate, with a corner without holes, at an excitation frequency inside the band gap and with wave propagation along the [110] direction. As can be seen in Fig. 4(a), the wave does not penetrate but is strongly reflected by the structure. If an impulsive signal is launched and the reflected signal is processed, overtones of the resonance are separated by regular steps of about 150 kHz which correspond to an acoustic path free of holes of about 4 cm, which coincides with the real path. We have also scanned the plane normal to the propagation direction [100] for a frequency of 1.2 MHz, which is outside the gap. Figure 4(b) shows in gray the levels of the amplitude of the transmitted wave at the receiving plane. The brightest zones correspond to the spacing between holes, separated by the lattice parameter a, while other bright zones are related to diffraction phenomena. The same type of scanning was performed for a frequency of 750 kHz at the middle of the gap corresponding to the [100] direction. In this case, as expected, no pattern was found by the acquisition data system.

When the propagation was emitted along the [100] direction and the sample had a length of 5 cm, a final accurate measurement of the amplitude at a single point at the receiving plane was performed by means of the nonautomatic hydrophonic technique. First, a monochromatic signal of 1.2 MHz, clearly outside the band gap, was launched and locally recorded at the receiving plane. The computed FFT of the received signal also showed an evident monochromatic profile. Then, another monochromatic signal of 750 kHz, at the middle of the gap, was emitted and recorded at the same receiving position. The latter signal was received with a huge attenuation, about 50 dB below the former signal, and moreover, after processing the monochromatism had disappeared. Some bound states were confined to the gap [17,18]. Such states may be due to the noninfinite size of the sample and to the disorder introduced by small defects during the plate mechanization. The width of the discrete spectrum of the

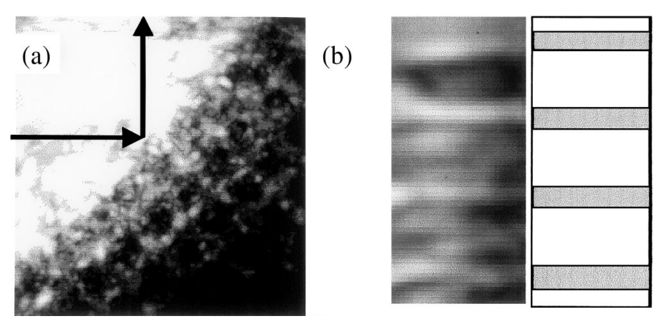

FIG. 4. Vibration amplitude scans showing the reflection by the structure in the [110] direction inside the band gap (a), and the transmission in the [100] direction outside the band gap (b). In (b), holes are schematically indicated in white.

mentioned bound states agrees with the width of the gap along the [100] direction.

In conclusion, we have experimentally found and characterized an ultrasonic full band gap for the longitudinal wave mode in a metallic bidimensional composite. This work can be extended to systems of higher frequency range using micromachining techniques.

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