

Interference Fragmentation Functions and the Nucleon's Transversity

R. L. Jaffe, Xuemin Jin, and Jian Tang

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We introduce twist-two quark interference fragmentation functions in helicity density matrix formalism and study their physical implications. We show how the nucleon's transversity distribution can be probed through the final state interaction between two mesons ($\pi^+\pi^-$, $K\bar{K}$, or πK) produced in the current fragmentation region in deep inelastic scattering on a transversely polarized nucleon. [S0031-9007(97)05179-X]

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The quark transversity distribution in the nucleon is one of the three fundamental distributions which characterize the state of quarks in the nucleon at leading twist. Measurements of the other two have shed considerable light upon the quark-gluon substructure of the nucleon. The transversity distribution measures the probability difference to find a quark polarized along versus opposite to the polarization of a nucleon polarized transversely to its direction of motion [1–4]. It is identical to the helicity difference distribution in the nonrelativistic limit where rotations and boosts commute. However, we have learned from $g_A/g_V \neq 5/3$ and the recent measurement of the spin fraction carried by quarks in the nucleon, $\Sigma \approx 0.2$ [5], that the quarks inside the nucleon cannot be nonrelativistic. The difference between the transversity and helicity distributions is a further and more detailed measure of the relativistic nature of the quarks inside the nucleon.

The transversity distribution measures the correlation of quarks with opposite chirality in the nucleon. Since hard scattering processes in QCD preserve chirality at leading twist, transversity is difficult to measure experimentally. For example, it is suppressed like $\mathcal{O}(m_q/Q)$ in totally inclusive deep inelastic scattering (DIS). Ways have been suggested to measure the transversity distribution. These include transversely polarized Drell-Yan [1], twist-three pion production in DIS [2,6], the so-called ‘‘Collins effect’’ as defined in single particle fragmentation [7], and polarized Λ production in DIS [3,8]. However, each of these has drawbacks [9].

In this Letter we develop another way to isolate the quark transversity distribution in the nucleon that is free from many of these shortcomings. We study semi-inclusive two-meson (e.g., $\pi^+\pi^-$, πK , or $K\bar{K}$) production in the current fragmentation region in deep inelastic scattering on a transversely polarized nucleon. The possibility of measuring the quark transversity distribution via such a process was first suggested by Collins and collaborators [10] (see also Ref. [6]). Our analysis focuses on the *interference* between the s and p wave of the two-meson system around the ρ (for pions), K^* (for πK), or the ϕ (for kaons). We make explicit use of two-meson

phase shifts to characterize the interference. Such an interference effect allows the quark's polarization information to be carried through $\vec{k}_+ \times \vec{k}_- \cdot \vec{S}_\perp$, where \vec{k}_+ , \vec{k}_- , and \vec{S}_\perp are the three-momenta of π^+ (K), π^- (\bar{K}), and the nucleon's transverse spin, respectively. This effect is at the leading twist level, and the production rates for pions and kaons are large in the current fragmentation region. However, it would vanish by T invariance in the absence of final state interactions, or by C invariance if the two-meson state were an eigenstate of C parity. Both suppressions are evaded in the ρ ($\pi^+\pi^-$), K^* (πK), and ϕ ($K\bar{K}$) mass regions.

The final state interactions of $\pi\pi$, πK , and $K\bar{K}$ are known in terms of meson-meson phase shifts. From these phase shifts we know that s - and p -wave production channels interfere strongly in the mass region around the ρ , K^* , and ϕ meson resonances. Since the s and p waves have opposite C parity, the interference provides exactly the charge conjugation mixing necessary. Combining perturbative QCD, final state interaction theory, and data on the meson-meson phase shifts, we can relate this asymmetry to known quantities, the transversity distribution we seek, and to a new type of fragmentation function that describes the s - and p -wave interference in the process $q \rightarrow \pi^+\pi^-(\pi K, K\bar{K})$. Unless this fragmentation is anomalously small, the measurement of this asymmetry may be the most promising way to measure the quark transversity distribution.

Earlier works [7] have explored angular correlations of the form $\vec{k}_1 \times \vec{k}_2 \cdot \vec{S}_\perp$, where \vec{k}_1 and \vec{k}_2 are vectors characterizing the final state in DIS. The simplest example would be $\vec{k}_1 = \vec{q}$ and $\vec{k}_2 = \vec{k}_\pi$, the momentum of a pion. These asymmetries, however, require that the final state interaction phase between the observed hadron(s) and the rest of the hadronic final state must not vanish when the unobserved states are summed over. We believe this to be unlikely. We utilize a final state phase generated by the two-meson final state interaction, which is well understood theoretically and well measured experimentally.

We consider the semi-inclusive deep inelastic scattering process with two-pion final states being detected: $e\vec{N}_\perp \rightarrow$

$e'\pi^+\pi^-X$. The analysis to follow applies as well to πK or $K\bar{K}$ production. The nucleon target is transversely polarized with polarization vector S_μ . The electron beam is unpolarized. The four-momenta of the initial and final electron are denoted by $k = (E, \vec{k})$ and $k' = (E', \vec{k}')$, and the nucleon's momentum is P_μ . The momentum of the virtual photon is $q = k - k'$, and $Q^2 = -q^2 = -4EE' \sin^2 \theta/2$, where θ is the electron scattering angle. We adopt the standard variables in DIS, $x = Q^2/2Pq$ and $y = Pq/Pk$. The $\sigma[(\pi\pi)_{i=0}^J]$ and $\rho[(\pi\pi)_{i=1}^J]$ resonances are produced in the current fragmentation region with momentum P_h and momentum fraction $z = P_h q/q^2$. We recognize that the $\pi\pi$ s wave is not resonant in the vicinity of the ρ and our analysis does not depend on a resonance approximation. For simplicity we refer to the nonresonant s wave as the “ σ ”. In their work on the two-pion system, Collins and Ladinsky [11] made the unphysical assumption of a narrow s -wave resonance interfering with a (real) continuum p wave, neither of which appears in $\pi\pi$ -scattering data.

The invariant squared mass of the two-pion system is $m^2 = (k_+ + k_-)^2$, with k_+ and k_- the momentum of π^+ and π^- , respectively. The decay polar angle in the rest frame of the two-meson system is denoted by Θ , and the

azimuthal angle ϕ is defined as the angle of the normal of two-pion plane with respect to the polarization vector \vec{S}_\perp of the nucleon, $\cos \phi = \vec{k}_+ \times \vec{k}_- \cdot \vec{S}_\perp / |\vec{k}_+ \times \vec{k}_-| |\vec{S}_\perp|$. This is the analog of the “Collins angle” defined by the $\pi^+\pi^-$ system [7].

To simplify our analysis we make a collinear approximation, i.e., $\theta \approx 0$, in referring the fragmentation coordinate system to the axis defined by the incident electron (the complete analysis will be published elsewhere [12]). At SLAC, HERMES, and COMPASS energies, a typical value for θ is less than 0.1. Complexities in the analysis of fragmentation turn out to be proportional to $\sin^2 \theta$ and can be ignored at fixed target facilities of interest. In this approximation the production of two pions can be viewed as a collinear process with the electron beam defining the common \hat{e}_3 axis. Also we take \vec{S}_\perp along the \hat{e}_1 axis.

Since we are only interested in a result at the leading twist, we follow the helicity density matrix formalism developed in Refs. [8,13], in which all spin dependence is summarized in a *double* helicity density matrix. We factor the process at hand into basic ingredients: the $N \rightarrow q$ distribution function, the hard partonic $eq \rightarrow e'q'$ cross section, the $q \rightarrow (\sigma, \rho)$ fragmentation, and the decay $(\sigma, \rho) \rightarrow \pi\pi$, all as density matrices in helicity basis:

$$\left[\frac{d^6 \sigma}{dx dy dz dm^2 d \cos \Theta d \phi} \right]_{H'H} = \mathcal{F}^{h_1 h'_1} \left[\frac{d^2 \sigma(eq \rightarrow e'q')}{dx dy} \right]_{h'_1 h_1} \left[\frac{d^2 \hat{\mathcal{M}}}{dz dm^2} \right]_{h'_2 h_2} \left[\frac{d^2 \mathcal{D}}{d \cos \Theta d \phi} \right]_{H'_1 H_1}, \quad (1)$$

where $h_i(h'_i)$ and $H(H')$ are indices labeling the helicity states of quark and nucleon, and $H_1(H'_1)$ labeling the helicity state of the resonance (σ, ρ); see Fig. 1. In order to incorporate the final state interaction, we have separated the $q \rightarrow \pi^+\pi^-$ fragmentation process into two steps. First, the quark fragments into the resonance (σ, ρ), then the resonance decays into two pions, as shown at the top part of Fig. 1.

We first discuss two-meson fragmentation, first examined in Ref. [10]. Here we introduce only those pieces

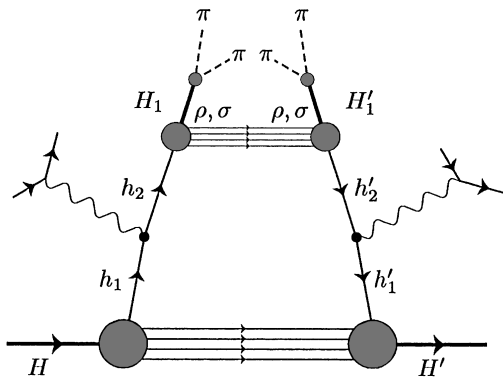


FIG. 1. Hard scattering diagram for two-meson production in the current fragmentation region of electron scattering from a target nucleon. Helicity density matrix labels are shown explicitly.

necessary to describe s - p interference in $\pi^+\pi^-$ production. A full account of these fragmentation functions will be given in Ref. [12]. A two-meson fragmentation function can be defined by a natural generalization of the single particle case. Using the light-cone formalism of Collins and Soper [14], the following replacement suffices,

$$|hX\rangle_{\text{out}} \langle hX| \rightarrow |\pi^+\pi^-X\rangle_{\text{out}} \langle \pi^+\pi^-X|. \quad (2)$$

The resulting two-meson fragmentation function depends on the momentum fraction of each pion, z_1, z_2 , the $\pi\pi$ invariant mass, m , and the angles Θ and ϕ . The subscript “out” places outgoing wave boundary conditions on the $\pi\pi X$ state. Two types of final state interactions can generate a nontrivial phase: (i) those between the two pions, and (ii) those between the pions and the hadronic state X . We ignore the latter because we expect the phase to average to zero when the sum on X is performed— $|\pi^+\pi^-X\rangle_{\text{out}} \rightarrow |(\pi^+\pi^-)_{\text{out}}X\rangle$. Furthermore, if the two-pion system is well approximated by a single resonance, then the resonance phase cancels in the product $|(\pi^+\pi^-)_{\text{out}}X\rangle \langle (\pi^+\pi^-)_{\text{out}}X|$. This leaves only the interference between two partial waves as a potential source of an asymmetry. The final state phase of the two-pion system is determined by the $\pi\pi$ \mathcal{T} -matrix [15]. We

separate out the phase for later consideration and analyze the (real) ρ - σ interference fragmentation function as if the two particles were stable.

The s - p interference fragmentation function describes the emission of a $\rho(\sigma)$ with helicity H_1 from a quark of helicity h_2 , followed by absorption of $\sigma(\rho)$, with helicity H'_1 forming a quark of helicity h'_2 . Conservation of angular momentum along the \hat{e}_3 axis requires

$$H_1 + h'_2 = H'_1 + h_2. \quad (3)$$

Parity and time reversal restrict the number of independent components of $\hat{\mathcal{M}}$:

$$\hat{\mathcal{M}}_{H'_1 H_1, h_2 h'_2}^{sp(ps)} = \hat{\mathcal{M}}_{-H'_1 -H_1, -h_2 -h'_2}^{sp(ps)} \quad (\text{parity}), \quad (4)$$

$$\hat{\mathcal{M}}_{H'_1 H_1, h_2 h'_2}^{sp} = \hat{\mathcal{M}}_{H_1 H'_1, h'_2 h_2}^{ps} \quad (T\text{-reversal}). \quad (5)$$

Note that Eq. (5) holds only after the T reversal violating final state interaction between two pions is separated out.

After these symmetry restrictions, only two independent components remain,

$$\hat{\mathcal{M}}_{00,++}^{sp} \propto \hat{q}_I, \quad \hat{\mathcal{M}}_{01,+ -}^{sp} \propto \delta \hat{q}_I, \quad (6)$$

and they can be identified with two novel interference fragmentation functions, \hat{q}_I , $\delta \hat{q}_I$, where the subscript I stands for interference. Here, to preserve clarity, the flavor, Q^2 , and z have been suppressed. The helicity $\pm \frac{1}{2}$ states of quarks are denoted \pm , respectively. Hermiticity and time reversal invariance guarantee \hat{q}_I and $\delta \hat{q}_I$ are real. From Eq. (6) it is clear that the interference fragmentation function, $\delta \hat{q}_I$, is associated with quark helicity flip and is therefore chiral-odd. It is this feature that enables us to access the chiral-odd quark transversity distribution in DIS.

Encoding this information into a double density matrix notation, we have

$$\begin{aligned} \frac{d^2 \hat{\mathcal{M}}}{dz dm^2} &= \Delta_0(m^2) \{ I \otimes \bar{\eta}_0 \hat{q}_I(z) + (\sigma_+ \otimes \bar{\eta}_- + \sigma_- \otimes \bar{\eta}_+) \delta \hat{q}_I(z) \} \Delta_1^*(m^2) \\ &+ \Delta_1(m^2) \{ I \otimes \eta_0 \hat{q}_I(z) + (\sigma_- \otimes \eta_+ + \sigma_+ \otimes \eta_-) \delta \hat{q}_I(z) \} \Delta_0^*(m^2), \end{aligned} \quad (7)$$

where $\sigma_{\pm} \equiv (\sigma_1 \pm i\sigma_2)/2$ with $\{\sigma_i\}$ the usual Pauli matrices. The η 's are 4×4 matrices in (σ, ρ) helicity space with nonzero elements only in the first column, and the $\bar{\eta}$'s are the transpose matrices ($\bar{\eta}_0 = \eta_0^T$, $\bar{\eta}_+ = \eta_+^T$, $\bar{\eta}_- = \eta_-^T$), with the first rows $(0, 0, 1, 0)$, $(0, 0, 0, 1)$, and $(0, 1, 0, 0)$ for $\bar{\eta}_0$, $\bar{\eta}_+$, and $\bar{\eta}_-$, respectively.

The final state interactions between the two pions are included explicitly in

$$\Delta_0(m^2) = -i \sin \delta_0 e^{i\delta_0}, \quad \Delta_1(m^2) = -i \sin \delta_1 e^{i\delta_1}, \quad (8)$$

where δ_0 and δ_1 are the strong interaction $\pi\pi$ phase shifts. Here we have suppressed the m^2 dependence of the phase shifts for simplicity. The decay process, $(\sigma, \rho) \rightarrow \pi\pi$, can be easily calculated and encoded into the helicity matrix formalism. The result for the interference part is

$$\begin{aligned} \frac{d^2 \mathcal{D}}{d \cos \Theta d\phi} &= \frac{\sqrt{6}}{8\pi^2 m} \sin \Theta [ie^{-i\phi} (\eta_- - \bar{\eta}_-) \\ &+ ie^{i\phi} (\eta_+ - \bar{\eta}_+) \\ &- \sqrt{2} \cot \Theta (\bar{\eta}_0 + \eta_0)]. \end{aligned} \quad (9)$$

Here we have adopted the customary conventions for the ρ polarization vectors, $\vec{\epsilon}_{\pm} = \mp(\hat{e}_1 \pm i\hat{e}_2)/\sqrt{2}$ and $\vec{e}_0 = \hat{e}_3$ in its rest frame with e 's being the unit vectors.

In the double density matrix notation, the quark distribution function \mathcal{F} can be expressed as [8]

$$\begin{aligned} \mathcal{F} &= \frac{1}{2} q(x) I \otimes I + \frac{1}{2} \Delta q(x) \sigma_3 \otimes \sigma_3 \\ &+ \frac{1}{2} \delta q(x) (\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+), \end{aligned} \quad (10)$$

where the first matrix in the direct product is in the nucleon helicity space and the second in the quark helicity space. Here q , Δq , and δq are the spin average, helicity difference, and transversity distribution functions, respectively, and their dependence on Q^2 has been suppressed.

The hard partonic process of interest here is essentially the forward virtual Compton scattering as shown in the middle of Fig. 1. For an unpolarized electron beam, the resulting cross section is [8]

$$\frac{d^2 \sigma(eq \rightarrow e'q')}{dx dy} = \frac{e^4 e_q^2}{8\pi Q^2} \left[\frac{1 + (1-y)^2}{2y} (I \otimes I + \sigma_3 \otimes \sigma_3) + \frac{2(1-y)}{y} (\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+) \right], \quad (11)$$

in the collinear approximation. Here e_q is the charge fraction carried by a quark. We have integrated out the azimuthal angle of the scattering plane.

Combining all the ingredients together, and integrating over Θ to eliminate the \hat{q}_I dependence, we obtain the transversity dependent part of the cross section for the production of two pions (kaons) in the current fragmentation region for unpolarized electrons incident on a transversely polarized nucleon as follows:

$$\frac{d^5 \sigma_{\perp}}{dx dy dz dm^2 d\phi} = -\frac{e^4}{64\pi^2 Q^2 m} \frac{1-y}{y} \sqrt{6} \cos \phi \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1) \sum_a e_a^2 \delta q_a(x) \delta \hat{q}_I^a(z). \quad (12)$$

Here the sum over a covers all quark and antiquark flavors.

An asymmetry is obtained by dividing out the polarization independent cross section,

$$\begin{aligned} \mathcal{A}_{\perp\top} &\equiv \frac{d\sigma_{\perp} - d\sigma_{\top}}{d\sigma_{\perp} + d\sigma_{\top}} \\ &= -\frac{\pi}{4} \frac{\sqrt{6}(1-y)}{1+(1-y)^2} \cos\phi \sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1) \frac{\sum_a e_a^2 \delta q_a(x) \delta \hat{q}_I^a(z)}{\sum_a e_a^2 q_a(x) [\sin^2 \delta_0 \hat{q}_0^a(z) + \sin^2 \delta_1 \hat{q}_1^a(z)]}, \end{aligned} \quad (13)$$

where \hat{q}_0 and \hat{q}_1 are spin-average fragmentation functions for the σ and ρ resonances, respectively. The crucial “figure of merit” for this asymmetry, $\sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)$, is shown in Fig. 2. This asymmetry can be measured either by flipping the target transverse spin or by binning events according to the sign of the crucial azimuthal angle ϕ . Note that this asymmetry only requires a transversely polarized nucleon target, but not a polarized electron beam.

The flavor content of the asymmetry $\mathcal{A}_{\perp\top}$ can be revealed by using isospin symmetry and charge conjugation restrictions. For $\pi^+\pi^-$ production, isospin symmetry gives $\delta\hat{u}_I = -\delta\hat{d}_I$ and $\delta\hat{s}_I = 0$. Charge conjugation implies $\delta\hat{q}_I^a = -\delta\hat{q}_I^{\bar{a}}$. Thus there is only one independent interference fragmentation function for $\pi^+\pi^-$ production, and it may be factored out of the asymmetry, $\sum_a e_a^2 \delta q_a \delta \hat{q}_I^a = [4/9(\delta u - \delta \bar{u}) - 1/9(\delta d - \delta \bar{d})] \delta \hat{u}_I$. Similar application of isospin symmetry and charge conjugation to the ρ and σ fragmentation functions that appear in the denominator of Eq. (13) leads to a reduction in the number of independent functions: $\hat{u}_i = \hat{d}_i = \hat{\bar{u}}_i = \hat{\bar{d}}_i$ and $\hat{s}_i = \hat{\bar{s}}_i$ for $i = \{0, 1\}$. For other systems the situation is more complicated due to the relaxation of Bose symmetry restrictions.

Finally, a few comments can be made about our results. First, the final state phase generated by the s - p interfer-

ence is crucial to this analysis. If the data are not kept differential in enough kinematic variables, the effect will almost certainly average to zero. We are particularly concerned about the two-meson invariant mass, m , where we can see explicitly that the interference averages to zero over the ρ as shown in Fig. 2. Second, the transversity distribution is multiplied by the fragmentation function $\delta\hat{q}_I$. Note that the transversity distribution *always* appears in a product of two soft QCD functions due to its chiral-odd nature. In order to disentangle the transversity distribution from the asymmetry, one may invoke the process $e^+e^- \rightarrow (\pi^+\pi^-X)(\pi^+\pi^-X)$ to measure $\delta\hat{q}_I$ [16], or use QCD inspired models to estimate it [12].

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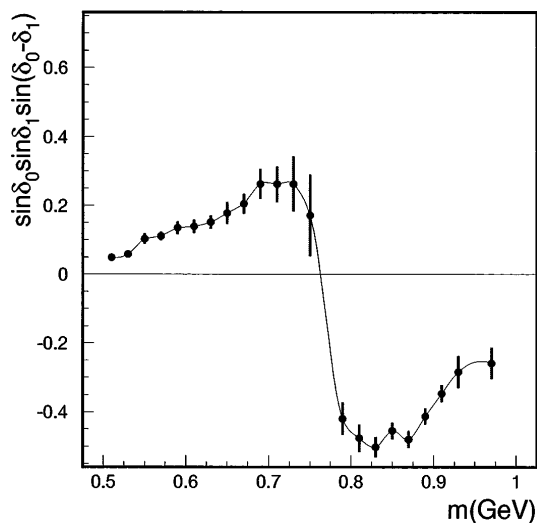


FIG. 2. The factor, $\sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1)$, as a function of the invariant mass m of the two-pion system. The data on $\pi\pi$ phase shifts are taken from Ref. [17].

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