Gravitational Constant Measured by Means of a Beam Balance

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We present a new method to measure the gravitational constant G. A beam balance compares the weight of two 1-kg test masses and measures the gravitational force of two field masses with a statistical uncertainty of 10 ng. Two vessels in a refined arrangement are used as field masses. They have been filled with water as a test. G has been determined with an uncertainty of 240 ppm. The next step is to fill the vessels with mercury. Because of the larger signal and further refinement of our experiment, we hope to reach the design uncertainty of 10 ppm. [S0031-9007(97)05223-X]

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The gravitational constant G is known with a relative uncertainty of only 128 ppm (part per million) while the uncertainty of all other fundamental constants is rapidly decreasing to values considerably below 1 ppm [1–3]. The most precise device for measuring the gravitational force is the torsion balance, first applied by Cavendish 200 years ago. Until 1890 beam balances were also used to measure the gravitational force [4–6]. The accuracy was limited to 0.16% at best [6], and these measurements were given up in favor of the torsion balance. During past few decades, no essential progress of the technique for measuring G has been made. Recent attempts with different techniques failed to improve the uncertainty of G [7–9]. The current results, as well as several former results, show unexplained discrepancies. No significant deviations from Newton's gravitational law were found experimentally. The most obvious explanation is, therefore, systematic errors even for the traditional torsion balance technique [10,11].

In order to overcome this situation, we proposed a new experiment. It is based on a beam balance which is a suitable device for measuring the gravitational force [12]. The idea of this experiment and the experience on how to operate the balance is based on our successful storage lake experiment, where the gravitational force of water has been measured in order to test Newton's inverse-square law [13,14]. This new experiment is a modern variation of the method used by von Jolly [5] and Richarz and Krigar-Menzel [6]. Our first results have already provided a significant accuracy improvement over the earlier beam-balance experiments. This is possible because of a different method and the general progress grained in mechanics, automation, and weighing techniques.

The principle of our new experiment is shown in Fig. 1: To large masses are moved in the vertical direction and alternately positioned in one of two states. Their gravitational field acts on two small test masses which are separately suspended on wires. The suspension devices of the test masses are alternately connected to a single-pan beam balance which measures the weight change due to the gravitational force. The arrangement of two test and field masses is symmetrical and enables a differential measurement such that many disturbing forces and drift effects cancel each other out.

The field masses have a cylindrical shape. They have an axial bore in order to let the test masses pass through the field masses. The bore has the beneficial effect in that the gravitational field has an extremum at each end of the bore. The field masses are alternately arranged in two states in such a way that both tests masses are located at an extremum of the gravitational field. This clearly reduces the accuracy requirements to the relative positions. In our case, a 1-ppm uncertainty corresponds to 0.2 mm which is three orders of magnitude larger than for a pure inverse-square dependence.

The gravitational force has to be calculated numerically. A simple but useful approximation should be discussed here. The test masses are pointlike objects with mass m. The field masses are ideal cylinders with a central bore, having a homogeneous density ρ and mass M. The gravitational field at the position of a test mass is taken as the surface field of the close field mass while the gravitational field of the far field mass is neglected. Substituting mass and density for the dimensions of the field masses and optimizing the shape, the amplitude of the differential weight signal is

$$A \simeq 4GmM/(M/\pi\rho)^{2/3}. \tag{1}$$

The factor 4 is due to the use of two test masses and two field masses. The term which appears instead of the $1/r^2$ term of Newton's gravitational law is the mean-field-mass dimension. The weight signal depends only on mass and density. No measurements of the field mass dimensions or the relative positions of field and test masses are necessary.

An advantage of our method is a low sensitivity to disturbing forces and systematics. Many effects cancel in the weight difference as exactly as they are equal for both test masses. Examples are tidal forces, the thermal drift of the balance, and tilt effects. Tilt of the ground due to the motion of the field masses can be a serious problem in a measurement of G. This is not expected to occur in our experiment because the total load of the ground is independent of the field mass position and possible effects

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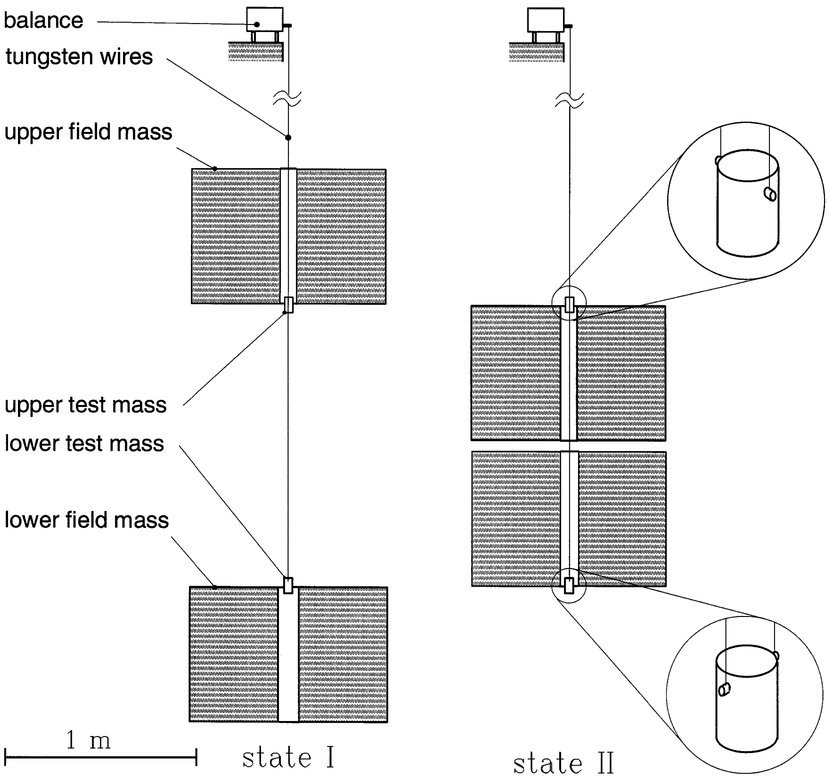

FIG. 1. Two large masses are alternately positioned in two states are shown. They generate a gravitational field which acts on two test masses and changes their weight. This effect is measured by means of a balance to which the suspension devices of the test masses are alternately connected.

cancel in the weight difference. Nevertheless, the tilt of the balance was measured and found to be insignificant.

In our experiment we use a liquid for the field masses because its density is much more homogenous than the density of typical solids. Mercury, with its high and well-known density, will be used for the final measurements. Two stainless-steel vessels, each with a volume of 500 l and a weight of 800 kg, contain 13.5 tons of mercury. The components of the vessels are very precisely machined and surveyed. The precision required for a 1-ppm uncertainty of G is 1 μm for the diameter of the central bore and 0.5 mm for the outer diameter of the vessel. The gravitational signal of the mercury-filled vessels amounts to 760 μg while, for water-filled vessels, the signal is still 110 μg. To get the same signal with water instead of mercury, more than 2000 tons of water would be necessary.

The vessels move vertically and in opposite directions by means of three spindles which are coupled by a gear drive. The precisely ground threads of the spindles are left handed in the upper section and right handed in the lower section. Thereby, the vessel moving down pulls the other vessel up.

A good location with appropriate mechanical and thermal stability was found at the Paul-Scherrer-Institute. The experiment is set up in a 4.8 m deep pit inside a hall as shown in Fig. 2. The upper part contains the balance, the vacuum system, the motor drive of the field masses, and the computer control of the whole experiment. The temperature in this part is roughly stabilized. The lower part of the pit contains the field and test masses. To avoid temperature gradients, a ventilating system circulates the air and controls the mean temperature. The thick concrete walls of the pit have been isolated with foam glass and the setup has been covered with radiation shields. Because of these provisions, the temperature is constant in space and time within 50 mK and independent of the field mass position within 10 mK.

In our experiment we use a special type of balance which compares 1-kg masses with high precision. It was made available to us by Mettler-Toledo and modified especially for our purposes. It is a single-pan balance, using flexure strips as pivots instead of knife edges. A counterweight is fixed to the beam of the balance while the pan is coupled to the other end of the beam. A special coupling mechanism eliminates unwanted torques, and the weight can be attached to the pan with reasonable precision. The weight to be measured is compensated mainly by the counterweight and up to 11 g is electrically compensated. Therefore, the balance is only able to compare 1-kg masses, but it has an unusually high resolution of 100 ng. In fact, the resolution is further increased to 10 ng by averaging many readings. The balance and the basic weighting technique is discussed in more detail in Refs. [12–16], for example.

The test masses have a cylindrical shape and are suspended by two wires in order to avoid slow torsion oscillations (see the detail in Fig. 1). They are made of gold-plated copper to exclude ferromagnetic forces. To ensure that the used material is free of ferromagnetic contaminations, the susceptibility of small samples and complete test masses has been measured. Also, the magnetic field at the test mass position has been measured,

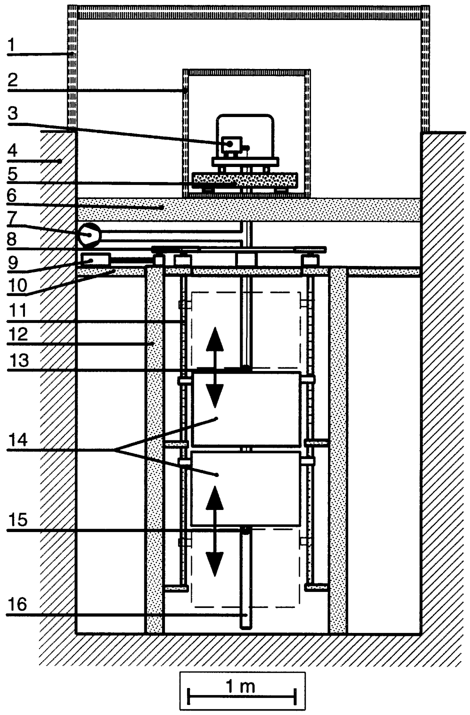

FIG. 2. The experimental setup. 1: chamber; 2: thermally insulated chamber; 3: balance; 4: concrete walls of the pit; 5: granite plate; 6: steel girder; 7: vacuum pumps; 8: gear drive; 9: motor; 10: working platform; 11: spindle; 12: steel girder of the main support; 13: upper test mass; 14: field masses; 15: lower test mass; 16: vacuum tube.

and any significant influence in the measurement of the gravitational force is clearly excluded.

The balance can be calibrated *in situ* with a precisely known standard mass of 1 g. This calibration is of great importance: It enables us to compare the strength of the gravitational force directly with a precisely known standard force (i.e., the weight of the standard mass). No further conversion is necessary. This is a valuable advantage in comparison to other traditional G measurements.

Precise weighing requires operation in vacuum (1×10^{-4} Pa) in order to avoid bouyancy, convection, and other gas pressure effects. Additionally, the temporal and spatial temperature gradients at the balance must be minute. This is achieved by the use of passive temperature stabilization which causes smaller spatial gradients than active stabilization.

The test masses are suspended inside a vacuum tube which passes through the central bore of the field masses (see Fig. 2). The weight of a test mass is found to depend on the temperature of the vacuum tube with a typical sensitivity of 1 μg/K. This effect is most probably due to a monolayer field of adsorbed water. Sorption effects are also observed by other groups [16,17], and further investigations are necessary if future standard weights should be kept in vacuum.

The balance is not very sensitive to vibrations or pendulum oscillations. It is sufficient to mount the balance on a massive granite plate with simple damping elements. No influence of the motor drive of the field masses or the vacuum pumps has been seen. Also seismic, human, or industrial vibrations were not observed.

In order to measure the weight difference of the two test masses, the suspension devices are alternately connected to the bottom side of the pan by means of a hydraulic device. This mechanism lifts the suspension device of the weight by 1 mm while the suspension device of the other weight is lowered onto the pan. The suspension devices are constructed to reproduce the position of a weight with a self-centering device to better than 10 μm. This guarantees that the reproducibility of a weighing is not appreciably affected by positioning errors. Relaxation of mechanical stress in the components of the balance, especially in the flexure strips, can be released by abrupt variations of the load of the balance [16]. Therefore, the exchange is performed in such a way that the total load is held constant within ± 1 g during the exchange [14].

A complete cycle of weighing both test masses lasts about 10 min. The standard deviation of the weight difference is 300 ng, dominated by internal low-frequency noise of the balance. The long-term drift of the weight difference has been reduced to 60 ng per day.

The procedure of a G measurement is as follows: The field masses alternate between the two states indicated in Fig. 1. They remain in each state for 4 h. During this time interval, the balance compares the weight of the test masses. Before the field masses are moved again, the balance is calibrated. This procedure is repeated periodically and results in a modulated time series (Fig. 3). Noise and drift can be further suppressed if the time series is demodulated and integrated for a few weeks.

First measurements of the gravitational force of the empty vessels have been performed as a test. The vessels were then filled with pure water. In order to avoid losses during the filling procedure, a closed device was used to fill the water from 20-1 containers through a pumping line into the vessels. To prevent air bubbles from being locked in the vessels, the air was pumped out before the filling started and, for a very short time, before the last container was filled. To determine the amount of water in

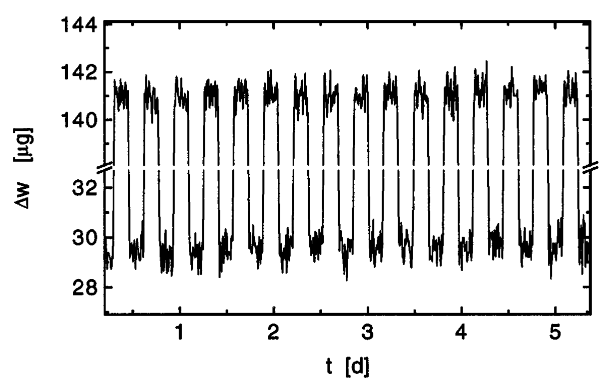

FIG. 3. A time series of the weight difference of the test masses, modulated by the gravitational force of the water-filled vessels. Notice the axis break.

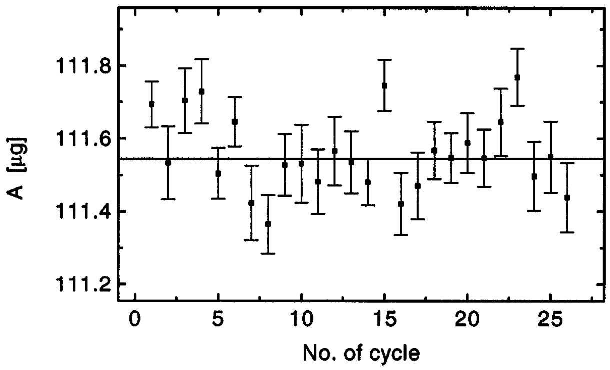

FIG. 4. The modulation amplitude of successive cycles of a 9-day run with water-filled vessels. The statistical uncertainty ifs about 90 ng for a single cycle and due mainly to low-frequency noise of the balance. It can be averaged out with an integration time of a few weeks.

the vessels, the containers have been weighed with high accuracy before and after the filling. As a consistency check, the volume of the water and the surveyed volume of the vessels were compared and agreed very well within their relative uncertainties of 30 ppm.

The gravitational force of full and empty vessels was measured with a statistical uncertainty of 10 ng, using an integration time of about 20 days. The results can be compared or subtracted, which allows a valuable consistency check. Figures 3 and 4 show examples of a measurement performed with water-filled vessels. The corresponding results for the gravitational constant agree very well within their statistical uncertainties and yield a mean value of

$$G = (6.6754 \pm 0.0005 \pm 0.0015) \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}.$$

The statistical uncertainty is 75 ppm and the systematic uncertainty is 230 ppm (Table I). We emphasize that the largest systematic uncertainties reflect the present state of our experiment and are not a fundamental limitation. Nevertheless, our result differs only sightly from the present standard value. It does not agree with the current results reported in Refs. [7–9], especially with the result of Michaelis *et al.* [9].

TABLE I. The systematic uncertainty of G. The first group pertains to upper limits of systematic effects whose investigations are not yet completed. The second group includes the uncertainty of the parameters which enter into the calculation of G.

Source	Contribution to uncertainty of G (ppm)
Systematic effects of the balance	$\leq$130
Sorption effects	$\leq$45
Integration of mass distribution	$\leq$180
Mass of each test mass	0.27
Test mass dimensions	2.7
Test mass position	2.5
Weight of each vessel	8
Density of the stainless steel vessels	$\leq$10
Weight of water in each vessel	8
Density of water	29
Standard mass for calibration	5
Local gravity	0.06
Total	230

The next step is to fill the vessels with mercury. We expect a larger signal of about 760 μg. With further progress of our experiment, we hope to reach a significant improvement in the knowledge of G. A more detailed report about this experiment is in preparation.

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