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Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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We report on a quantum optical experimental implementation of teleportation of unknown pure quantum states. This realizes all of the nonlocal aspects of the original scheme proposed by Bennett *et al.* and is equivalent to it up to a local operation. We exhibit results for the teleportation of a linearly polarized state and of an elliptically polarized state. We show that the experimental results cannot be explained in terms of a classical channel alone. The Bell measurement in our experiment can distinguish between all four Bell states *simultaneously* allowing, in the ideal case, a 100% success rate of teleportation. [S0031-9007(97)05275-7]

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In Ref. [1], Bennett *et al.* showed that an unknown quantum state can be "disassembled into, then later reconstructed from, purely classical information and purely nonclassical Einstein-Podolsky-Rosen (EPR) correlations." They called this process *teleportation.* In their scheme, a sender, traditionally called Alice, is given a state unknown to her. She also has one of two particles prepared in an EPR state (such as a singlet state). She performs a Bell measurement on the combined system of the unknown state and her EPR particle, and transmits the result of this measurement by a classical channel to Bob, who has the second of the EPR particles. Depending on the result of the measurement, Bob performs one of four possible unitary transformations on his particle and it will now be in the unknown state.

In the experiment reported in this paper we take an approach first suggested in [2] in which a total of two photons, rather than three, are used. The EPR state is realized by **k**-vector (or path) entanglement, and the polarization degree of freedom of one of the photons is employed for preparing the unknown state [3]. This avoids the difficulties associated with having three photons and, as will be seen below, makes the Bell measurement much more straightforward. However, this approach does place a restriction on us in that the preparer must prepare his state on one of the EPR photons, and so the unknown state cannot come from outside (i.e., this means that Alice is presented with an unknown pure state rather than with part of an entangled state). Nevertheless, the scheme described here realizes all the nonlocal aspects of the original teleportation scheme, and is equivalent to the original scheme up to a local operation (since, in principle, any unknown state of a particle from outside could be swapped onto the polarization degree of freedom of Alice's EPR particle by a local unitary operation [4]). In particular, as in the original scheme, we emphasize that if the preparer does not tell Alice what state he has prepared then there is no way Alice can find out what the state is.

It is worthwhile mentioning that this leads to a 100% success rate for the Bell measurement in the ideal case rather than 50% as in previously suggested schemes [5].

No experiment is perfect, so we need to known how good the experiment has to be before we can say we have quantum teleportation. Our objective is to show that the experimental results cannot be explained by a classical channel alone (that is, without an EPR pair). Thus, consider the following scenario. With a probability of $\frac{1}{3}$ the preparer prepares one of the states $|\phi_a\rangle$ ($a = 1,2,3$) which corresponds to equally spaced linearly polarized states at 0°, 120°, and −120°. He then gives this state to Alice (without telling her which one it is). Alice makes a measurement on it in an attempt to gain some information about the state. The most general measurement she can make is a positive operator valued measure [6]. She will never obtain more information if some of the positive operators are not of rank one, and thus we can take them all to be of rank one, that is proportional to projection operators. Let Alice's measurement have L outcomes labeled $l = 1, 2, \ldots, L$ and let the positive operator associated with outcome l be $|\varepsilon_l\rangle\langle\varepsilon_l|$. We require that

$$\sum_{l=1}^{L} |\varepsilon_l\rangle\langle\varepsilon_l| = I\,. \tag{1}$$

Note that, in general, the states $|\varepsilon_l\rangle$ are neither orthogonal to each other or normalized but rather form an overcomplete basis set. The probability of getting outcome l given that the state prepared is $|\phi_a\rangle$ is $|\langle\phi_a|\varepsilon_l\rangle|^2$. Alice sends the information l to Bob over the classical channel and Bob prepares a state $|\phi_l^c\rangle$ (the c denotes that the state has been "classically teleported"). This state is chosen so as to give the best chance of passing a test for the original state. Bob now passes this state onto a verifier. We suppose that the preparer has told the verifier which state he prepared and the verifier sets his apparatus to measure the projection operator, $|\phi_a\rangle\langle\phi_a|$, onto this state. The probability that the classically teleported state will pass the test in this case is $|\langle\phi_a|\phi_l^c\rangle|^2$. The average probability S of passing the test

$$S = \sum_{a,l} \tfrac{1}{3}|\langle\phi_a|\phi_l^c\rangle|^2\,|\langle\phi_a|\varepsilon_l\rangle|^2. \tag{2}$$

In the Appendix we show that this classical teleportation protocol must satisfy

$$S \leq \tfrac{3}{4}\,. \tag{3}$$

To show that we have quantum teleportation we must show that the experimental results violate this inequality [7].

The experiment is shown in Fig. 1. Pairs of polarization entangled photons are created directly using type-II degenerate parametric down-conversion by the method described in Refs. [8,9]. The β-barium borate (BBO) crystal is pumped by a 200 mW UV cw argon laser with wavelength 351.1 nm. The down-converted photons have a wavelength of 702.2 nm. The state of the photons at this stage is $\frac{1}{\sqrt{2}}(|v\rangle_1|h\rangle_2 + |h\rangle_1|v\rangle_2)$. However, we want a **k**-vector entangled state so next we let each photon pass through a calcite crystal (C), after which the state becomes

$$\frac{1}{\sqrt{2}}\,(|a_1\rangle\,|a_2\rangle + |b_1\rangle\,|b_2\rangle)\,|v\rangle_1|h\rangle_2\,. \tag{4}$$

By this method a polarization entangled state has been converted into a **k**-vector entangled state. Here, $|a_1\rangle\,|v\rangle_1$,

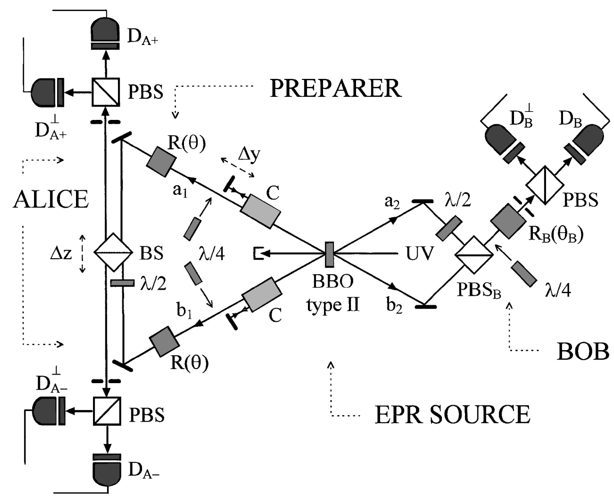

FIG. 1. Diagram of experimental setup showing the separate roles of the preparer, Alice, and Bob.

for example, represents the state of photon 1 in path a_1 and having vertical polarization. Since each photon has the same polarization in each of the two paths it can take, the polarization part of the state factors out of the **k**-vector entanglement. The EPR pair for the teleportation procedure is provided by this **k**-vector entanglement. By means of (zero order) quarter-wave plates oriented at some angle γ to the horizontal and Fresnel rhomb polarization rotators (R) acting in the same way on paths a_1 and b_1 as shown in Fig. 1, the polarization degree of freedom of photon 1 is used by the preparer to prepare the general state: $|\phi\rangle = \alpha|v\rangle_1 + \beta|h\rangle_1$. This is the state to be teleported. The state of the whole system is now

$$\frac{1}{\sqrt{2}}\,(|a_1\rangle\,|a_2\rangle + |b_1\rangle\,|b_2\rangle)\,(\alpha|v\rangle_1 + \beta|h\rangle_1)\,|h\rangle_2\,. \tag{5}$$

We now introduce four orthonormal states which are directly analogous to the Bell states considered in [1]:

$$|c_\pm\rangle = \frac{1}{\sqrt{2}}\,(|a_1\rangle\,|v\rangle_1 \pm |b_1\rangle\,|h\rangle_1)\,, \tag{6}$$

$$|d_\pm\rangle = \frac{1}{\sqrt{2}}\,(|a_1\rangle\,|h\rangle_1 \pm |b_1\rangle\,|v\rangle_1)\,. \tag{7}$$

We can rewrite (5) by using these states as a basis:

$$\begin{aligned}\tfrac{1}{2}|c_+\rangle\,(\alpha|a_2\rangle + \beta|b_2\rangle)\,|h\rangle_2 &+ \tfrac{1}{2}|c_-\rangle\,(\alpha|a_2\rangle - \beta|b_2\rangle)\,|h\rangle_2\\ &+ \tfrac{1}{2}|d_+\rangle\,(\beta|a_2\rangle + \alpha|b_2\rangle)\,|h\rangle_2\\ &+ \tfrac{1}{2}|d_-\rangle\,(\beta|a_2\rangle - \alpha|b_2\rangle)\,|h\rangle_2\,.\end{aligned} \tag{8}$$

For Alice, it is simply a question of measuring on the basis $|c_\pm\rangle$, $|d_\pm\rangle$. To do this we first rotate the polarization of path b_1 by a further 90° (in the actual experiment this was done by setting the angle of the Fresnel rhomb in path b_1 at $\theta + 90°$ rather than by using a separate plate as shown in the figure) so that the state $|b_1\rangle|v\rangle_1$ becomes $-|b_1\rangle|h\rangle_1$ and the state $|b_1\rangle|h\rangle_1$ becomes $|b_1\rangle|v\rangle_1$. Thus,

$$|c_\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \pm |b_1\rangle)|v\rangle_1, \quad (9)$$

$$|d_\pm\rangle \rightarrow \frac{1}{\sqrt{2}}(|a_1\rangle \mp |b_1\rangle)|h\rangle_1. \quad (10)$$

Paths a_1 and b_1 now impinge on the two input ports of an ordinary 50:50 beam splitter (BS). At this BS each of the two polarizations h and v interfere independently. After BS, there are two sets of detectors $D_{A_\pm}$ and $D^\perp_{A_\pm}$ which are coupled, respectively, to the h and v polarizations, being the ones selected by two polarizing beam splitters (PBS). A click registered at $D^\perp_{A_\pm}$ corresponds to a measurement onto $|c_\pm\rangle$ while a click registered at $D_{A_\pm}$ corresponds to a measurement onto $|d_\mp\rangle$. Furthermore, the position Δz of BS is set in such a way that the detectors D_{A_+} and $D^\perp_{A_+}$ are excited by the photon in the state $\frac{1}{\sqrt{2}}(|a_1\rangle + |b_1\rangle)$ while D_{A_-} and $D^\perp_{A_-}$ are excited by state $\frac{1}{\sqrt{2}}(|a_1\rangle - |b_1\rangle)$. In this way each of the four Bell states [(6) and (7)] can be measured.

The paths a_2 and b_2, taken by the photon sent to Bob, originate from backreflections at the end of the two calcite crystals and are transmitted through the BBO crystal. The path a_2 is rotated through 90° by a half-wave plate and is combined with b_2 at a polarizing beamsplitter (PBS_B) oriented to transmit horizontal and to reflect vertical polarizations. The state in (8) becomes

$$\tfrac{1}{2}|c_+\rangle(\beta|h\rangle_2 + \alpha|v\rangle_2) + \tfrac{1}{2}|c_-\rangle(-\beta|h\rangle_2 + \alpha|v\rangle_2) + \tfrac{1}{2}|d_+\rangle(\alpha|h\rangle_2 + \beta|v\rangle_2) + \tfrac{1}{2}|d_-\rangle(-\alpha|h\rangle_2 + \beta|v\rangle_2). \quad (11)$$

Bob's photon can be transformed back to the original state $\alpha|v\rangle + \beta|h\rangle$ by applying an appropriate unitary transformation, depending on the outcome of Alice's measurement. However, we are simply interested in verifying that the appropriate state has been produced at Bob's end so, rather than performing these unitary transformations, we will simply orient (by acting on a polarization rotator R_B and on a quarter-wave plate) the measuring apparatus at end 2 appropriately for each of Alice's outcomes (the transformations can either be seen as *active* transformations or as a *passive* reorientation of our reference system, with respect to which the verification measurements are made). In the case of linear polarization the verification measurements can be accomplished by rotating the polarization of the state through an angle θ_B by means of a Fresnel rhomb device (R_B), then letting it impinge on a polarizing beam splitter followed by two detectors $D(\theta_B)$ (a photon originally incident with polarization θ_B would certainly be detected at this detector) and $D(\theta_B^\perp)$. In the more general case of elliptical polarization we can add, before R_B, a quarter-wave plate oriented at an angle γ_B with respect to the horizontal direction.

Wide filters ($\Delta\lambda = 20$ nm) were placed just before each detector. Wide, rather than narrow, filters were used so that the count rate was high enough to allow measurements to be made in a few seconds in order that problems associated with phase drift were minimized.

The beam splitter and the back reflecting mirror acting on path b_2 were each mounted on a computer controlled micrometrical stage which could be incremented in 0.1 μm steps. To align the system, a half-wave plate oriented at 45° to the vertical was placed before one of the calcite crystals to rotate the polarization by 90°. This had the effect of sending both photons to Alice's end or Bob's end. The correct values of Δz and Δy were found by looking for an interference dip in the coincidence count rates between D_{A_+} and D_{A_-} at Alice's end, and between D(45°) and D(−45°) at Bob's end. After alignment, the half-wave plate was rotated 0° to the vertical so that it had no effect on the polarization of photons passing through it. The distance between the Alice's and Bob's apparatuses was about 2.5 m. The coincidence time window was 1.6 ns and each measurement was taken over runs lasting 10 s. Typically, there were about 5×10^2 coincidence counts during each run.

We will report separately on two aspects of the experiment. First, using three equally spaced linear polarization settings (0°, +120°, −120°), we will see that the classical teleportation limit in Eq. (3) is surpassed. Second we will see that, for some arbitrary states (linear and elliptically polarized), we see all of the expected features.

Let $I(\theta_B)$ be the coincidence count between $D(\theta_B)$ and one of Alice's detectors. Let $I_\parallel$ be the coincidence rate when θ_B is oriented so as to measure the projection operator onto the corresponding term in (11). For example, if $\theta = 120°$, then Alice's output $|d_+\rangle$ corresponds to $\theta_B = -30°$. (In the present work all polarization angles are referred to the horizontal direction.) Let $I_\perp$ be the count rate when θ_B is rotated through 90° from the value used to measure $I_\parallel$. We will have $I_\parallel = k|\langle\phi|\phi_{\rm tele}\rangle|^2$ and $I_\perp = k|\langle\phi_\perp|\phi_{\rm tele}\rangle|^2$, where $|\phi\rangle$ is the prepared state and $|\phi_{\rm tele}\rangle$ is the state that is actually produced at Bob's side by the teleportation process. If the state produced at Bob's end is not pure then this analysis is easily adapted by summing over a particular decomposition of the impure state. The normalization constant k depends on the detector efficiencies. Since $I_\parallel + I_\perp = k$, we show that $|\langle\phi|\phi_{\rm tele}\rangle|^2 = I_\parallel/(I_\parallel + I_\perp)$. To beat the classical teleportation limit the average value of this quantity must exceed $\frac{3}{4}$. This average is taken over three equally spaced linear polarizations (each weighted with probability $\frac{1}{3}$) and over each of the four possible outcomes of Alice's Bell state measurement (each weighted by $\frac{1}{4}$). With the average understood to be in this sense we can write $S = [I_\parallel/(I_\parallel + I_\perp)]_{\rm av}$. This quantity was measured, and we found that $S = 0.853 \pm 0.012$. This represents a violation of the classical teleportation limit by eight standard deviations. Note, if $I_{\parallel} = I_{\max}$ and $I_{\perp} = I_{\min}$ (as we would expect, and as is indeed true for the data to be discussed below), and if the quantity in (12) is greater than $\frac{3}{4}$, then the visibility is greater than 50% (and vice versa).

Now consider in more detail one linear polarization case. All of the quarter-wave plates were removed in this case. In Fig. 2, we show the count rates related to four *simultaneous* coincidence experiments between each outcome of Alice's Bell state measurement ($c_{\pm}$ and $d_{\pm}$) and Bob's detector $D(\theta_B)$ as a function of θ_B for the particular case where the preparer prepared linear polarization with $\theta = 22.5°$. Note that the displacements of the maxima are 22.5°, 67.5°, −67.5°, and −22.5°. These are consistent with Eq. (11), where $\alpha = \sin(\theta)$ and $\beta = \cos(\theta)$. In the present experiment the detector $D(\theta_B^{\perp})$ is used only for the alignment of Bob's apparatus.

To prepare an elliptical polarized state we set $\theta = 0°$ and insert the quarter-wave plates at angle γ equal to 20°. This produced the elliptically polarized state,

$$\frac{1}{\sqrt{2}}[(1 + i\cos(2\gamma))\,|v\rangle + \sin(2\gamma)\,|h\rangle]. \qquad (12)$$

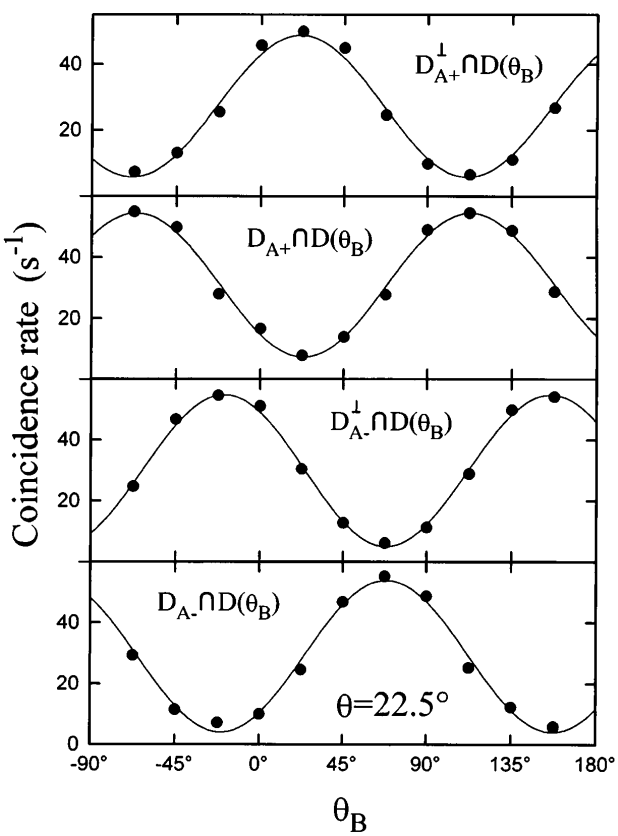

FIG. 2. Results for a linear polarized state at 22.5° to the horizontal obtained by four *simultaneous* coincidence experiments involving the four detectors D_A at Alice's site and Bob's detector $D(\theta_B)$. The graphs show the coincidence rates as a function of the angle θ_B. The error bars are smaller than the dots.

To verify that the state had been teleported according to Eq. (11) a quarter-wave plate was used at Bob's end, oriented at angle γ_B. A different setting of this was used corresponding to each of Alice's outcomes. For outcomes $|d_{\pm}\rangle$ we set $\gamma_B = \pm\gamma + 90°$. This converts the corresponding state at Bob's side to $|v\rangle$. For outcomes $|c_{\pm}\rangle$ we set $\gamma_B = \pm\gamma$. This converts the corresponding state at Bob's side to $|h\rangle$. The state was then analyzed in linear polarization over a range of values of θ_B. The results, shown in Fig. 3, demonstrate that the state after the quarter-wave plate is vertically or horizontally polarized, as required.

In this paper we have seen how a state, which is totally unknown to Alice, can be disassembled into purely classical and purely nonlocal EPR correlations and then reconstructed at a distant location. In the reconstruction procedure we took an essentially passive view of the unitary transformations. Work is currently in progress to implement the transformations in an active way by using fast Pockels cells fired by Alice's detectors.

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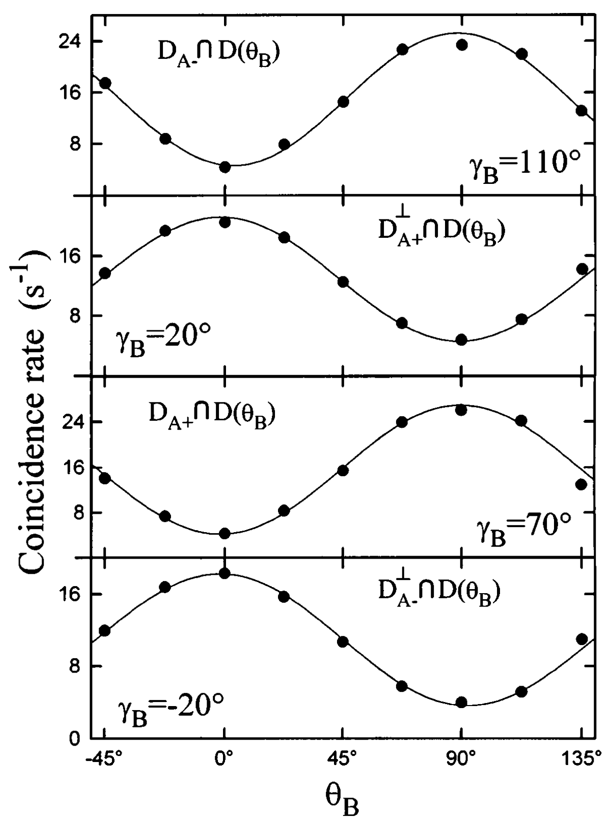

FIG. 3. Results of four coincidence experiments the same as for Fig. 2 but for an elliptically polarized case generated by using a quarter-wave plate at angle 20° with respect to the horizontal.

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Appendix.—Define the normalized state $|\Omega_l\rangle$ by $|\varepsilon_l\rangle = \sqrt{\mu_l}\,|\Omega_l\rangle$ where $\mu_l = \langle\varepsilon_l|\varepsilon_l\rangle$. By taking the trace of (1), we obtain $\sum_l \mu_l = 2$. The 2 here corresponds to the dimension of the Hilbert space. Define $T_l = \sum_a |\langle\phi_a|\phi_l^c\rangle|^2 |\langle\phi_a|\Omega_l\rangle|^2$. Then $S = \sum_l \frac{1}{3}\mu_l T_l$. By varying with respect to the vectors $|\phi_l^c\rangle$ and $|\Omega_l\rangle$ we obtain $T_l^{\max} = \frac{9}{8}$. Hence, $S \le \frac{3}{8}\sum_l \mu_l = \frac{3}{4}$. We obtain (3) as required.

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