# Continuum of Chiral Luttinger Liquids at the Fractional Quantum Hall Edge

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We study current versus voltage ($I$-$V$) when tunneling into the edge of the fractional quantum Hall effect over a continuum of filling factors ($\nu$) from $1/4$ to 1. Our devices manifest the power law $I$-$V$ behavior previously observed by Chang *et al.* at discrete fillings, but now with as many as six decades in current and over the whole range of filling factor suggesting the existence of a continuum of chiral Luttinger liquids. Surprisingly the exponent behaves approximately as $1/\nu$ and does not exhibit the strong plateau features predicted in recent theoretical works based on the intermixing of copropagating and counterpropagating multiple edge modes. [S0031-9007(97)05218-6]

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Since the prediction by Wen [1] that the edges of the fractional quantum Hall effect (FQHE) [2] would host a new interacting one-dimensional electronic state called the chiral Luttinger liquid [3,4], there has been a wealth of theoretical and experimental interest intent on characterizing this long-anticipated non-Fermi liquid. The Luttinger liquid occurs at special fractional filling $\nu$, corresponding to incompressible fluids. Subsequent theory described how tunneling measurements could reveal the characteristic power law energy density of states at these special filling factors [5–7], and recent experiments using two distinct tunneling geometries have succeeded to varying degrees in measuring a power law in the temperature dependence of the tunneling conductance [8–11] and in the current versus voltage ($I$-$V$) [9,10]. These experiments have investigated specific filling factors ($1/3$, 1 [8,9,11], $2/3$ [8,9]) where comparisons with theory can easily be drawn, and $\nu = 1/2$ [10] corresponding to the compressible composite Fermion fluid for which a definitive theory is yet unavailable. In this work we perform the first systematic characterization of the power law behavior over a continuum of fractional filling factors, spanning both compressible and incompressible liquids. Two major results have emerged: (1) there is a continuum of power law $I$-$V$ behavior observed, and (2) the $I$-$V$ exponent is approximately given by $1/\nu$, with the edge appearing to behave as a single mode Luttinger liquid with reduced conductance parameter $g \sim \nu$. Our results are surprising, first because the observation of Luttinger liquid behavior at all fillings is not fully expected [10] as incompressibility of the bulk fluid was considered crucial to the existence of a Luttinger liquid, and second because the power law exponent exhibits no plateau features, in direct contrast to theoretical analyses based on the intermixing of copropagating versus counterpropagating edge modes [5,7,12].

A one-dimensional (1D) interacting electron system is known to be a non-Fermi liquid [13], more recently termed a Luttinger liquid (LL) [3,4], with plasmonlike gapless elementary excitations instead of the quasiparticle excitations familiar to Fermi liquid theory. It is characterized by a reduced conductance $g \equiv \frac{G_{1D}}{e^2/h}$, with $g < 1$ for repulsive interactions. Remarkably, the same dimensionless parameter $g$ also characterizes the exponent in the tunneling density of states: $\rho(E) \sim E^{(1/g)-1}$. In general, it is difficult to calculate the exact value of $g$ since details of the interaction are rarely known. On the other hand, Wen showed that the edges of the FQHE are a special example of a 1D system with known interactions, a chiral Luttinger liquid ($\chi$LL) [1]. The edge trajectories that skip along the 1D periphery of a 2D sample are chiral in that the specularly bouncing edge orbits drift in only one direction. They also circumnavigate disorder without backscattering, thus making it possible to predict the exact value of the conductance parameter $g$. For the case of a single edge mode $g$ is equal to the reduced FQHE Hall conductance, so for the fundamental Laughlin fractions $\nu = f = \frac{1}{2n+1}$ ($n$ integer) where there is only one edge mode, the edge is expected to behave as a $\chi$LL with $g = f$. Note that at other hierarchical fractions with multiple edge modes the simple relation $g = f$ is no longer expected to hold because each mode now has its own $g_i$ and their mixing modifies the exponent [5,7,12]. Interestingly, copropagating daughter modes of an $f = \frac{1}{(2n+1)}$ mode should manifest the same exponent as the parent even after mixing.

Experimentally, previous works have probed the edge either with point-contact tunneling between two edges [8,11] or with edge tunneling to an entire FQHE edge from a metallic electrode [9,10]. In the former geometry, two counterflowing edge channels tunnel where they intersect at a gate-defined, pointlike tunnel junction. Only conductance versus temperature [8,11] and the temperature dependence of gate tuned resonances [8] can be measured since $I$-$V$ measurements require biases which distort the point-contact potential. The maximum temperature is limited by the small window in energy with roughly constant tunneling transmission coefficient. Consequently, the outcome is suggestive but controversial since in the limited temperature range many competing functional forms can be accommodated, like power law, $e^{-(T_o/T)^{1/2}}$ variable-range hopping, etc.

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The edge tunneling geometry circumvents these shortcomings by creating a sharp 100 mV barrier on the cleaved edge of a 2D quantum well that tolerates much larger biases ($\sim$10 mV) without appreciably changing the tunneling probability. Voltages 3 orders of magnitude larger than the base temperature $\frac{k_B T_{\mathrm{base}}}{q} \sim 2.3\ \mu$V can be applied to access the strongly coupled regime where the Hall conductance limits the conductance rather than the tunnel barrier. The expediency of $I$-$V$ measurements makes it feasible to survey a range of filling fractions.

The edge tunneling experiments by Chang *et al.* observed the power law in $I$-$V$ and $G$-$T$ at select fillings $1/3$, $2/3$, 1 [9], and $1/2$ [10], always measuring an exponent smaller than the expected value [5,7]. To better understand this reduced value, whether the power law behavior exists at all fillings, and the functional dependence of the exponent, we proceed to systematically examine edge tunneling as a function of $\nu$ as described below.

The devices in this experiment have a $90-225$ Å thick $Al_{0.1}Ga_{0.9}As$ tunneling barrier grown by cleaved-edge overgrowth [14] on the edge of a 250 Å wide GaAs quantum well, capped by bulk doped $n^+$ GaAs enabling tunneling spectroscopy of the density of states at the edge of the 2D electron system as above (Fig. 1, inset). Typically, the edge-tunnel barrier is about 1 mm in length. The growth parameters of the four samples are listed in Table I. This type of device structure has previously been characterized for zero magnetic field [15,16] and at select filling factors in the FQHE regime [9,10].

The Hall measurement for sample M in Fig. 1 shows a number of fractional Hall plateaus including $1/3$, $2/5$, and $2/3$. The two-point tunneling conductance reveals two different regimes. Below 7.5 T the conductance scales roughly as $1/B$, whereas above 7.5 T the conductance drops *exponentially* with increasing $B$. We understand these regimes by modeling the device as a 2D Hall system in series with the tunneling resistance (Fig. 1, inset). At low $B$ when the tunneling resistance is negligible, the measurement yields a two-point magnetoconductance measurement of the 2D system, $G_{2\mathrm{point}} \sim 1/R_{xy} = ne/B$, and shows quantized plateaus at the appropriate filling factors. At high $B$, on the other hand, the conductance is limited by the tunneling barrier. The tunneling probability, $P_{\mathrm{tunn}}$, drops exponentially with $B$ due to its dependence on the magnetic length, $l_B = (\hbar/eB)^{1/2}$: $P_{\mathrm{tunn}} \sim e^{-(x_0/l_B)^2} \sim e^{-B/B_0}$. The field at which the crossover occurs is a function of the barrier height and thickness (see Table I). For a $b =$ 90 Å $Al_{0.1}Ga_{0.9}As$ barrier, we measure crossover at $B = 7.5-9.0$ T or $l_B = 86-94$ Å for three different samples, roughly corresponding to $l_B \sim b$. This relation also holds to within 10% for barriers up to 125 Å thick [17]. The reproducible resonance structure at $B = 12.5$ T will be explored in detail in an upcoming paper [18].

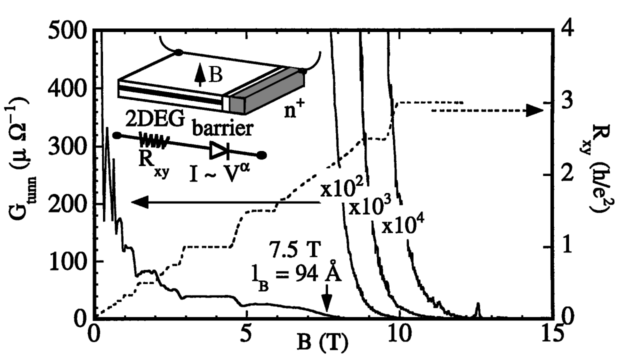


FIG. 1. $R_{xy}$ and $G_{\mathrm{tunn}}$ vs $B$ for sample M (see Table I), $T = 30$ mK. Inset shows microscopic device structure and simple circuit equivalent.

Since in previous work we have already demonstrated a consistency between the power laws observed in $I$-$V$ and in $G$-$T$, for the incompressible fluids at both $1/3$ and $2/3$ [9] as well as the compressible fluid at $1/2$ [10], here we focus on the $I$-$V$ relationship. To measure meaningful tunneling $I$-$V$ we concentrate on the high magnetic field regime above 7.5 T where the tunneling resistance dominates. After verifying the $I$-$V$ to be symmetric with respect to dc bias up to $\pm 50$ mV, subsequent measurement was performed using a four point AC lock-in (2.3 Hz) method wherein a symmetric square wave voltage was applied and the resulting square wave current was measured. The lock-in method increased the signal-to-noise ratio by 2 orders of magnitude yielding sensitivities of $10^{-15}$ A at $10^{-7}$ V bias.

Focusing on the continuous log-log $I$-$V$ trace in Fig. 2(a) for sample M at $B = 11.0$ T ($\nu = 1/3$) we observe three distinct regions. The power law region dominates the middle of the curve for over three decades in current and one in voltage, and at low excitations ($V < \frac{2\pi k_B T}{q}$) softens to the thermally limited linear behavior previously observed [9]. The power law part of the $I$-$V$ characteristic results from an integration of the power law tunneling density of states, predicted by Luttinger liquid theory at the edge of the FQHE.

$$I \sim \int \rho(eV)\, dV \sim V^{1/g} \sim V^{\alpha}, \tag{1}$$

where we introduce $\alpha$ as the experimentally measured power law exponent.

Extending into the *high* bias limit the power law again softens to a linear behavior, saturating at the maximum conductance of $\sim 1/R_{xy}$. Note that $R_{xx} \ll R_{xy}$ throughout. At the higher magnetic field ($B > 10.0$ T), the well developed power law enables us to deduce

TABLE I. Sample parameters.

| Sample | $n_e$ ($10^{11}$ cm$^{-2}$) | $\mu$ ($10^6$ cm$^{-2}$/V s) | QW[a] (Å) | Barrier (Å) | Crossover $l_B$(Å) |
|---|---|---|---|---|---|
| J | 1.16 | 2.9 | 250 | 225 | [b] |
| M | 0.89 | 1.8 | 250 | 90 | 94 |
| Q | 1.09 | 0.5 | 250 | 90 | 90 |
| R | 1.08 | 2.9 | 250 | 90 | 86 |

[a]Quantum well.
[b]See Ref. [17].

the exponent $\alpha$ directly from the slope of the log-log plot. When the power law region becomes less than two decades in current ($B < 10.0$ T), the crossover behavior obscures the underlying power law, and we make use of a model which provides an excellent empirical fit to reveal the underlying exponent. Here we utilize the theory of Chamon and Fradkin (ChF) [19] for a single mode $\chi$LL with $g = \nu$ which models the wide tunnel junction as a sequence of incoherent, pointlike tunnel junctions, while treating the 3D metal as a chiral Fermi liquid. Although at present justification for a single mode $\chi$LL at arbitrary $\nu$ is unavailable, this model is successful in fitting our data. The adjustable parameters are $\nu$: the bulk 2D filling factor which sets the saturation conductance at the high excitation limit; $T_S$: the saturation temperature [20], which defines the upper crossover; $\alpha$: the power law exponent; and $T$: the physical temperature of the sample, which defines the lower crossover to linear behavior. Since $T$ is determined by experimental conditions, and $\nu$ from the Hall measurement, the only adjustable parameters are $\alpha$ and $T_S$ ($\delta = \alpha - 1, \tau = \frac{2\pi T}{T_S}$),

$$I = \int \nu \frac{e^2}{h} \left( 1 - \frac{e^{-(1/2)\tau^\delta}}{[\frac{(V/\tau T_S)^\delta}{\Gamma^2[(\alpha+1)/2]} (1 - e^{-(\delta/2)\tau^\delta}) + 1]^{\alpha/\delta}} \right) dV . \tag{2}$$

Equation (2) is expected to be appropriate for a single mode Luttinger liquid with reduced conductance $g = 1/\alpha$. At $B = 11.0$ T ($\nu = 1/3$), it fits the data with remarkable precision [Fig. 2(a), dotted line]. For comparison we also plot the series resistance model used to guide our intuition [Fig. 2(a), dashed line] and see that the knee of the crossover region at high bias is far too soft.

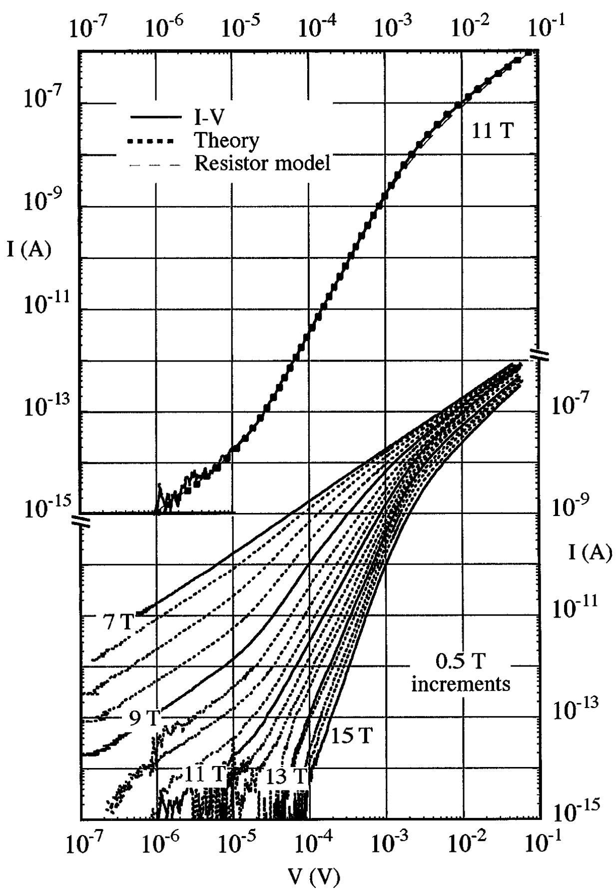


FIG. 2. (top) Log-log $I$-$V$ for sample M at 11.0 T, $\nu = 1/3$. Theory of Chamon and Fradkin (dotted line) and simple series resistance model (dashed line) are overlaid for comparison. (bottom) Log-log $I$-$V$ for sample M at different values of $B$ from 7.0 to 15.0 T in 0.5 T steps.

Next we examine the series of log-log $I$-$V$ curves over the whole range of $B$ field for sample M in Fig. 2(b). At the higher $B$ fields (13.0 T) we observe a power law up to 6 decades in current, whereas at the lowest field (7.0 T) the power law region nearly disappears, yielding an approximately linear conductance $G = 1/R_{xy}$ over the entire voltage range. At lower $B < 10.0$ T (corresponding to high $\nu > 2/5$, the fit of the ChF theory to each trace is still very good, and we are able to extract $\alpha$ and $T_S$. Nonetheless, the fit is not as exact as indicated by the larger error bars in Fig. 3.

Performing similar $I$-$V$ measurements on the three samples R, Q, and J (Table I), we summarize the full result of the exponent $\alpha$ versus $1/\nu$ in Fig. 3. Samples R and Q yielded sufficient decades of power law to fit to the ChF theory, whereas sample J exhibited a strong power law only at the highest magnetic fields, settling to the weak power law of about 1.1 over the $1/\nu$ range of 1 to 1.4. Error bars for representative data points are provided at various fillings. For samples M and R above $1/\nu > 2.8$ and sample Q above $1/\nu > 2.4$, the error is negligible.

At $1/\nu = 3$, the cumulative value for $\alpha$ is $2.85 \pm 0.15$ which lies below the predicted value of $\alpha = 3$. A classical electrostatic calculation of the electron gas density profile at zero $B$ based on an electron density of $10^{18}$ cm$^{-3}$ in the $n^+$ GaAs metal ($E_F = 54$ meV) predicts an edge density enhancement of about 20% above the bulk within 0.2 $\mu$m of the edge due to the electrochemical equilibration. Adjusting for this density enhancement would shift the whole curve to the left, bringing the exponent closer to the predicted value. Even though samples R and Q exhibit a hint of a plateau near $\nu = 1/3$, there is no evidence of any plateau for sample M. This contrasts with the saturation conductance at high excitations which does exhibit the expected $R_{xy} = 3h/e^2$ plateau 1.5 T wide in $B$ (0.3 on the $1/\nu$ axis). These contrasting behaviors suggest that the power law associated with the edge does not directly reflect the bulk transport properties.

Based on our results we make the following observations. First, the plot shows a remarkable continuum of power law exponent values spanning the entire range

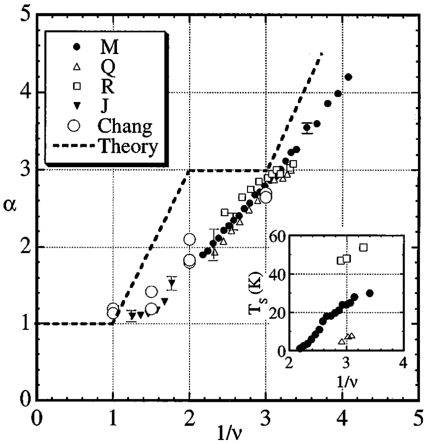


FIG. 3. Power law exponent $\alpha$ vs $1/\nu$, the reciprocal of the filling factor, for four samples. The work of Chang *et al.* [9,10] is included for reference. (Inset) $T_S$ vs $1/\nu$ for three samples whose traces spanned high excitations.

$1 < \alpha < 4$. This is the first experimental evidence that the characteristic LL coupling constant $g$ may, in fact, assume a whole continuum of values. Second, the trend in $\alpha$ versus $1/\nu$ is linear for $1/\nu > 1.4$ with $\alpha \simeq 1.16/\nu - 0.58$. This linear behavior appears to characterize all four samples studied regardless of electron mobility, carrier density, and tunneling barrier thickness. It is in striking contrast to theoretical expectations that $\alpha$ would reflect the bulk transport and therefore exhibit plateaus whenever the Hall conductance is quantized. Finally, for $1/\nu < 1.4$, the exponent saturates at a lower limit, $\alpha = 1.1$, indicating an approach to Fermi liquid behavior. The shift in the knee for Fermi-liquid behavior is too large to be accounted for by the edge density enhancement described above.

Two current theories predict values for the tunneling exponent at a sharp edge as a function of filling factor, namely, the hierarchical construction of the Luttinger liquid edge [5,7], and the tunneling of composite fermions (CF) into a compressible edge [12,21]. The first predicts $\alpha$ only at the discrete hierarchical filling fractions, whereas the second spans the whole range of $1/\nu$, and both theories are consistent when they overlap. We plot the latter CF theory with a dashed line in Fig. 3.

$\chi$LL theory predicts that at filling fractions $\nu = \frac{1}{p+1}$ for $p$ even, the $I$-$V$ tunneling exponent is $\alpha = 1/\nu = p + 1$ [1]. The hierarchical daughters of this fraction are of the form $\nu = \frac{n}{np+1}$ for positive integer $n$, and are predicted to show the same exponent $\alpha = p + 1$ as the parent due to the fact that all edge channels are copropagating, giving rise to the plateau regions in the dashed curve on Fig. 3. For fractions with $n$ negative, *counterpropagating* modes can reduce the exponent upon scattering with each other. Kane *et al.* [7] find a universal value for the exponent $\alpha = p + 1 - 2/|n|$ reached in the limit of large scattering, leading to the linearly sloped portion in the dashed curve of Fig. 3 [12]. The plateau structure is clearly not observed in our experiment.

In conclusion, we have performed the first systematic investigation of the power law behavior observed when tunneling into the edge of the FQHE over a sizable span of filling factors from $\nu = 1/4$ to 1. Utilizing an empirical fit of the $I$-$V$ characteristic to a multiple point-tunneling model [19], we observe a linear dependence of the power law exponent $\alpha$ on $1/\nu$ in contrast to existing theories.

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